7.84965	1.2535	-4.68051	0.966109	-3.22775	-1.40967	1.04987	2.20804	2.05859	-2.74987	-1.64543	3.35408	4.29874	-5.02125	6.52605	2.01761	3.59206	0.64543	-2.00992	4.93398
1.2535	1.65206	8.33673	-4.71971	3.60494	-4.71429	6.40789	-4.49126	2.55534	6.63397	-0.945923	3.215	5.98604	-1.60141	-0.129235	2.42979	7.49154	4.0145	-1.19705	-1.52836
-4.68051	8.33673	-3.01479	-2.46841	-3.29238	-6.61109	0.74621	-3.43432	-1.03865	-0.30422	2.40855	3.57606	-1.11509	3.20497	7.20177	8.1657	-0.116565	-3.14732	0.511673	3.94111
0.966109	-4.71971	-2.46841	-7.8243	5.14281	-7.19343	-1.20332	1.84345	4.16636	2.39665	0.0208252	-3.57817	-4.14458	-0.47492	0.914608	-6.33641	-3.01087	-1.87663	-0.174213	-2.4841
-3.22775	3.60494	-3.29238	5.14281	-4.03947	2.43619	0.455571	-2.78473	2.88738	-2.08562	7.88945	-4.85517	-5.51583	4.88178	-6.15698	3.06304	-4.25845	-1.87652	-7.20465	-7.66405
-1.40967	-4.71429	-6.61109	-7.19343	2.43619	2.12356	-2.86321	-0.540309	-4.98338	-2.60091	5.26163	-1.38449	-0.263283	-2.83463	-3.98221	-2.21576	6.16042	4.27227	-8.09404	3.51651
1.04987	6.40789	0.74621	-1.20332	0.455571	-2.86321	9.5065	-3.2708	2.67314	-1.98862	-2.10908	-2.58889	-1.19255	6.32787	2.66516	3.28843	4.48772	-3.68105	-0.878843	-4.6014
2.20804	-4.49126	-3.43432	1.84345	-2.78473	-0.540309	-3.2708	4.58452	0.494782	4.74231	-5.31854	-5.67096	-0.676868	-0.998946	7.88187	3.24483	8.09262	1.62552	-2.49092	0.803143
2.05859	2.55534	-1.03865	4.16636	2.88738	-4.98338	2.67314	0.494782	0.75548	-6.04531	0.171377	-0.348583	0.707132	-1.81468	0.103862	-0.244199	-5.99486	6.5466	-4.51996	2.17515
-2.74987	6.63397	-0.30422	2.39665	-2.08562	-2.60091	-1.98862	4.74231	-6.04531	8.67484	0.672978	4.46651	-2.23802	3.16206	4.47344	-5.73083	0.68381	0.833792	3.15598	-2.09861
-1.64543	-0.945923	2.40855	0.0208252	7.88945	5.26163	-2.10908	-5.31854	0.171377	0.672978	3.75263	8.33161	3.57465	1.43892	1.29786	-7.85634	5.08369	5.64605	-0.514373	-0.00994157
3.35408	3.215	3.57606	-3.57817	-4.85517	-1.38449	-2.58889	-5.67096	-0.348583	4.46651	8.33161	0.959226	-1.41935	5.91457	0.00283354	0.251408	0.857021	7.1394	-2.96233	3.37396
4.29874	5.98604	-1.11509	-4.14458	-5.51583	-0.263283	-1.19255	-0.676868	0.707132	-2.23802	3.57465	-1.41935	-2.47487	-8.46501	-2.72423	4.34547	0.031314	-0.903284	-4.46197	5.46396
-5.02125	-1.60141	3.20497	-0.47492	4.88178	-2.83463	6.32787	-0.998946	-1.81468	3.16206	1.43892	5.91457	-8.46501	7.71372	-2.68265	2.2555	6.73738	-0.807175	3.81351	0.675944
6.52605	-0.129235	7.20177	0.914608	-6.15698	-3.98221	2.66516	7.88187	0.103862	4.47344	1.29786	0.00283354	-2.72423	-2.68265	7.29527	1.967	-5.45142	0.385776	-0.539797	1.64131
2.01761	2.42979	8.1657	-6.33641	3.06304	-2.21576	3.28843	3.24483	-0.244199	-5.73083	-7.85634	0.251408	4.34547	2.2555	1.967	2.29342	-3.08668	0.949119	-1.41291	4.10748
3.59206	7.49154	-0.116565	-3.01087	-4.25845	6.16042	4.48772	8.09262	-5.99486	0.68381	5.08369	0.857021	0.031314	6.73738	-5.45142	-3.08668	-1.8623	-4.18897	0.899056	0.191952
0.64543	4.0145	-3.14732	-1.87663	-1.87652	4.27227	-3.68105	1.62552	6.5466	0.833792	5.64605	7.1394	-0.903284	-0.807175	0.385776	0.949119	-4.18897	8.12945	3.17942	-5.45775
-2.00992	-1.19705	0.511673	-0.174213	-7.20465	-8.09404	-0.878843	-2.49092	-4.51996	3.15598	-0.514373	-2.96233	-4.46197	3.81351	-0.539797	-1.41291	0.899056	3.17942	8.12623	2.05487
4.93398	-1.52836	3.94111	-2.4841	-7.66405	3.51651	-4.6014	0.803143	2.17515	-2.09861	-0.00994157	3.37396	5.46396	0.675944	1.64131	4.10748	0.191952	-5.45775	2.05487	2.86362

Una passeggiata tra le matrici random e dintorni

Luca Guido Molinari, 30 maggio 2023

7.84965	1.2535	-4.68051	0.966109	-3.22775	-1.40967	1.04987	2.20804	2.05859	-2.74987	-1.64543	3.35408	4.29874	-5.02125	6.52605	2.01761	3.59206	0.64543	-2.00992	4.93398
1.2535	1.65206	8.33673	-4.71971	3.60494	-4.71429	6.40789	-4.49126	2.55534	6.63397	-0.945923	3.215	5.98604	-1.60141	-0.129235	2.42979	7.49154	4.0145	-1.19705	-1.52836
-4.68051	8.33673	-3.01479	-2.46841	-3.29238	-6.61109	0.74621	-3.43432	-1.03865	-0.30422	2.40855	3.57606	-1.11509	3.20497	7.20177	8.1657	-0.116565	-3.14732	0.511673	3.94111
0.966109	-4.71971	-2.46841	-7.8243	5.14281	-7.19343	-1.20332	1.84345	4.16636	2.39665	0.0208252	-3.57817	-4.14458	-0.47492	0.914608	-6.33641	-3.01087	-1.87663	-0.174213	-2.4841
-3.22775	3.60494	-3.29238	5.14281	-4.03947	2.43619	0.455571	-2.78473	2.88738	-2.08562	7.88945	-4.85517	-5.51583	4.88178	-6.15698	3.06304	-4.25845	-1.87652	-7.20465	-7.66405
-1.40967	-4.71429	-6.61109	-7.19343	2.43619	2.12356	-2.86321	-0.540309	-4.98338	-2.60091	5.26163	-1.38449	-0.263283	-2.83463	-3.98221	-2.21576	6.16042	4.27227	-8.09404	3.51651
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2.20804	-4.49126	-3.43432	1.84345	-2.78473	-0.540309	-3.2708	4.58452	0.494782	4.74231	-5.31854	-5.67096	-0.676868	-0.998946	7.88187	3.24483	8.09262	1.62552	-2.49092	0.803143
2.05859	2.55534	-1.03865	4.16636	2.88738	-4.98338	2.67314	0.494782	0.75548	-6.04531	0.171377	-0.348583	0.707132	-1.81468	0.103862	-0.244199	-5.99486	6.5466	-4.51996	2.17515
-2.74987	6.63397	-0.30422	2.39665	-2.08562	-2.60091	-1.98862	4.74231	-6.04531	8.67484	0.672978	4.46651	-2.23802	3.16206	4.47344	-5.73083	0.68381	0.833792	3.15598	-2.09861
-1.64543	-0.945923	2.40855	0.0208252	7.88945	5.26163	-2.10908	-5.31854	0.171377	0.672978	3.75263	8.33161	3.57465	1.43892	1.29786	-7.85634	5.08369	5.64605	-0.514373	-0.00994157
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4.29874	5.98604	-1.11509	-4.14458	-5.51583	-0.263283	-1.19255	-0.676868	0.707132	-2.23802	3.57465	-1.41935	-2.47487	-8.46501	-2.72423	4.34547	0.031314	-0.903284	-4.46197	5.46396
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4.93398	-1.52836	3.94111	-2.4841	-7.66405	3.51651	-4.6014	0.803143	2.17515	-2.09861	-0.00994157	3.37396	5.46396	0.675944	1.64131	4.10748	0.191952	-5.45775	2.05487	2.86362

1 MATRIX MODELS (DOTTORATO)
- SVILUPPO 1/N IN TEORIA DEI CAMPI
- MECCANICA STATISTICA SU SUPERFICI RANDOM

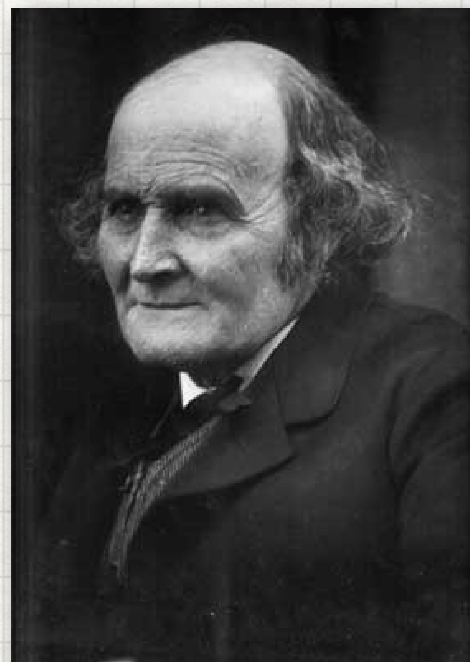
2 QUANTUM CHAOS (POSTDOC)
- IL KICKED ROTATOR
- STATISTICA DEI LIVELLI IN SISTEMI COMPLESSI
- MATRICI RANDOM A BANDA

3 LOCALIZZAZIONE DI ANDERSON
- TRASPORTO IN AMBIENTE DISORDINATO
- UNA DUALITA` SPETTRALE

NANOCONI E MATRICI DI PASCAL

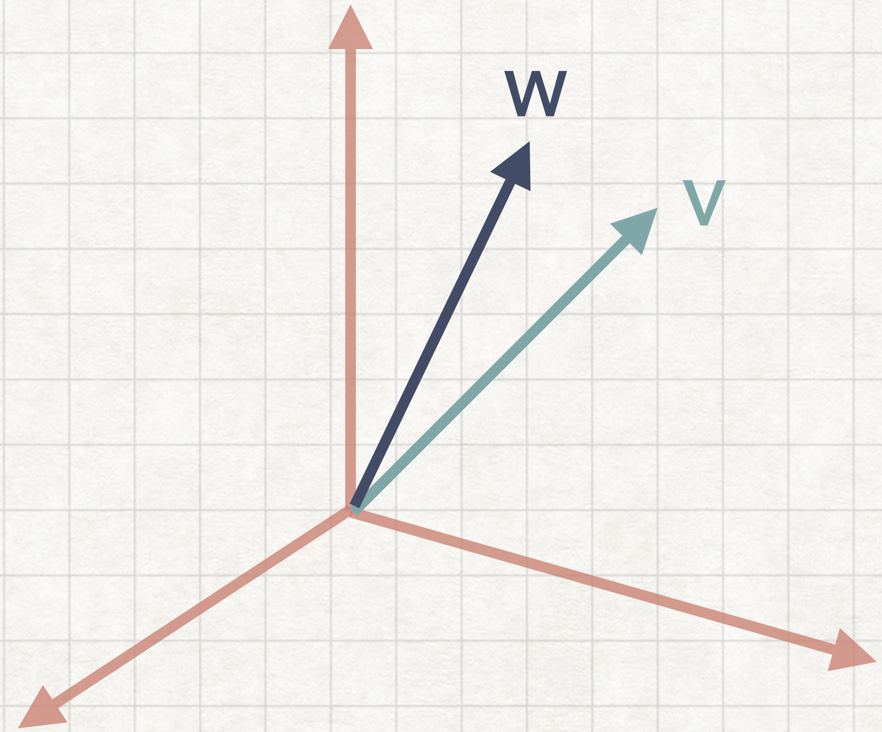
TEORIA MOLTI CORPI, RELATIVITA` GENERALE

MATRICI e mappe lineari sui VETTORI



Arthur Cayley

James Sylvester



$$\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ A_{n1} & \dots & \dots & A_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \dots \\ V_n \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ \dots \\ W_n \end{bmatrix}$$

Av=w, Bw=z, (BA)v=z
Cayley, 1855

Assiomi del campo complesso (1833)
e quaternionico (1843)

Matrici reali, complesse, quaternioniche
a a+ib a+ib+jc+kd

Sir William Rowland Hamilton
1805-1865

I GRUPPI CLASSICI

Adolf Hurwitz

SO(N) = gruppo delle matrici reali NxN che conservano la lunghezza del vettore (rotazioni)

Parametrizzazione con Angoli di Eulero (Hurwitz, 1897)
Matrice di rotazione = punto in uno spazio di angoli
SO(3): 3 angoli (rotazioni dello spazio)
SO(N): N(N-1)/2 angoli
Misura invariante per azione del gruppo

SU(N) = gruppo delle matrici complesse NxN che conservano la norma del vettore (m. unitarie)

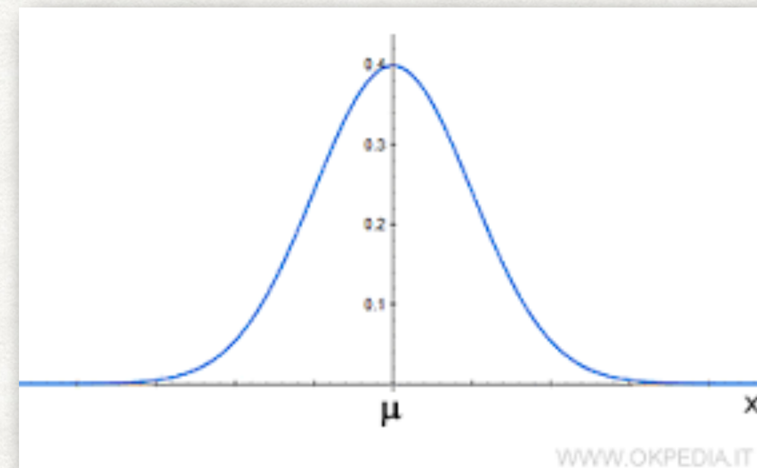
Sp(2N) = gruppo delle matrici di quaternioni 2Nx2N che conservano la norma del vettore (1939)

Hermann Weyl

LE MATRICI RANDOM GAUSSIANE

THE THREEFOLD WAY. Algebraic Structure of Symmetry Groups and Ensembles in Quantum Mechanics (Freeman Dyson, 1962)

GOE: matrici reali simmetriche Gaussiane
invarianti per S $\longrightarrow$ RSR*, R in SO(N)
GUE: matrici complesse Hermitiane Gaussiane
invarianti per H $\longrightarrow$ UHU*, U in SU(N)
GSE: matrici simplettiche Gaussiane
invarianti per K $\longrightarrow$ JKJ*, J in Sp(2N)

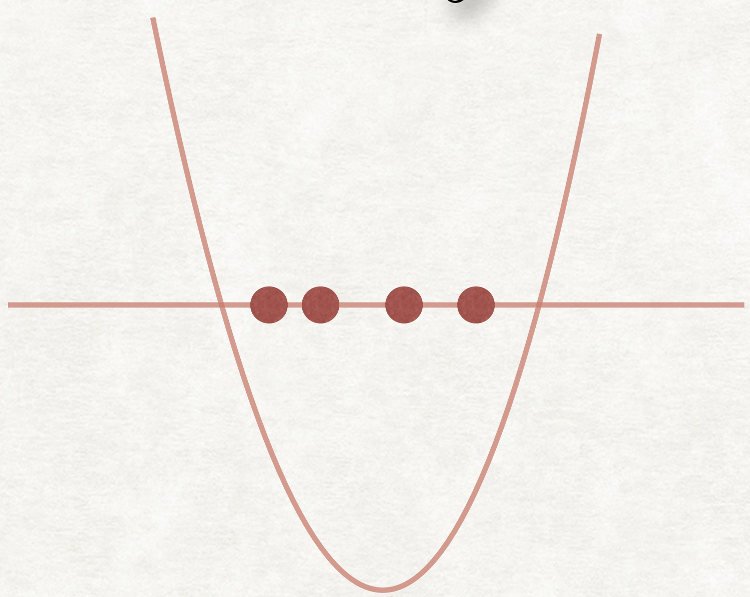


$$H = UXU^\dagger$$

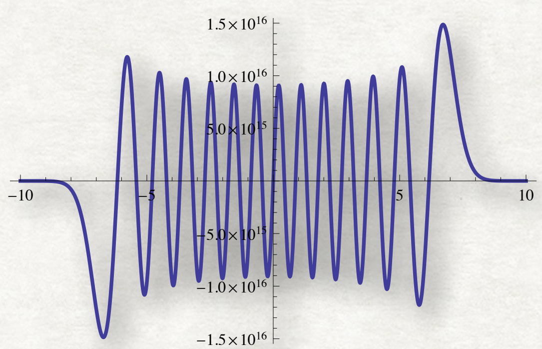
$$\prod_{i\leq j} e^{-C|H_{ij}|^2} dH_{ij} = \prod_{i<j} |x_i - x_j|^\beta e^{-k\sum_n x_n^2} dx_1 \ldots dx_N \, dU_{Haar}$$

GAS DI AUTOVALORI INTERAGENTI
(ELETTROSTATICA 2D SULLA RETTA)
(Stieltjes, Jacobi, … , Riesz)

$$Z = \int d\mathbf{x} e^{-\beta \sum_j V(x_j) - 2\sum_{i<j} \log|x_i - x_j|}$$



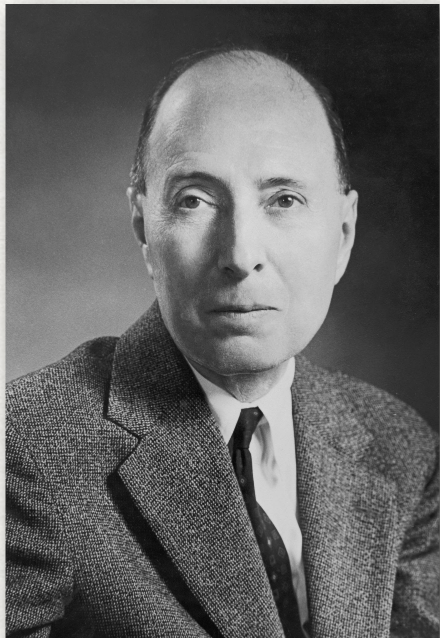
N particelle in potenziale armonico
con interazione log |x-y|



N=25

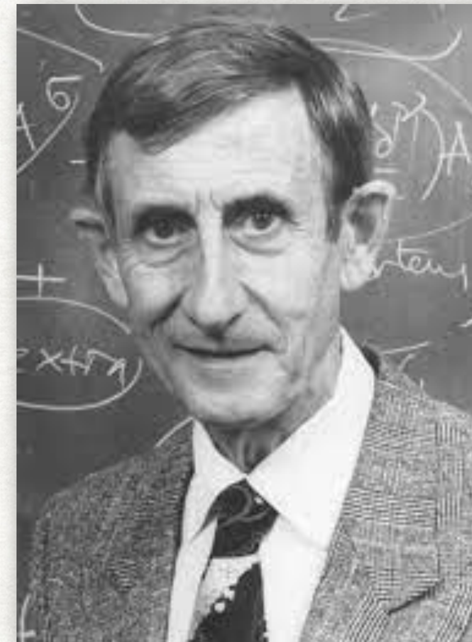
Posizione di equilibrio: gli zeri
del polinomio di Hermite H_N(x)

EUGENE WIGNER

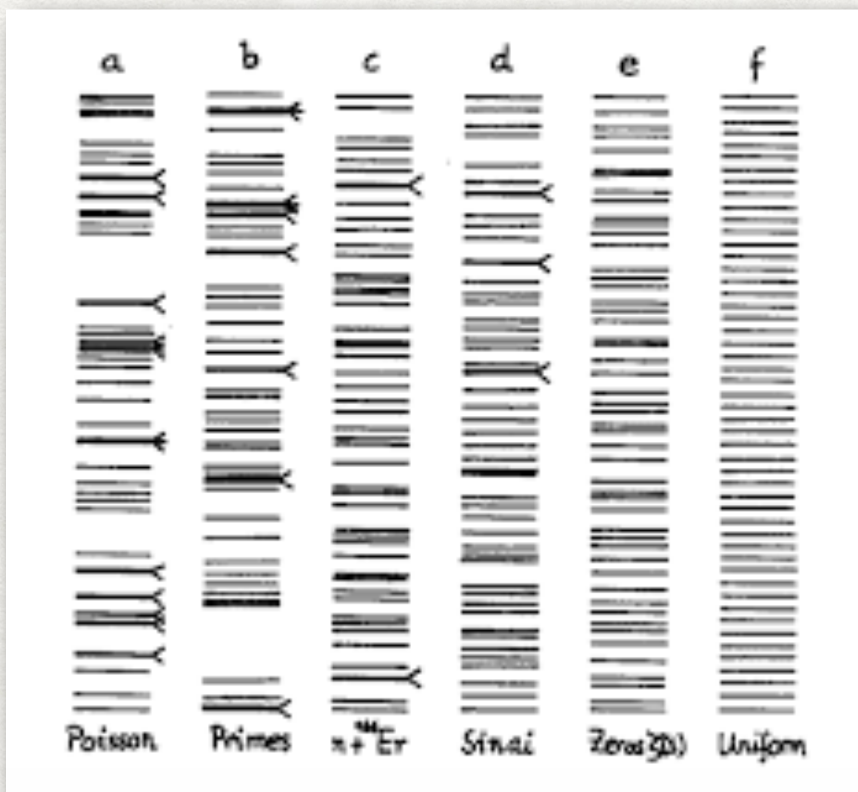


Nobel 1963: “for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles”

FREEMAN DYSON

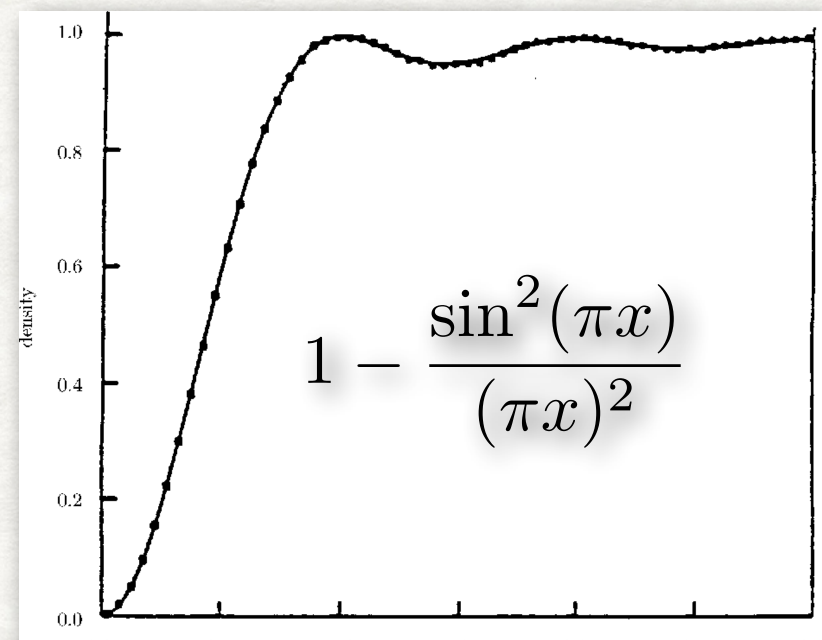


Fisico e matematico eclettico. Dimostro` l’equivalenza della teoria elettrodinamica (QED) di Feynman e di Schwinger-Tomonaga



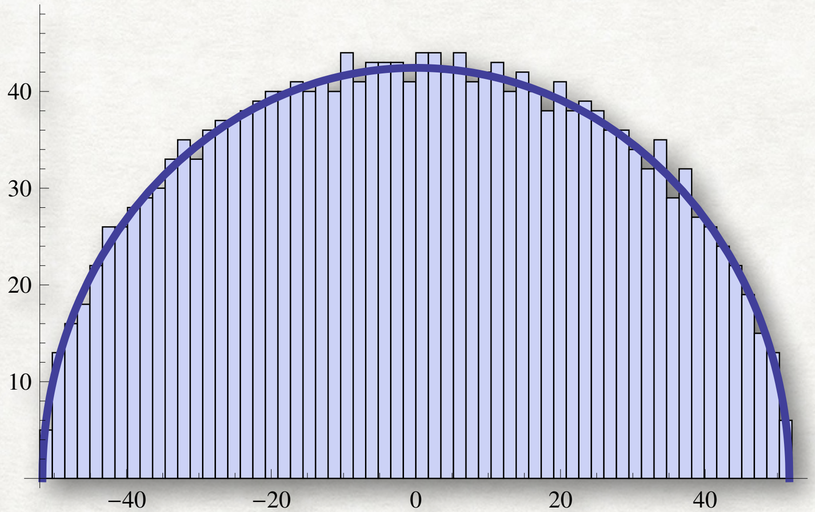
$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

$$\frac{1}{\zeta(s)} = \prod_{P} \left(1 - \frac{1}{P^s}\right)$$

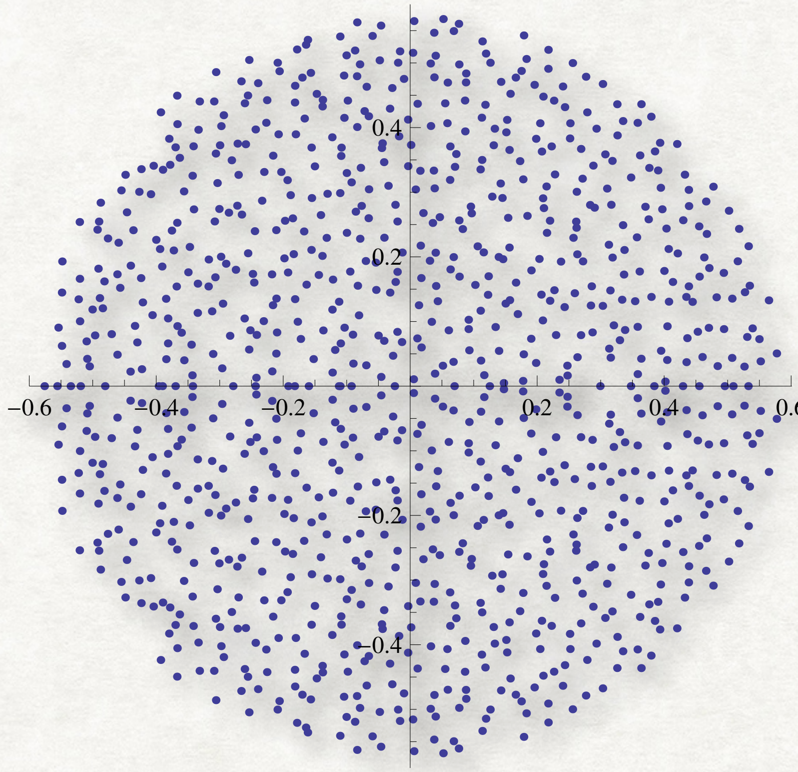


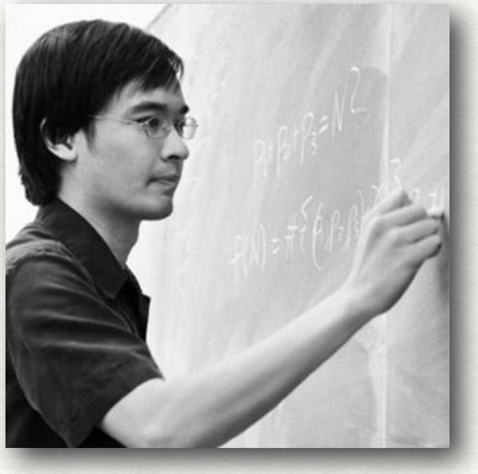
Pair correlation per 8 milioni di zeri di Riemann attorno allo zero # 10^29 e la curva GUE

LE MATRICI RANDOM GAUSSIANE



Wigner semicircle





Terence Tao

Ginibre (nessuna simmetria)

LE PROPRIETA` STATISTICHE DEGLI AUTOVALORI SONO UNIVERSALI. NON DIPENDONO DALLA DISTRIBUZIONE DELLA MATRICE

Esempi:
distribuzione delle spaziature di livelli vicini P(s)
distribuzione dell'autovalore massimo (statistica di Tracy-Widom per eventi rari)

Showing 1–50 of 1,588 results for title: random matrices

random matrices

Title

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1. arXiv:2305.08167 [pdf, other] cs.IT

   Random Generator of Orthogonal Matrices in Finite Fields

   Authors: Lasha Ephremidze, Ilya Spitkovsky

   Submitted 14 May, 2023; **originally announced** May 2023.

   Comments: 10 pages

   MSC Class: 15B10; 20G40; 42C40; 94A60

2. arXiv:2305.07505 [pdf, other] hep-th cond-mat.stat-mech quant-ph

   Sparse random matrices and Gaussian ensembles with varying randomness

   Authors: Takanori Anegawa, Norihiro Iizuka, Arkaprava Mukherjee, Sunil Kumar Sake, Sandip P. Trivedi

   Submitted 12 May, 2023; **originally announced** May 2023.

   Comments: 82 pages, 38 figures

3. arXiv:2305.06390 [pdf, other] hep-th

   BPS Structure Constants and Random Matrices

   Authors: Adolfo Holguin

   Submitted 10 May, 2023; **originally announced** May 2023.

   Comments: 2 figures

4. arXiv:2305.04687 [pdf, ps, other] math.PR

   Universality for Random Matrices

   Authors: Simona Diaconu

   Submitted 8 May, 2023; **originally announced** May 2023.

5. arXiv:2305.02753 [pdf, other] math-ph math.CA math.PR

   Large deviations and fluctuations of real eigenvalues of elliptic random matrices

   Authors: Sung-Soo Byun, Leslie Molag, Nick Simm

   Submitted 4 May, 2023; **originally announced** May 2023.

   Comments: 36 pages

   MSC Class: 60F05; 60F10; 41A60; 60B20; 30E15

6. arXiv:2304.13047 [pdf, ps, other] math.PR math-ph math.OA

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1 2 3 4 5 … Next

1. arXiv:2305.06009 [pdf, ps, other] math.DS

   Continuity of the Lyapunov exponents of random matrix products

   Authors: Artur Avila, Alex Eskin, Marcelo Viana

   Submitted 10 May, 2023; **originally announced** May 2023.

   MSC Class: 37H15 37D25

2. arXiv:2305.01055 [pdf, ps, other] math.OC

   An Augmented Lagrangian Approach for Problems With Random Matrix Composite Structure

   Authors: Dan Greenstein, Nadav Hallak

   Submitted 1 May, 2023; **originally announced** May 2023.

3. arXiv:2304.09319 [pdf, other] math-ph math.NA math.PR

   The conditional DPP approach to random matrix distributions

   Authors: Alan Edelman, Sungwoo Jeong

   Submitted 3 May, 2023; **v1** submitted 18 April, 2023; **originally announced** April 2023.

   Comments: 23 pages, 6 figures

4. arXiv:2304.09055 [pdf, ps, other] math.PR

   A large deviation inequality for the rank of a random matrix

   Authors: M. Rudelson

   Submitted 18 April, 2023; **originally announced** April 2023.

   MSC Class: 60B20

5. arXiv:2304.05714 [pdf, ps, other] math.PR math.GR math.OA

   Norm of matrix-valued polynomials in random unitaries and permutations

   Authors: Charles Bordenave, Benoit Collins

   Submitted 12 April, 2023; **originally announced** April 2023.

   Comments: 70 pages

6. arXiv:2304.04580 [pdf, other] cs.IT eess.SP

   Matrix Factorization Based Blind Bayesian Receiver for Grant-Free Random Access in mmWave MIMO mMTC

   Authors: Zhengdao Yuan, Fei Liu, Qinghua Guo, Xiaojun Yuan, Zhongyong Wang, Yonghui Li

   Submitted 10 April, 2023; **originally announced** April 2023.

7. arXiv:2304.02153 [pdf, ps, other] math-ph math.NT

   Real moments of the logarithmic derivative of characteristic polynomials in random matrix ensembles

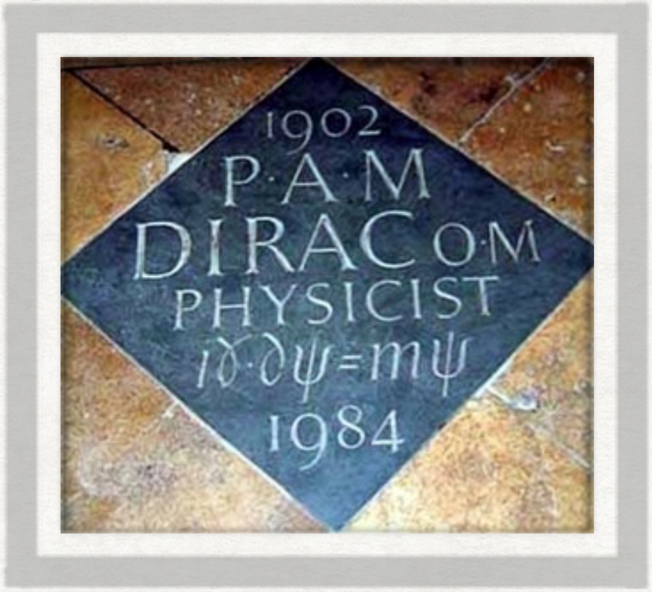
   Authors: Fan Ge

   Submitted 12 May, 2023; **v1** submitted 4 April, 2023; **originally announced** April 2023.

   Comments: rewrote the introduction

LE TEORIE DI GAUGE PER LE INTERAZIONI FONDAMENTALI

In meccanica quantistica la descrizione di una **una particella libera** e` data da:
- **osservabili** X, P, S, m, e, …
- **stato** (un campo d'onda)
- **dinamica** (eqz. di Schrodinger, Dirac, …)



$$\psi'_j(x) = U_{jk}(x)\psi_k(x)$$

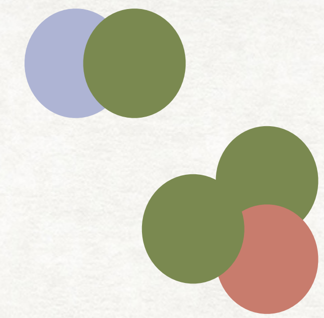
X'=U*XU=X
Pertanto U=U(x)
P'=U*(x)(-id)U(x)= -iU*(x)dU(x) + P

$$(D_\mu\psi)_j = \partial_\mu\psi_j - qA_{\mu,jk}\psi_k$$

U(1) = elettro-magnetismo
SU(2)xU(1) = interazione elettrodebole
SU(3) = interazione forte

1979: S. Glashow, A. Salam, S. Weinberg

A bassa energia i quark non sono mai isolati ma formano doppietti qq* (mesoni), tripletti (p,n, …) etc.



Non vale la teoria perturbativa (partirebbe da quark liberi).

- TEORIA DI RETICOLO

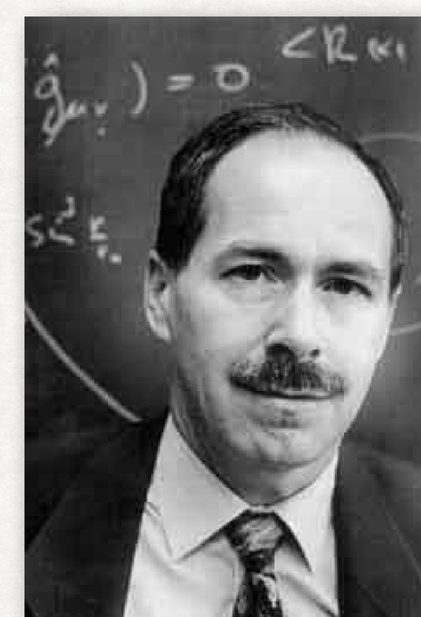
- SVILUPPO 1/N

In MQ un problema con simmetria sferica e\` riducibile a un problema radiale nel sistema rotante (en. cinetica con termine centrifugo).
Soluzione esatta solo in pochi casi (atomo H, oscillatore armonico, …)
Alternativo allo sviluppo perturbativo: SO(3) ⟶ SO(N).
L'equazione radiale e\` risolta in sviluppo 1/N (poi N=3):

$$E = E_0 + \frac{1}{N} E_1 + \frac{1}{N^2} E_2 + \dots$$

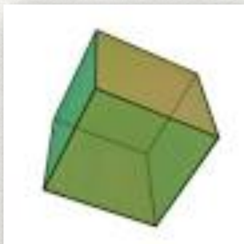
SU(3) (3 quark, 8 gluoni) ⟶ SU(N)

j,k

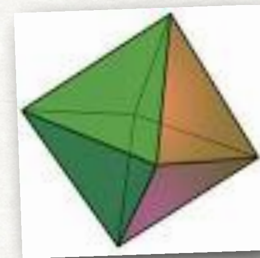


Gerard 'tHooft

EULERO: 2 - 2H = F + V - E

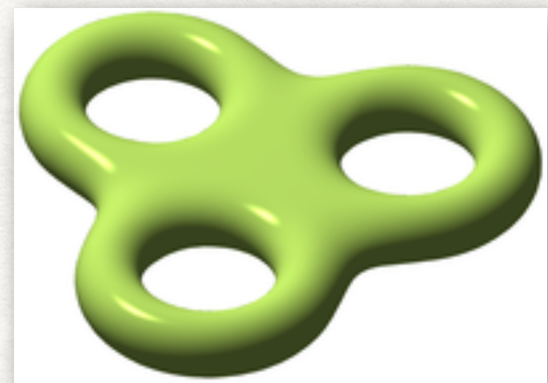


Cubo
F=6, V=8. E=12
h=0



Ottaedro
F=8, V=6. E=12
h=0

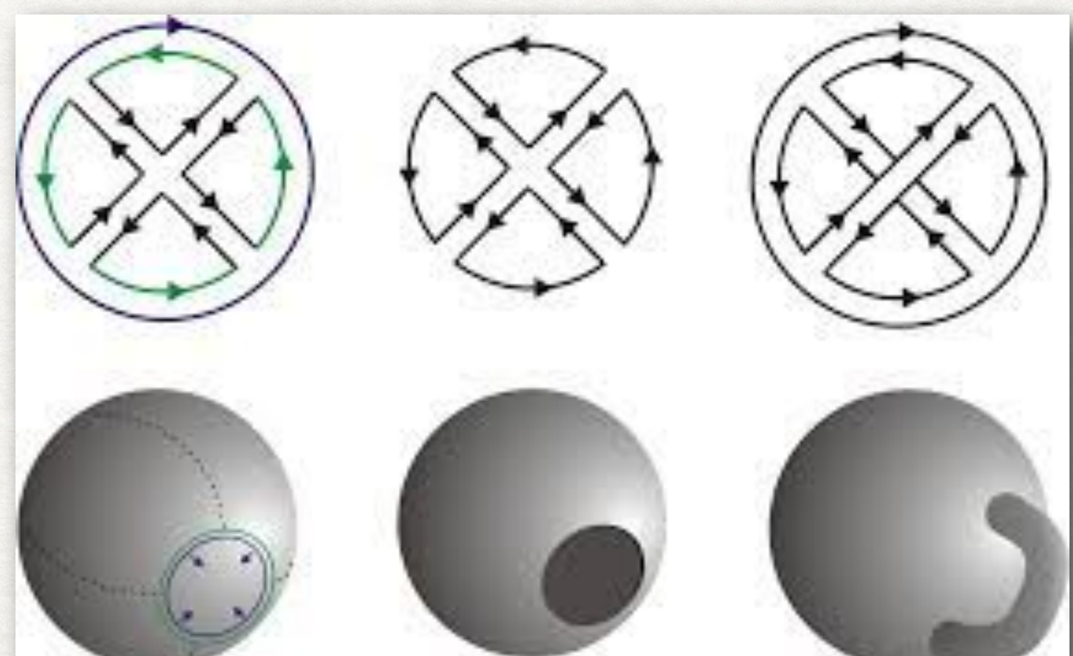
h=1



h=3

LO SVILUPPO 1/N classifica i grafi
di Feynman in base alla loro topologia:

N^2: grafi PLANARI (sulla sfera)
N^0 grafi sul toro
1/N^2 grafi sul 2-toro
....



In dimensione 4 e` un problema difficile (equaz. di Migdal-Makeenko per i gluoni)
In dimensione 0 si riduce a un integrale multiplo ordinario (su MATRICI)
(Brezin, Itzykson, Parisi, Zuber, 1975)

$$Z = \int \prod_{i \leq j} dH_{ij}\, e^{-NTr(M^2 + gM^4)}$$

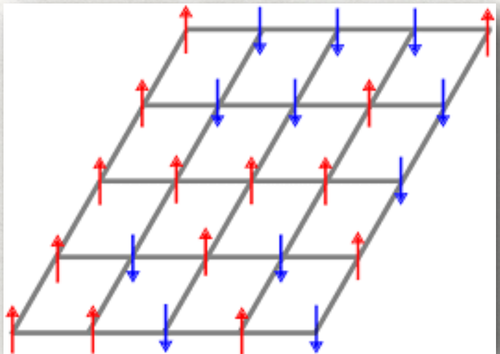
$$dH = dU \prod_k dx_k \prod_{j<k} (x_j - x_k)^2$$

Integrazione con punto di sella (N grande), Polinomi ortogonali (integrazione esatta)

IN DIMENSIONE 0: Z[G,N] CONTA I DIAGRAMMI.

MECCANICA STATISTICA !

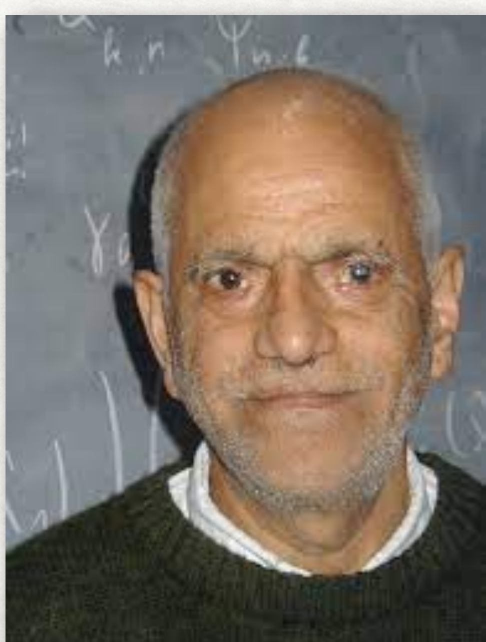
MECCANICA STATISTICA SUI GRAFI PLANARI RANDOM
(gravita` quantistica 2D con materia)



$$E = -J\sum_{\langle i,j\rangle} S_i S_j + H\sum_{j} S_j \qquad S_i = \pm 1$$

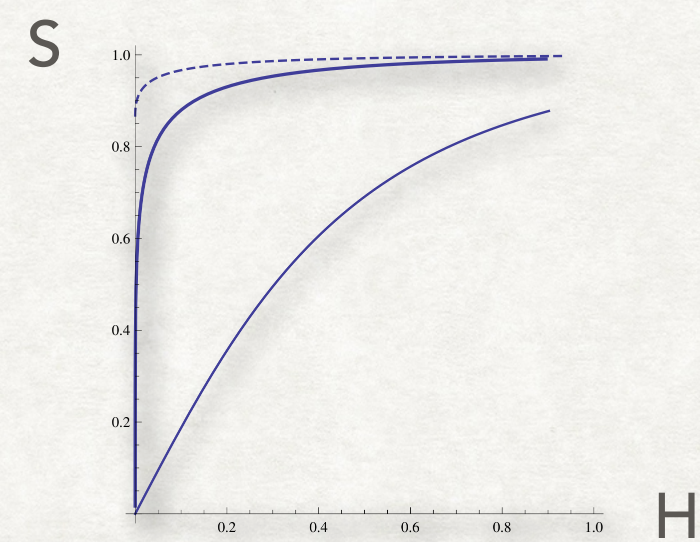
Modello di Ising per il ferromagnetismo.
Soluzione esatta: Onsager (1944) con H=0
Magnetizzazione spontanea: Chen-Ning Yang (1952)

Modello di Ising con campo esterno **su grafi random** =
Modello a 2 matrici: Tr[A^2+B^2 +gA^4 + gB^4 -2c AB]



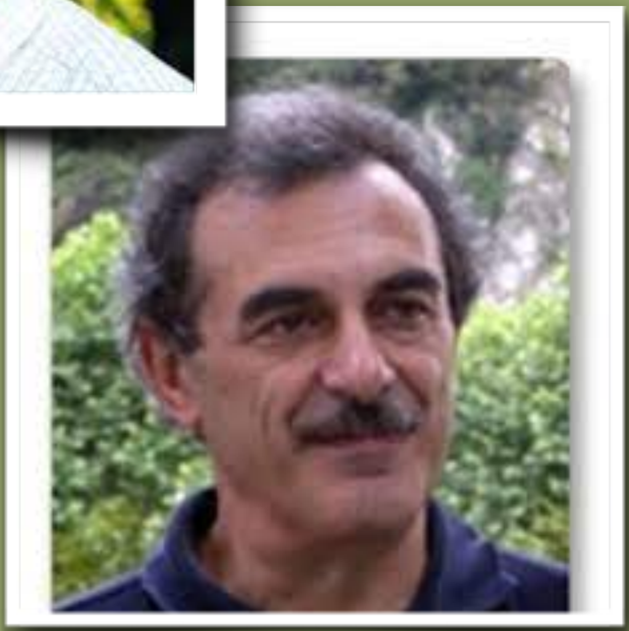
Madan Lal Mehta

Yuri Kazakov

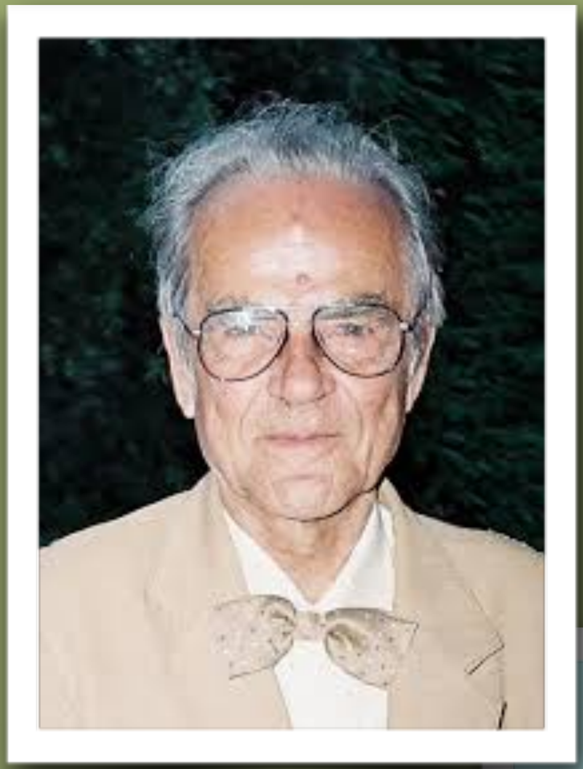


ANNI '80: CAOS QUANTISTICO Milano - Novosibirsk

Giulio Casati

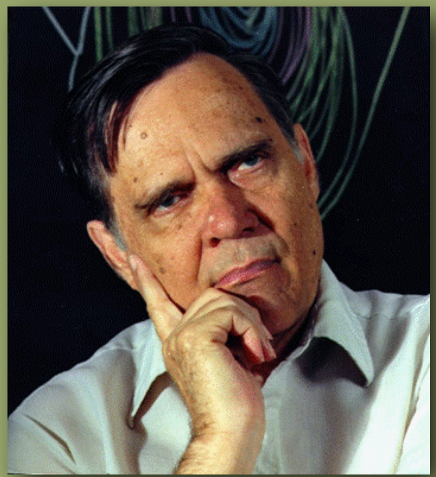


Italo Guarneri

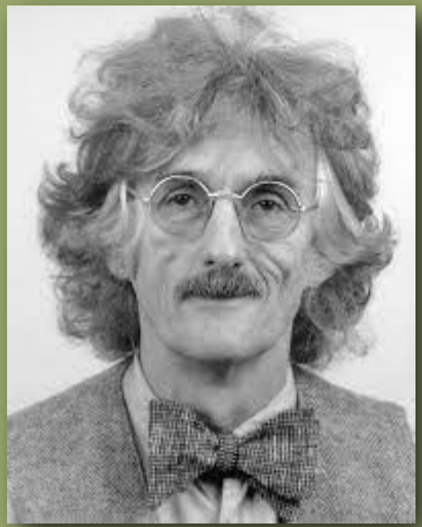


Boris Chirikov

Felix Izrailev



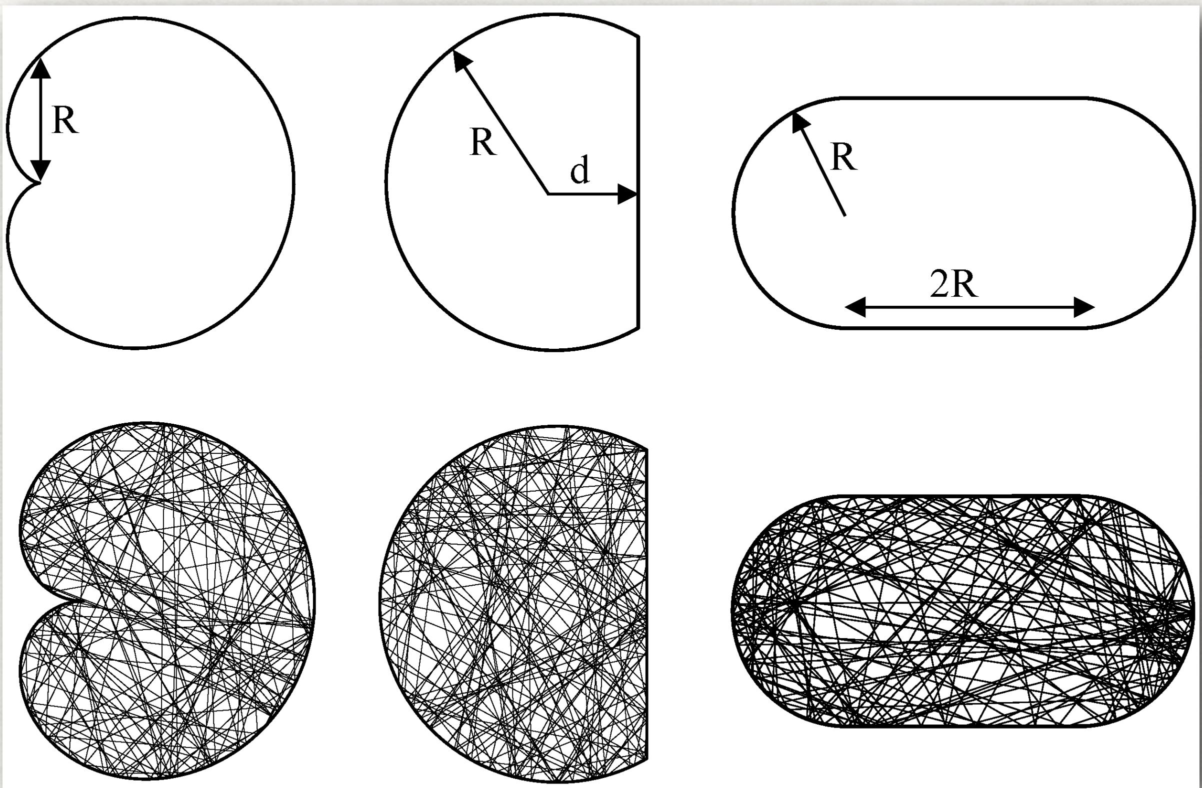
Joe Ford



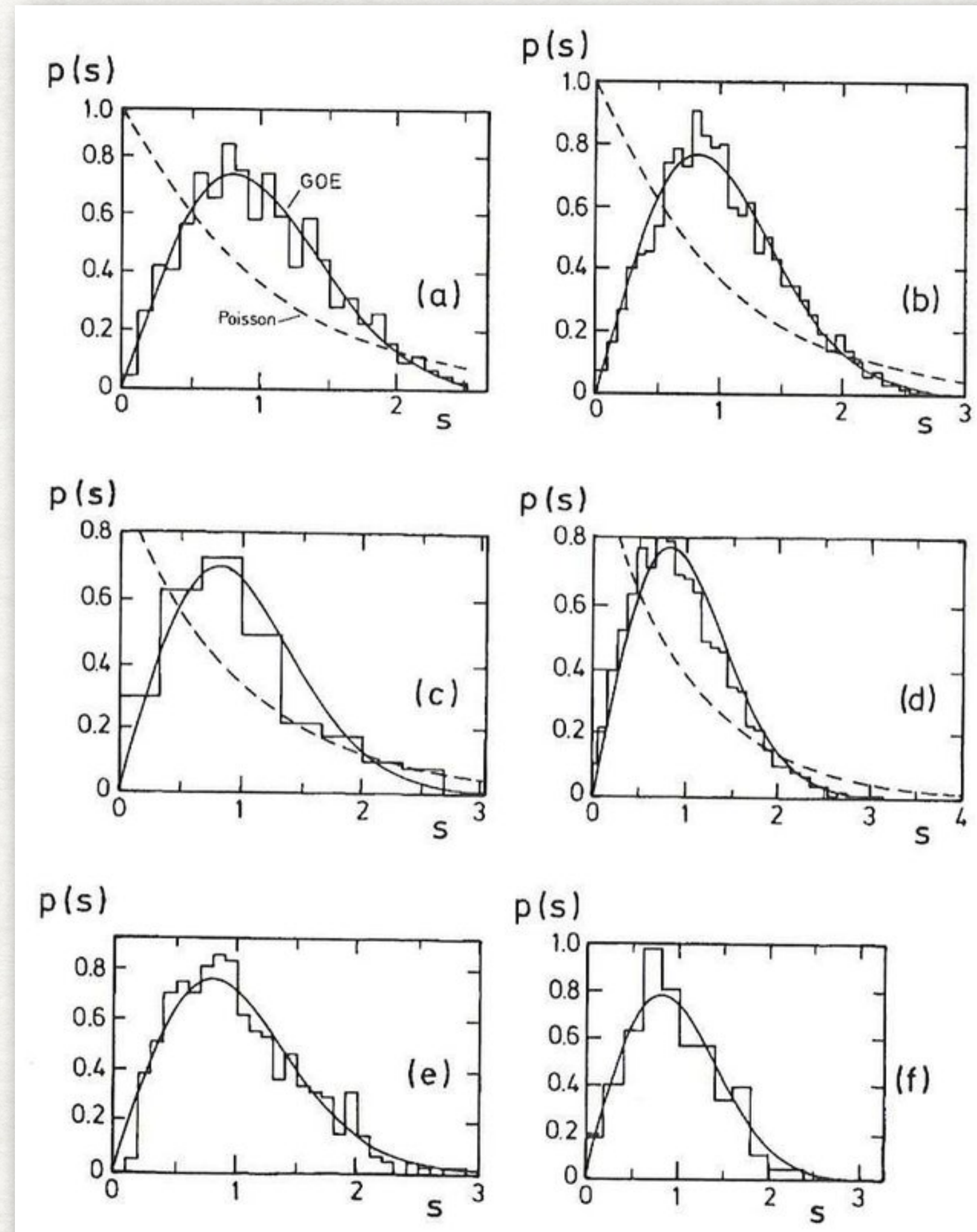
Fritz Haake

CAOS QUANTISTICO

(come si manifesta il caos classico in MQ?)



Biliardi caotici



A) Biliardo di Sinai,
B) Atomo H in campo H forte
C) Spettro NO2
D) …
E) Spettro cavita` 3D microonde
F) Frequenze di una lamina 1/4 stadio di Sinai

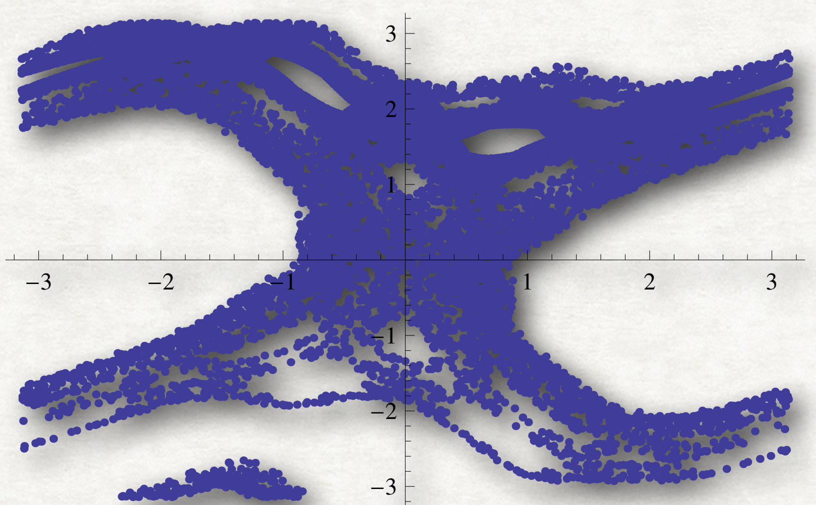
IL KICKED ROTATOR

$$H = \frac{1}{2}P^2 + K\cos\theta \sum_n \delta(t-n)$$

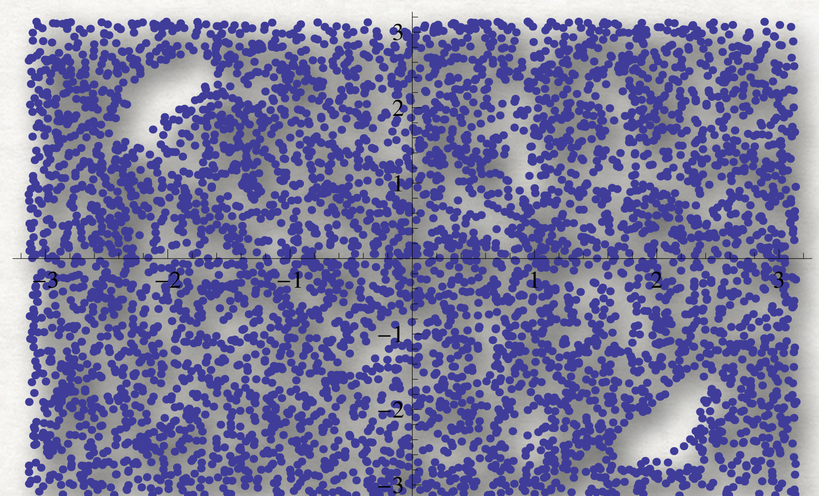
$$P' = P - K\sin\theta'$$
$$\theta' = \theta + P'$$

$$U = e^{-iK\cos\theta} e^{-iP^2/2}$$

Standard Map (Chirikov-Taylor, 1979)



K=1



caos classico K=5

In meccanica quantistica, in regime caotico, l’energia <p^2> cresce come radice t e poi si arresta. Autostati di U esponenzialmente localizzati (localizzazione dinamica) <m|U|m’> e` una matrice ``pseudo-random” a BANDA

PHYSICAL REVIEW LETTERS

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Scaling Properties of Band Random Matrices

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(Received 22 January 1990)

It is shown on the basis of numerical data that the normalized localization length of eigenvectors of band random matrices follows a scaling law. The scaling parameter is b^2/N, where b measures the bandwidth and N is the size of the matrix.

PACS numbers: 05.45.+b, 72.15.Rn

In a previous investigation[1,2] on the statistical properties of quantum "chaotic" systems, it was found that in the particular model of the kicked rotator on the torus, localization effects display a scaling behavior. This scaling is connected with the quantum suppression of classical dynamical chaos[3,4] and has a counterpart in the scaling theory of localization for one-dimensional disordered systems of finite size.[5]

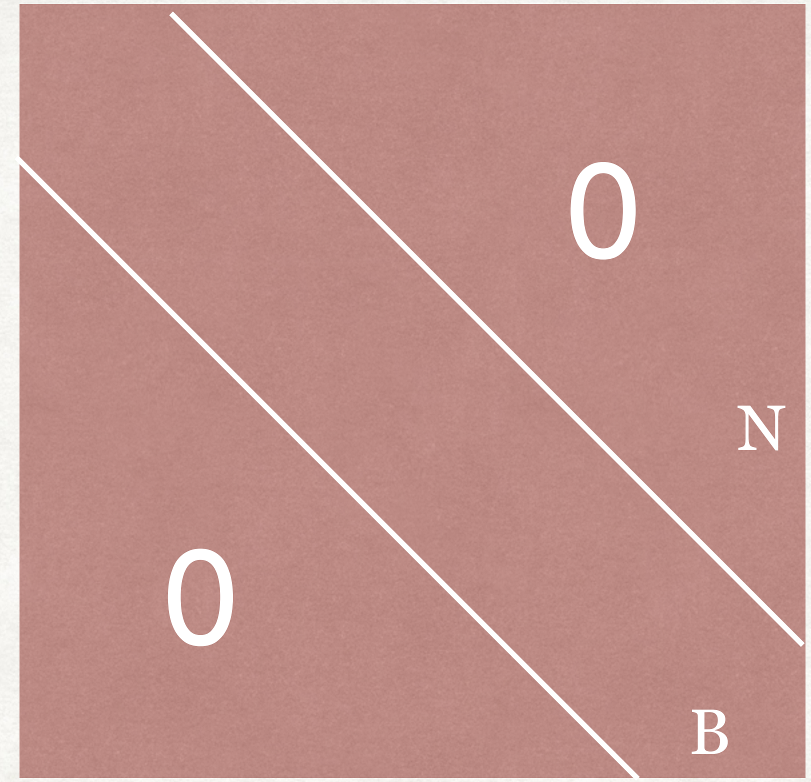
The one-period evolution of the kicked rotator, in the angular momentum representation, is given by a unitary $N \times N$ matrix. The matrix elements are appreciably different from zero only inside a band of size $2k$, where k is the strength of the perturbation. Outside the band they decay exponentially. In the case of classical strong chaos, the matrix elements can be considered as pseudorandom numbers, and when k is large enough, the unitary matrix exhibits the statistical properties of the circular-orthogonal ensemble.[6] The scaling parameter which describes the statistical properties in the regime of full classical chaos is the ratio k^2/N. The quantity k^2 is proportional to ξ_∞, the localization length measured through the rate of exponential decay of the eigenvectors in the limit of infinite size ($N \to \infty$).

The natural question then arises whether scaling behavior is a general property of random matrices with a band structure. This is an interesting mathematical problem, which is also relevant for physics. Indeed, band random matrices may be regarded as models for quantum systems whose states are only partially coupled to each other by the interaction. For example, this feature is common to many models of solid-state physics[7] and may be relevant for several problems in atomic and nuclear physics.[8]

In this paper, we consider ensembles of real symmetric band random matrices (BRM). The statistical properties of such matrices are poorly understood, except in the limiting cases of Gaussian-orthogonal-ensemble (GOE) (Refs. 9–11) and tridiagonal matrices. In the latter case, the eigenvectors display an exponential localization in the large-N limit: $\psi_n \approx \exp(-|n-n_0|/\xi)$ (Ref. 7), where ξ is the inverse of the Lyapounov exponent, computed, for example, by means of Thouless's formula[12] or by the transfer-matrix method.[13] The investigation of the intermediate situation, with particular reference to the localization properties of eigenvectors, is the object of this paper. Unlike the random-matrix theory, for which many analytical treatments are available, the lack of rotational invariance of BRM ensembles makes the use of computer simulation unavoidable at this stage.

Before presenting our results, we would like to mention that a particular class of band matrices ("bordered matrices") has been considered by Wigner.[14] They are characterized by integer diagonal entries $\dots, -2, -1, 0, 1, \dots$ and a band of size b of matrix elements $a_{ij} = \pm h$, where h is constant and the sign is random; outside the band $a_{ij} = 0$. The model is analytically solvable in the tridiagonal case, and exhibits a semicircle distribution of the eigenvalues in the limit b and $h \gg 1$, with

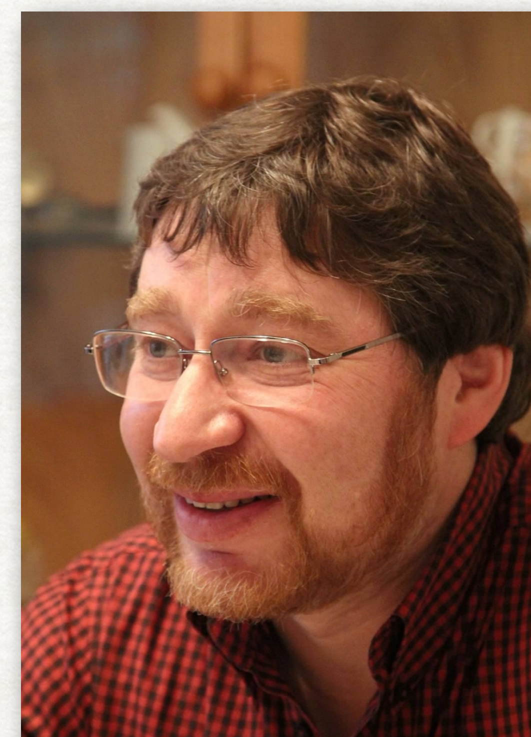
Banded
Random
Matrices



Medie di disordine con variabili di Grassmann

$$\left\langle \frac{\det(M-z)}{\det(M-w)} \right\rangle \quad \left\langle \frac{\det(M-z)\det(M-z')}{\det(M-w)\det(M-w')} \right\rangle$$

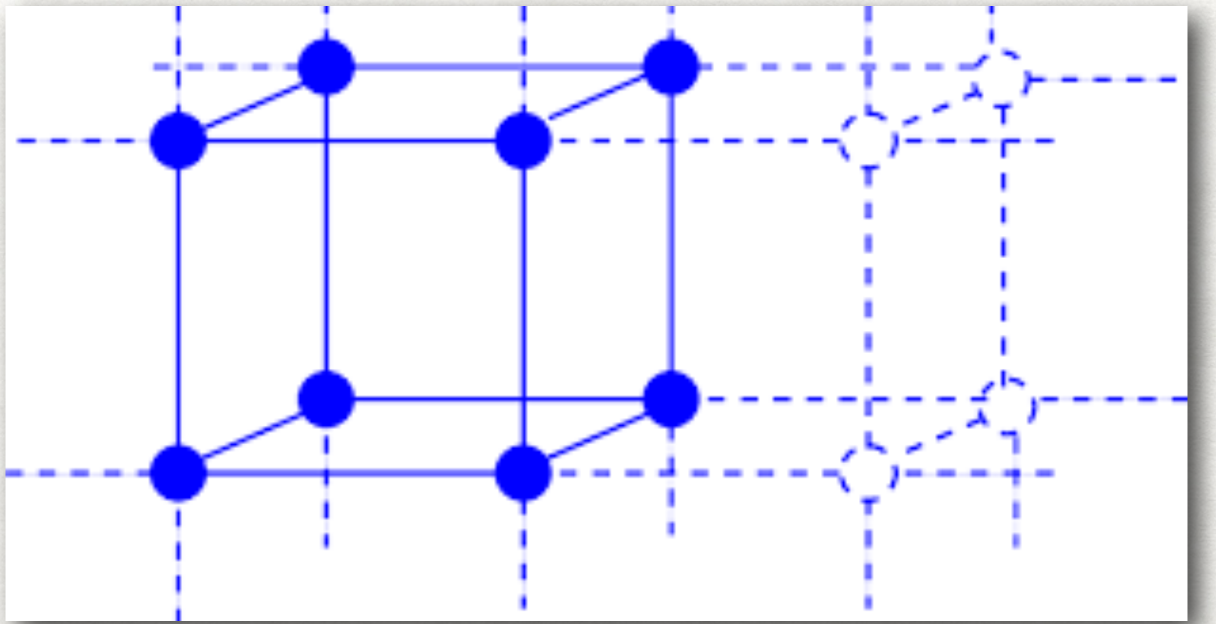
$$\psi_a \psi_b = -\psi_b \psi_a$$



Yan Fyodorov

LA LOCALIZZAZIONE DI ANDERSON

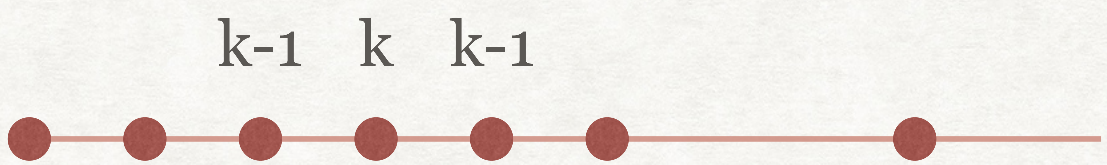
Absence of Diffusion in Certain Random Lattices
Philip Warren Anderson (1957)



Particella nel reticolo cubico. In ogni sito il potenziale e` diverso (un numero random in [-W,W])
Nel reticolo infinito:
per W<Wc gli autovettori sono estesi (conduttore)
per W>Wc gli autovettori sono exp localizzati (isolante)

LA LOCALIZZAZIONE DI ANDERSON

E LA TRANSIZIONE METALLO - ISOLANTE



$$u_{k+1} + u_{k-1} + \epsilon_k u_k = E u_k$$

Disordine uniforme in [-W,W]

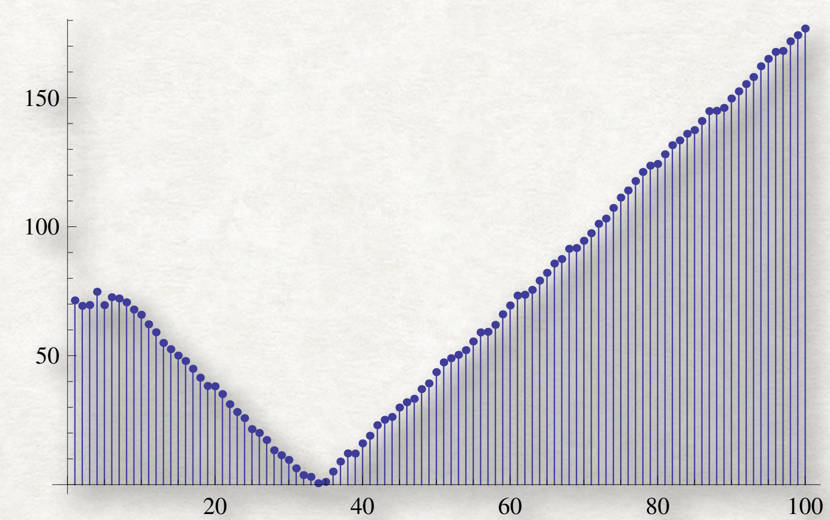
$$\begin{bmatrix} \epsilon_1 & 1 & & \\ 1 & \epsilon_2 & & \\ & & \ddots & 1 \\ & & 1 & \epsilon_N \end{bmatrix}$$

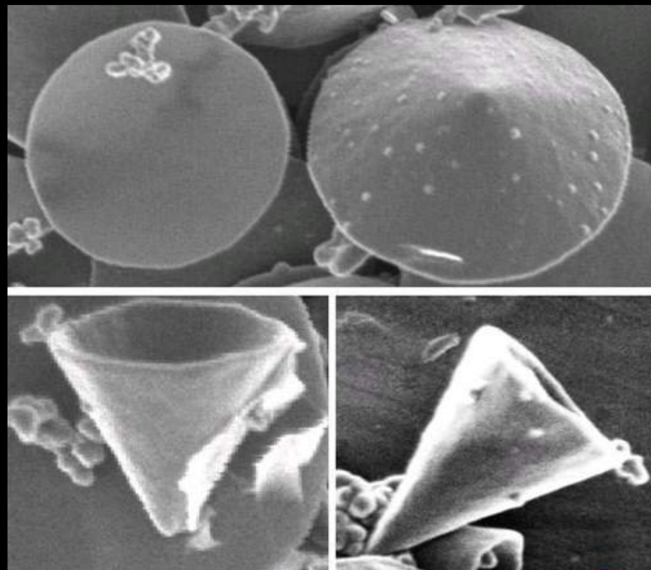
Matrice random tridiagonale

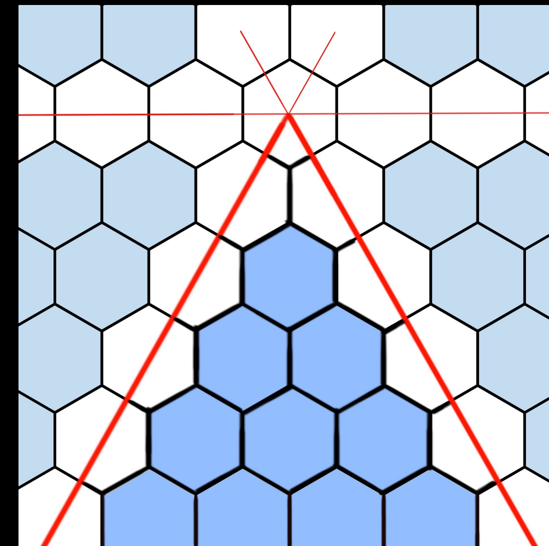
$$\begin{bmatrix} u_{k+1} \\ u_k \end{bmatrix} = \begin{bmatrix} E - \epsilon_k & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_k \\ u_{k-1} \end{bmatrix} = T(E) \begin{bmatrix} u_1 \\ u_0 \end{bmatrix}$$

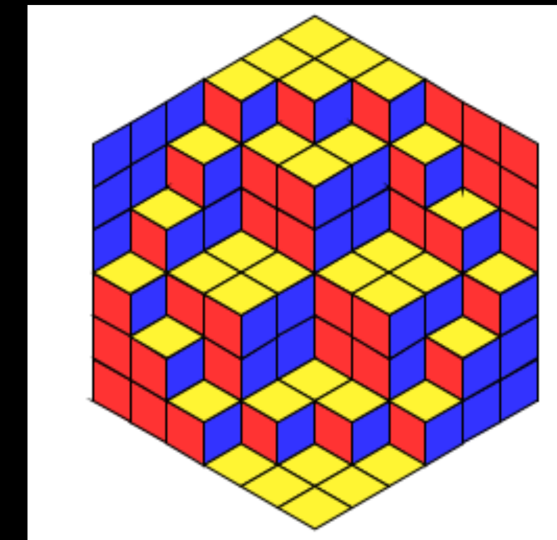
Teorema di Furstenberg: gli autovalori di T(E) sono exp[n t(E)] e exp[-n t(E)]

Autovettori esponenzialmente localizzati
t(E) = formula di Thouless









Graphene nanocones and Pascal matrices

A surprising connection with famous models in combinatorics (plane partitions, lozenge tilings, dense loops on cylinder)

$$H_2 = \left[\begin{array}{c|ccc|ccccc} x+y & 0 & 1 & & & & & & \\ \hline 0 & 0 & 1 & y & 0 & 1 & & & \\ 1 & 1 & 0 & 1 & 0 & & 0 & & \\ & x & 1 & 0 & 0 & & & 1 & \\ \hline & 0 & 0 & 0 & 0 & 1 & & & y \\ & 1 & & & 1 & 0 & 1 & & \\ & & 0 & & & 1 & 0 & 1 & \\ & & & 1 & & & 1 & 0 & 1 \\ & & & & x & & & 1 & 0 \end{array}\right]$$

Adjacency matrix of a nanocone (size n^2)

$$\det H_n(e^{-i\theta}, e^{i\theta}) = e^{-i(n+1)\theta} \det(Q_n + e^{2i\theta})$$

DETERMINANT?

$$Q_5 = \left[\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ \hline 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{array}\right]$$

Pascal Matrix (size n)

n	$\theta = 0$	$\pi/6$	$\pi/3$	$\pi/2$	$\pi/4$
2	20	$3^2\sqrt{3}$	7	0	$8\sqrt{2}$
3	132	10^2	42	2^4	70
4	1452	$25^2\sqrt{3}$	429	0	$526\sqrt{2}$
5	26741	140^2	7436	7^4	13167
6	826540	$588^2\sqrt{3}$	218348	0	$280772\sqrt{2}$

L.G. Molinari, arXiv:2206.14428

Giacomo Livan, Marcel Novaes, Pierpaolo Vivo,
Introduction to Random Matrices - Theory and Practice
(arXiv:1712.07903). Printed by Springer

http://wwwteor.mi.infn.it/~molinari/RMT/molinari_RMT.html