Milano W mass workshop report

October 28, 2009

1 Introduction

The setup proposed here should help to tune the different codes and to compute benchmark numbers for cross sections and distributions. In a second step we will compare the best predictions, as produced by each group, and provide an estimate of theoretical uncertainties due to *missing* and *unknown* higher-order corrections.

For completeness, we propose that each group will provide a short description of their code which will be made available in the appendix of the workshop report.

Timeline:

- By the end of September: preliminary results for the tuned comparison, mainly to check that there are no trivial errors in setting up our codes for the final runs.
- By the end of October: final results for the tuned comparison and the best results as well as contributions to the discussion of the theoretical uncertainty (see Section 5).
- By Christmas: first draft of the workshop report.

2 Setup for the tuned comparison

1.) For the numerical evaluation of the cross sections at the Tevatron ($\sqrt{s} = 1.96$ TeV) and the LHC ($\sqrt{s} = 10$ TeV) we choose the following set of Standard Model input parameters [1]:

$$
\begin{aligned}
G_\mu &= 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2}, & \alpha &= 1/137.035999679, & \alpha_s &\equiv \alpha_s(M_Z^2) = 0.118 \\
M_Z &= 91.1876\ \mathrm{GeV}, & \Gamma_Z &= 2.4952\ \mathrm{GeV} \\
M_W &= 80.398\ \mathrm{GeV}, & \Gamma_W &= 2.141\ \mathrm{GeV}
\end{aligned}
$$

$$
\begin{aligned}
M_H &= 115 \text{ GeV}, \\
m_e = 0.51099891 \text{ MeV}, & \quad m_\mu = 0.1056583668 \text{ GeV}, \quad m_\tau = 1.77684 \text{ GeV} \\
m_u = 0.06983 \text{ GeV}, & \quad m_c = 1.2 \text{ GeV}, \quad m_t = 171.2 \text{ GeV} \\
m_d = 0.06984 \text{ GeV}, & \quad m_s = 0.15 \text{ GeV}, \quad m_b = 4.6 \text{ GeV} \\
|V_{ud}| = 0.975, & \quad |V_{us}| = 0.222 \\
|V_{cd}| = 0.222, & \quad |V_{cs}| = 0.975 \\
|V_{cb}| = |V_{ts}| = |V_{ub}| &= |V_{td}| = |V_{tb}| = 0
\end{aligned} \tag{1}
$$

We work in the constant width scheme and fix the weak mixing angle by $c_w = M_W/M_Z$, $s_w^2 = 1 - c_w^2$. The Z and W-boson decay widths given above are used in both the LO and NLO evaluations of the cross sections. The fermion masses only enter through loop contributions to the vector boson self energies and as regulators of the collinear singularities which arise in the calculation of the QED contribution. The value of the running electromagnetic coupling at the Z resonance is given by $\alpha(M_Z) = \alpha(0)/(1-\Delta\alpha), \Delta\alpha = \Delta\alpha_{lep} + \Delta\alpha_{top} + \Delta\alpha_{had}^{(5)}$. The light quark masses are chosen in such a way, that the value for the hadronic five-flavour contribution to the photon vacuum polarization, $\Delta\alpha_{had}^{(5)}(M_Z^2) = 0.027572$ [2], is recovered, which is derived from low-energy e^+e^- data with the help of dispersion relations.

2.) To compute the hadronic cross section we use the CTEQ6.6M set of parton distribution functions [3] and take the renormalization scale, μ_r, and the QCD factorization scale, $\mu_{\rm QCD}$, to be $\mu_r = \mu_{\rm QCD} = M_{l\nu}$ in the W boson case and $\mu_r = \mu_{\rm QCD} = M_{l^+l^-}$ in the Z boson case. The invariant masses, $M_{l\nu}$ and $M_{l^+l^-}$ are calculated after applying the recombination procedure described in item 5 below.

All numerical evaluations require the subtraction of QED initial state collinear divergences, which is performed using the QED DIS scheme. It is defined analogously to the usual DIS [4] scheme used in QCD calculations, i.e. by requiring the same expression for the leading and next-to-leading order structure function F_2 in deep inelastic scattering, which is given by the sum of the quark distributions. Since F_2 data are an important ingredient in extracting PDFs, the effect of the $\mathcal{O}(\alpha)$ QED corrections on the PDFs should be reduced in the QED DIS scheme [1]. The QED factorization scale is chosen to be equal to the QCD factorization scale, $\mu_{QED} = \mu_{QCD}$.

[1] The subtraction of the QED initial state collinear divergences is a necessary step to obtain a finite partonic cross section. The absence of a QED evolution in the PDF set CTEQ6.6 has little phenomenological impact on the distributions, much smaller than the change from the massless-charm parametrizations like MRST2004QED to the massive charm sets CTEQ6.6 or MSTW2008. See Section 5.5 for further discussion.

Table 1: Two-loop running of $\alpha_s(\mu_r^2)$.

μ_r [GeV]	α_s
91.1876	0.117981588
50	0.129786654
100	0.116361764
200	0.105509842
500	0.0939820525

3) We work in the on-shell renormalization scheme and use the following Z and W mass renormalization constants:

$$\delta M_Z^2 = \mathcal{R}e\Big(\Sigma^Z(M_Z^2) - \frac{(\hat{\Sigma}^{\gamma Z}(M_Z^2))^2}{M_Z^2 + \hat{\Sigma}^\gamma(M_Z^2)}\Big), \quad \delta M_W^2 = \mathcal{R}e\Sigma^W(M_W^2) \tag{2}$$

where $\Sigma^V(\hat{\Sigma}^V)$ denote the transverse parts of unrenormalized(renormalized) vector boson self energies. Using our choice for the EW input parameters one finds $\hat{\Sigma}^{\gamma Z}(M_Z^2) = (-165.16896, -49.3808933)$ and $\hat{\Sigma}^\gamma(M_Z^2) = (-494.132427, 134.841466)$ (please see [5, 6] for details). This choice of the Z mass renormalization constant is motivated by the LEP-I treatment and that LEP-I measurements of the Z mass may be used for detector calibration at hadron colliders.

For the sake of simplicity and to avoid additional sources of discrepancies in the tuned comparison we suggest to use the finestructure constant $\alpha(0)$ throughout in both the calculation of CC and NC cross sections. We will discuss the impact of using different EW input schemes in Section 5.2.

In the course of the calculation of the W observables the Kobayashi-Maskawa-mixing has been neglected, but the final result for each parton level process has been multiplied with the square of the corresponding physical matrix element V_{ij}. From a numerical point of view, this procedure does not significantly differ from a consideration of the Kobayashi-Maskawa-matrix in the renormalisation procedure as it has been pointed out in [7].

4.) We choose to evaluate the running of the strong coupling constant at the two-loop level, with five flavours, using as reference value $\alpha_s(M_z) = 0.118$, which is consistent with the choice made in the PDF set CTEQ6.6. In Table 1 we provide $\alpha_s(\mu_r^2)$ for several choices of the QCD renormalization scale μ_r.

5.) The detector acceptance is simulated by imposing the following transverse momentum (p_T) and pseudo-rapidity (η) cuts:

$$p_T(\ell) > 25 \text{ GeV}, \qquad |\eta(\ell)| < 1, \qquad \ell = e,\, \mu, \tag{3}$$

$$\not{p}_T > 25 \text{ GeV}, \tag{4}$$

where $\not{p}_T$ is the missing transverse momentum originating from the neutrino. These cuts approximately model the acceptance of the CDF II and DØdetectors at the Tevatron, and the ATLAS and CMS detectors at the LHC. In the case of γ/Z production, in addition to the separation cuts of Eq. 3 we apply a cut on the invariant mass of the final-state lepton pair of $M_{ll} > 50$ GeV.

Ilija provided a C++ routine *simplesim.cc* (a fortran routine is work in progress) that provides a prescription for photon merging and deals with the MIP energy of the muon. This routine is only needed when calculating observables for the *calo* setup.

Here is Ilija's description of the routine:
The idea is to slice the central ($|\eta| < 1.1$) electromagnetic (CEM) calorimeter into 20 slices in η ($\Delta\eta = 0.11$) and 24 slices in ϕ and then: first check if the photons are not falling into a crack between towers (routine cracks()). For electrons we merge the photons to the electron if they are one tower away in η and same ϕ, where we check which of the two neighbouring towers the lepton is closer to. We don't do that for the towers at η of 0 because there is a crack between the two halves of the detector there. We knock out (not included in the recoil) a region of 7 towers around the electron, looking like this (knocked-out towers are 0):
1 0 0
0 0 0
1 0 0

where $\Delta\phi$ ($\Delta\eta$) is on the $x-$($y-$)axis, and is defined such that the electron is always closest to the tower on the right. The remaining photons go to the recoil.

For Muons, the EM energy deposit is estimated from cosmic events, to which we add the underlying event (UE) and the photons. To decide which photons to add, see the explanation below.

Minimum ionising contribution:
The MIP contribution is estimated from cosmics and has an approximate Gaussian shape in $Log_{10}E_T$, so I fitted the distribution we use and give the parameters in Mip::mip_avg and mip_sig, the mip_zero is the fraction of events that leave no energy in CEM. I add to it the average UE contribution of 149 MeV (in our simulation this contribution has a η dependence).

Adding photons:
The function MipE() estimates these two contributions. Then you need to loop through the photons in the event and add their energy if: EM energy is in the same tower as muon, add its energy to the muon CEM energy. We regard a muon to be in two towers, if it is closer than 1.58 cm from the next tower in

z direction (η). Then you use function MipCutFail with the total muon CEM energy and its p_T to see if it failed the MIP cut.

We knock out a region of 3 towers around the muon, looking like this (Towers with 0 are knocked out):
1 0 1
1 0 1
1 0 1
The rest of the photons go to recoil.

So, in practise you loop through photons, use the routines ElectronPhoton(electron4mom,photon4mom) and MuonPhoton(muon4mom,photon4mom) to see if the photons fall in a crack, need to be merged, knocked out or added to the recoil. The Muon CEM energy (without the photons) is estimated using MipE(). If the muon with this MIP energy with photon contribution fails the MIP cut is checked using MipCutFail(...).

The simulation of the leakage of the showers into the hadronic calorimeter and the energy losses in the coil have not been simulated, as they require more detailed parameterisations. I have also not added the nonlinear response parameterisation of the calorimeter, since it depends on the modelling of the leakage.

6.) Since we consider predictions inclusive with respect to QCD radiation, we do not impose any jet definition.

3 W and Z boson observables

In the following we provide a list of observables which will be evaluated in the benchmarking and the comparison of the best results for EW and QCD predictions. If not stated otherwise, we consider the following charged (CC) and neutral current (NC) processes: $pp(p\bar{p}) \to W^+ \to l^+\nu_l$ and $pp(p\bar{p}) \to \gamma, Z^0 \to l^+l^-$ with $l = e, \mu$. See Section 6 for the specifications of the histograms used in the fitting procedure.

To facilitate a quick and easy comparison of histograms, please use the lower value of the bin range to label the bin (e.g., for a range of 100 GeV and a bin size of 1 GeV the first bin is labeled 0 GeV, the second bin 1 GeV etc). Please provide the histograms in form of an ASCII file including a bin-by-bin Monte Carlo integration error.

W boson observables:

- σ_W: total inclusive cross section of W boson production.

- $\frac{d\sigma}{dM_T(l\nu)}$: transverse mass distribution of the lepton lepton-neutrino pair. The transverse mass is defined as

$$M_T = \sqrt{2p_T(\ell)p_T(\nu)(1-\cos\phi^{\ell\nu})}\ , \tag{5}$$

where $p_T(\nu)$ is the transverse momentum of the neutrino, and $\phi^{\ell\nu}$ is the angle between the charged lepton and the neutrino in the transverse plane. The neutrino transverse momentum is identified with the missing transverse momentum, $\not{p}_T$, in the event. M_T range (bin size): 50-100 GeV (0.5 GeV)

- $\frac{d\sigma}{dp_T^l}$: transverse lepton momentum distribution. p_T^l range (bin size): 25-55 GeV (0.25 GeV)

- $\frac{d\sigma}{d\not{E}_T^l}$: missing transverse energy distribution. $\not{E}_T$ range (bin size): 25-55 GeV (0.25 GeV)

- $\frac{d\sigma}{d\eta_l}$: charged lepton pseudorapidity distribution. η_l range (bin size) -3 - 3 (0.1)

- $\frac{d\sigma}{d\cos\theta_l^*}$: lepton scattering angle distributions where θ_l^* is the emission angle of the charged lepton in the partonic c.m.s.: -1-1 (0.02)

- $d\sigma_W/dQ_T(W)$: W transverse momentum distributions. Q_T range (bin size): 0-25 (0.25 GeV) and 0-100 GeV (1 GeV)

- $\frac{d\sigma}{dE_\gamma}$: photon energy distribution. E_γ range (bin size): 0-50 (0.5)

- $\frac{d\sigma}{dp_{T,\gamma}}$: photon transverse momentum distribution. $p_{T,\gamma}$ range (bin size): 0-100 (0.5)

- $\frac{d\sigma}{d\sqrt{R_{l\gamma}}}$: $R_{l,\gamma} = \sqrt{\Delta\Phi^2 + \Delta\eta^2}$. $\sqrt{R_{l,\gamma}}$ range (bin size): 0-1 (0.02)

- $\frac{d\sigma}{dy^{1/3}}$: $y = E_\gamma/(E_l + E_\gamma)$. $y^{1/3}$ range (bin size): 0-1 (0.02)

- $\frac{d\sigma}{d\log_{10}R_{l\gamma}}$: $R_{l,\gamma} = \sqrt{\Delta\Phi^2 + \Delta\eta^2}$. $\log_{10}R_{l,\gamma}$ range (bin size): -6 -0 (0.1)

- $\frac{d\sigma}{d\log_{10}y}$: $y = E_\gamma/(E_l + E_\gamma)$. $\log_{10}y$ range (bin size): -4 -0 (0.1)

- $\frac{d^2\sigma}{d\log_{10}R_{l\gamma}d\log_{10}y}$: same ranges as for the single 1D distributions

In the calculations of the photon observables the following default phase space slicing parameter is used where applicable: $E_\gamma > \delta_s\sqrt{\hat{s}}/2$ with $\delta_s = 0.0001$. In addition, we check the infrared safety of these observables by varying the slicing parameters.
Z **boson observables:**

- σ_Z: total inclusive cross section of Z boson production.
- $\frac{d\sigma}{dM_{ll}}$: invariant mass distribution of the lepton pair. M_{ll} range (bin size): 50-200 GeV (1 GeV)
- $\frac{d\sigma}{dp_T^l}$: transverse lepton momentum distribution (l is the positively charged lepton). p_T^l range (bin size): 25-65 GeV (0.25 GeV)
- $\frac{d\sigma}{d\eta_l}$: positively charged lepton pseudorapidity distribution. η_l range (bin size) -3 - 3 (0.1)
- $\frac{d\sigma}{dy_Z}$: lepton-pair rapidity distribution. y_Z range (bin size) -3 - 3 (0.1)
- $\frac{d\sigma}{d\cos\theta_l^*}$: lepton scattering angle distributions where θ_l^* is the emission angle of the (positively) charged lepton in the partonic c.m.s.: -1-1 (0.1)
- $d\sigma_Z/dQ_T(Z)$: Z transverse momentum distributions. Q_T range (bin size): 0-25 (0.25 GeV) and 0-100 GeV (1 GeV)

W/Z **Ratios:**

- $\frac{\sigma_W}{\sigma_Z}$: ratio of the total inclusive cross sections of the W and Z boson.
- $\frac{d\sigma_W/dX_M(W)}{d\sigma_Z/dX_M(Z)}$: ratio of W and Z transverse mass distributions, with $X_M(V) = M_T^V/M_V, V = W, Z$. X_M range (bin size): 0.6-1.2 (0.006)
  The transverse mass of the lepton pair in Z boson events is defined in complete analogy to Eq. (5):

$$M_T^Z = \sqrt{2p_T(\ell^+)p_T(\ell^-)(1-\cos\phi)} \,, \tag{6}$$

- $\frac{d\sigma_W/dX_p(W)}{d\sigma_Z/dX_p(Z)}$: ratio of the lepton transverse momentum distributions in W and Z boson production, with $X_p(V) = p_T^V(l)/M_V, V = W, Z$ (NC: l is the positively charged lepton). X_p range (bin size): 0.6-1.2 (0.006)
- $\frac{d\sigma_W/dQ_T(W)}{d\sigma_Z/dQ_T(Z)}$: ratio of the transverse momentum distributions of the W and Z bosons. Q_T range (bin size): 0-25 (0.25 GeV) and 0-100 GeV (1 GeV)

4 Benchmarks

For each observable (where applicable) listed in Section 3 we will compare predictions at NLO EW (Dittmaier *et al.*, HORACE, SANC, WZGRAD), NLO QCD (MC@NLO, MCFM, Resbos, Powheg, FEWZ), NLO QCD+resummation (MC@NLO,Resbos,Powheg), and at NNLO QCD (Catani *et al.* and FEWZ).

It seems that NLO QCD predictions can only be obtained with MC@NLO when no acceptance cuts are applied (this needs to be confirmed with the authors). Therefore, we suggested that we only compute total cross sections at NLO QCD without applying acceptance cuts (in the NC case only the invariant mass cut is applied which is necessary to avoid the Coulomb singularity of the photon-exchange diagram).

For the comparison of EW NLO, NLO QCD+resummation and NNLO QCD predictions for both the CC and NC channel, we compute total cross sections and kinematic distributions with the acceptance cuts described in Section 2 for both the *bare* and *calo* setup. *Bare* results are obtained without smearing and recombination, ie only separation cuts are applied. We also include in our study results that are obtained with smearing and recombination *calo*, so that we can estimate what survives from the observed effects under somewhat more realistic experimental conditions.

5 Best predictions and theoretical uncertainties

In this section each group provides their best prediction for the different observables listed in Section 3, which will still be based on the setup described in Section 2 but adjusted where necessary to obtain the best prediction. Each group will provide a description of these modifications, for instance, changes in the input scheme and the inclusion of higher-order corrections. In the following we distinguish the different kinds of effects which can be studied separately. As result of this study, we will provide an estimate of the theoretical uncertainty due to *missing* and *unknown* higher-order corrections. Most of the following is based on the workshop notes.

Where appropriate we will also present results from earlier studies (our own studies and see, e.g., recent work by Adam *et al.* [8]).

5.1 EW: Multiple photon radiation

Multiple photon radiation (mPR) is included in the NLO EW prediction either in the struction function or parton shower approach. In the structure function approach the scale is set equal to the parton-level center-of-mass energy $Q = \sqrt{\hat{s}}$.

Besides comparing prediction with and without mPR, we will also provide an estimate of the uncertainty due to unknown $\mathcal{O}(\alpha^2)$ corrections (two-photon radiation) beyond LL (is it a 0.1 MeV, 1 MeV shift in M_W or larger ?) by

- varying the scale in the structure function/PS,
- using the difference between exact $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha)$ LL (see Iliya's talk), but keep in mind that LL is tuned to the exact $\mathcal{O}(\alpha)$, and
- studying the radiation of two hard photons (tree-level $q\bar{q}' \to l\nu_l\gamma\gamma$ production with cuts, $E_\gamma > 100$ MeV and $\theta_{l\gamma} > 0.1$ rad)

Finally, we will study the effect of a "lost" fermion pair in $q\bar{q}' \to l\nu\gamma \to l\nu_l l^+ l-$ production on M_W, where "lost" approximately means fermion energy smaller than 500 MeV. This can be done with truncated structure functions integrated with different upper bounds.

5.2 EW: input scheme dependence

The scheme dependence can still be quite evident at NLO EW. The HORACE formula, which matches exact NLO results with QED multiple photon radiation to all orders, tends to reduce the sensitivity to the different input choices. Each group will provide a description of their preferred input scheme and an estimate of the residual uncertainty by studying different implementations of this scheme. For example, the effects of using different implementations of the G_μ scheme, ie only $\Delta r \sigma_{LO}$ vs. in addition $\Delta r \delta\sigma, \Delta r^2 \sigma_{LO}$ etc. See also the detailed discussion in Ref. [49].

5.3 QCD: matched (fixed order + resummed) results

Besides the tuned comparison of Resbos, POWHEG, and MC@NLO, the following study will provide an estimate of the theoretical uncertainty in the $Q_T(W)/Q_T(Z)$ ratio due to

- different resummation prescriptions by varying free parameters such as the resummation scale etc.,
- ambiguities in the threshold resummation (Borel vs. Mellin),
- the $q_\perp$ broadening in the $Q_T(Z)$ spectrum, and
- differences in parton showers by comparing POWHEG+HERWIG and POWHEG+PYTHIA.

5.4 QCD+EW interplay

- Tuned comparison of HORACE+MC@NLO and Resbos-A.
- Comparison of the full $\mathcal{O}(\alpha)$ EW and final-state QED radiation with and without initial-state QCD radiation (with HORACE+MC@NLO).

- Study of the effect of a joint (ISR) QED and QCD parton shower (see recent papers by S.Yost, B.Ward).

5.5 PDFs

- Study of the PDF uncertainty in the $Q_T(W)/Q_T(Z)$ ratio using the most recent CTEQ and MSTW PDFs as well as NNPDFs.
- Discussion of the impact of a joint $Q_T(Z)$ and PDF global fit, ie PDFs are extracted reproducing the measured $Q_T(Z)$.

If the MSTW2008QED set of PDFs becomes available within the timeframe of this workshop (we contacted Robert Thorne and work is in progress), we will include

- a study of the difference in the $Q_T(W)/Q_T(Z)$ ratio when including or not including QED in PDFs, and
- a discussion of the effects of photon induced processes.

Otherwise, only a summary of results of earlier studies with MSTR2004QED will be provided.

5.6 Treatment of the finite W and Z boson width

A description of the treatment of the W, Z widths in the available public codes with a re-clarification of using the two options fixed vs. running width as done for LEP EW precision physics.

6 W mass fits

Each group will provide their own study of shifts in M_W due to the various effects described here, using their own fitting routine.

Here are the specifications for the generation of distributions used in the M_W fits:

- Generation cuts
  Transverse mass : $50 < M_T < 100 GeV$
  Lepton transverse momentum : 25 GeV $< p_T <$ 55 GeV
  Missing transverse momentum : 25 GeV $< E_T <$ 55 GeV
  Lepton pseudo-rapidity: $-1.2 < \eta_l < 1.2$
  Lepton-pair transverse momentum: $p_T^{l\nu} <$ 30 GeV

- Transverse mass ($M_T(l\nu)$) distribution
  Fitting range: $60 < M_T < 90$ GeV
  Number of bins: 100
  $-1 < \eta_l < 1$

- Lepton p_T distribution
  Fitting range: $30 < p_T^l < 50$ GeV
  Number of bins: 100
  $-1 < \eta_l < 1$

- Missing E_T distribution
  Fitting range: $30 < p_T^\nu < 50$ GeV
  Number of bins: 100
  $-1 < \eta_l < 1$

For the electroweak and QCD studies respectively LO and Resbos templates obtained for several values of M_W in the range 80.398 ± 0.050 GeV will be used to determine the shift in M_W due to different sources of higher order corrections.

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