Feynman diagrams in Particle Physics

Dr Juan Rojo

VU Amsterdam and Nikhef Theory group

j.rojo@vu.nl

Outline

- Motivation: what **Feynman diagrams are useful for**?
- Feynman diagrams in **Quantum Electrodynamics**
- Feynman diagrams in **Quantum Chromodynamics**
- Feynman diagrams in **Electroweak Theory**
- Sample calculation at the **Born** and the **one-loop level**

Motivation

Feynman diagrams

At the quantum level, **fundamental interactions** look very different that at the classical level

Elementary particles interact by **exchanging force carriers** among them

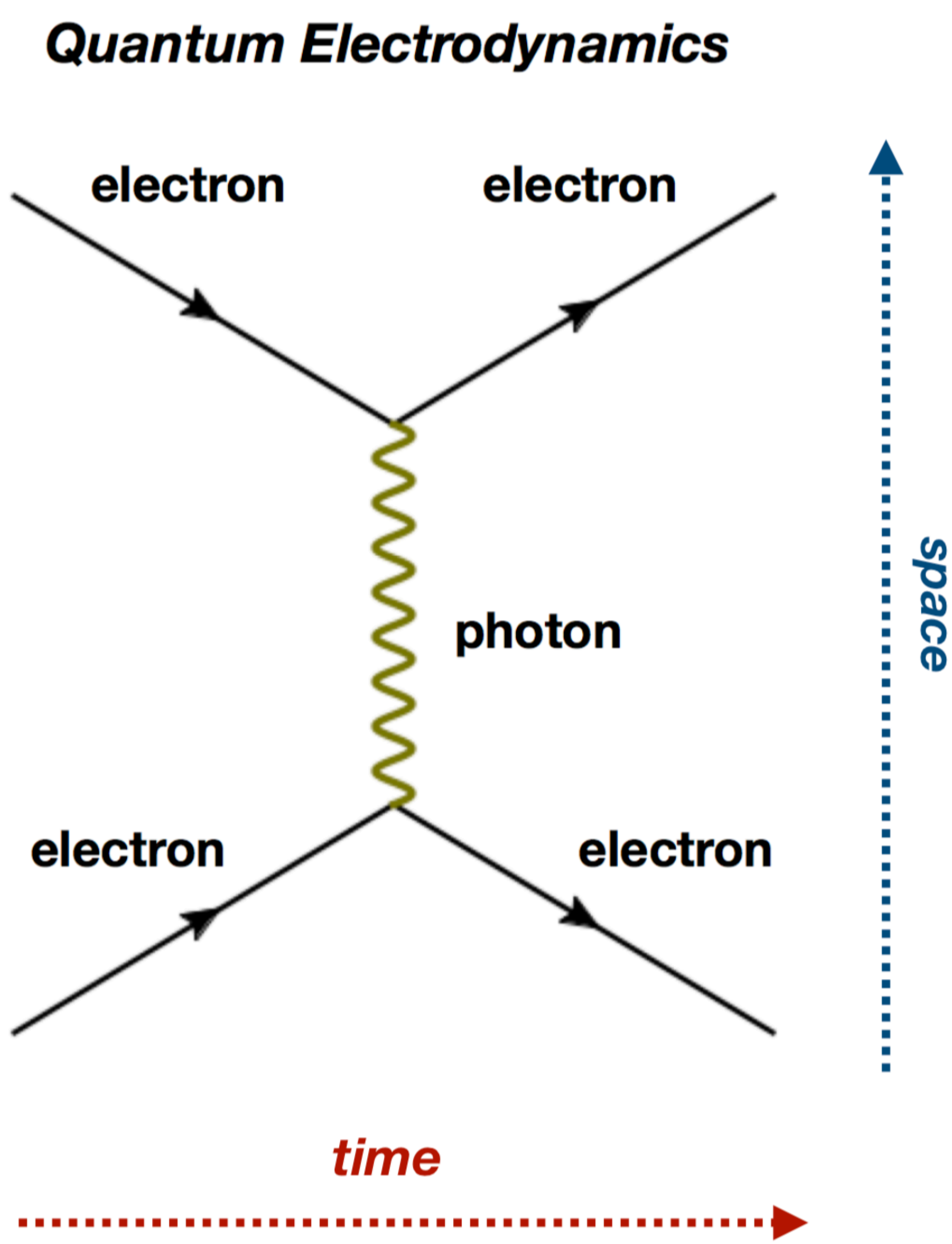
For example, the **photon** is the force carried particle of **Quantum Electrodynamics,** the quantum version of classical electromagnetic theory

A powerful tool to **visualise** interactions between elementary particles is known as **Feynman diagrams,** that represent the trajectories in space and time of the particles involved in a scattering reaction

Feynman diagrams

At the quantum level, **fundamental interactions** look very different that at the classical level

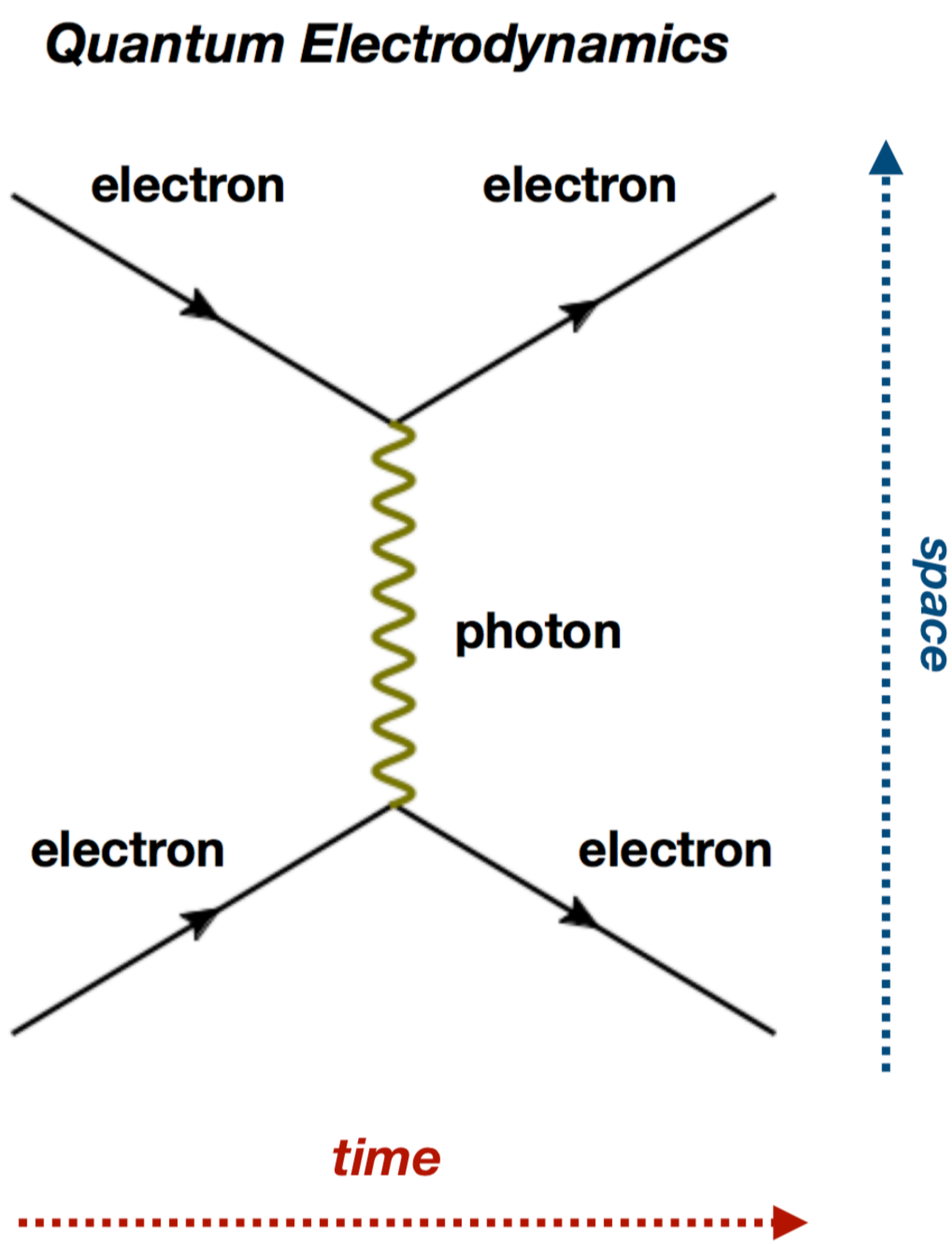
Elementary particles interact by **exchanging force carriers** among them

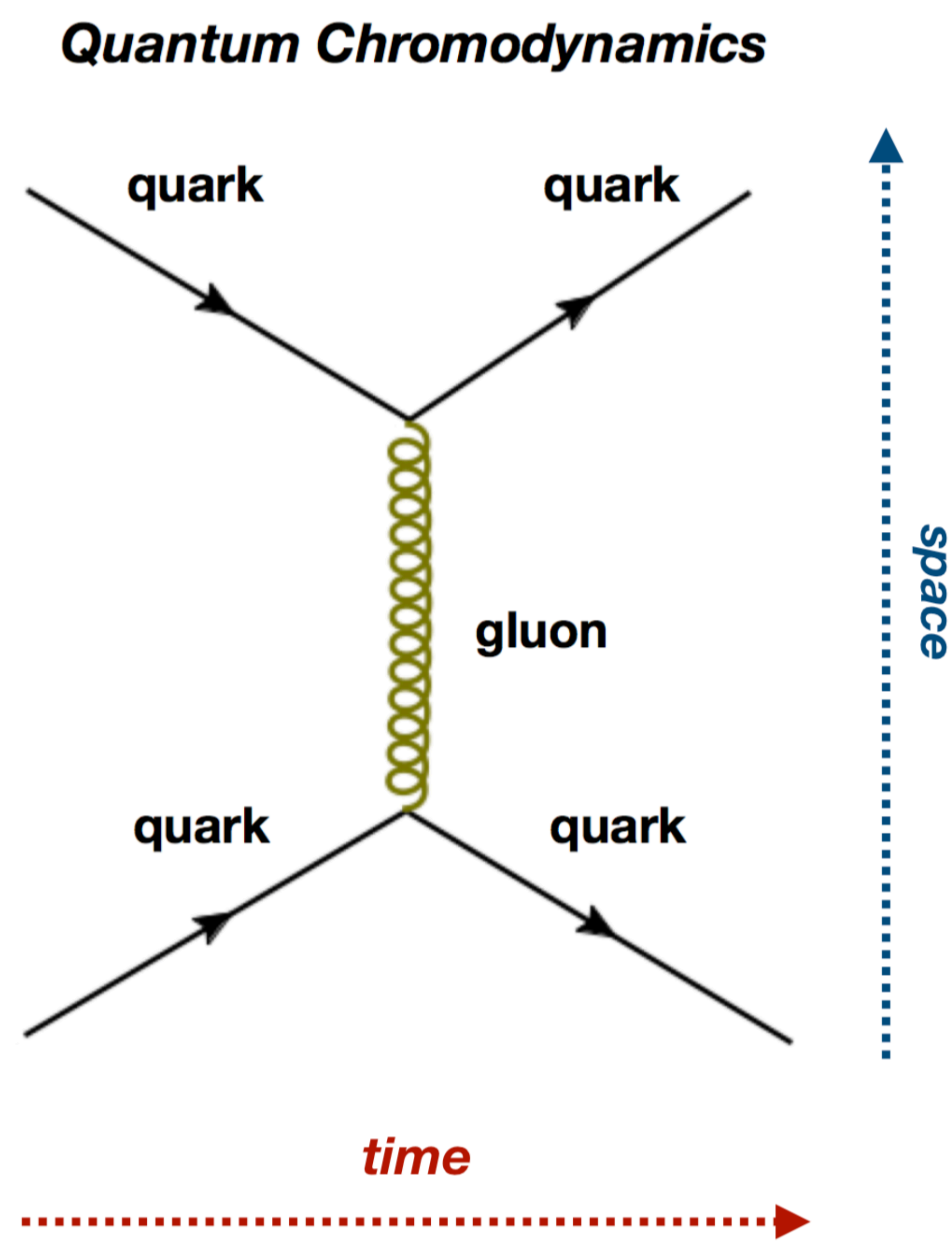


Feynman diagrams

At the quantum level, **fundamental interactions** look very different that at the classical level

Elementary particles interact by **exchanging force carriers** among them





Drawing Feynman diagrams

- If the scattering reaction involves composite particles (hadrons) first of all determine their **quark decomposition** making sure all quantum numbers add up consistently

Drawing Feynman diagrams

- ☑ If the scattering reaction involves composite particles (hadrons) first of all determine their **quark decomposition** making sure all quantum numbers add up consistently

- ☑ Then put at the **left** of the diagram the **initial-state particles** and at the **right** of the diagram the **final-state** particles

Drawing Feynman diagrams

- ☑ If the scattering reaction involves composite particles (hadrons) first of all determine their **quark decomposition** making sure all quantum numbers add up consistently

- ☑ Then put at the **left** of the diagram the **initial-state particles** and at the **right** of the diagram the **final-state** particles

- ☑ Attempt to connect the initial and final state particles among them. Note that some particles will not interact and will be just **spectators** in the reaction

Drawing Feynman diagrams

- If the scattering reaction involves composite particles (hadrons) first of all determine their **quark decomposition** making sure all quantum numbers add up consistently

- Then put at the **left** of the diagram the **initial-state particles** and at the **right** of the diagram the **final-state** particles

- Attempt to connect the initial and final state particles among them. Note that some particles will not interact and will be just **spectators** in the reaction

- Make sure that all interaction vertices **conserve the corresponding quantum numbers:** for example, if gluons or photons are conserved, then ***Q, B, S, C, b, …*** should be conserved

Why are Feynman diagrams useful?

- Determine whether a given scattering process is **possible** or **impossible**

Why are Feynman diagrams useful?

- Determine whether a given scattering process is **possible** or **impossible**

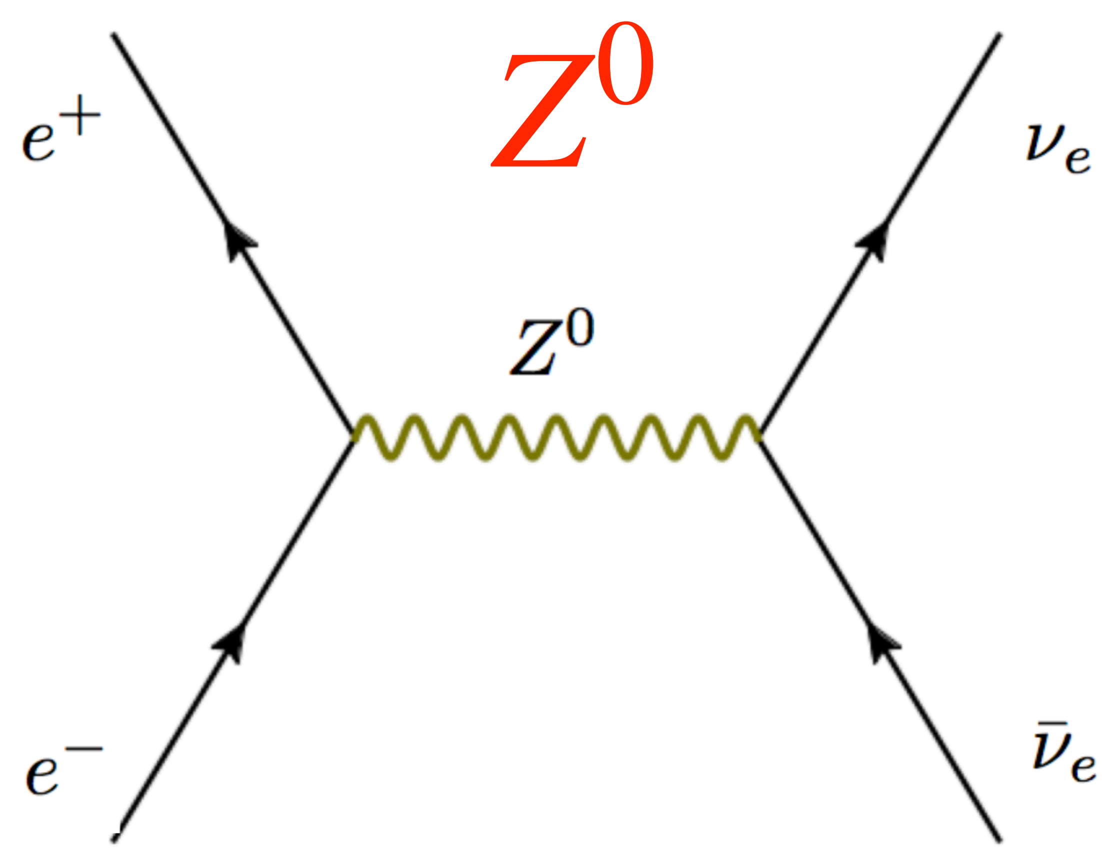
$$\mu^+ + \mu^- \rightarrow e^+ + e^- \qquad \text{allowed??}$$

Why are Feynman diagrams useful?

☑ Determine whether a given scattering process is **possible** or **impossible**

$$\mu^+ + \mu^- \rightarrow e^+ + e^- \qquad \text{allowed??}$$

Yes since I can draw a Feynman diagram with a **known particle mediating the interaction** and where **all conservation laws are satisfied!**



Why are Feynman diagrams useful?

- Determine whether a given scattering process is **possible** or **impossible**

- Determine which of two processes has a **higher likelihood of taking place** based on **power counting** of coupling constants

Why are Feynman diagrams useful?

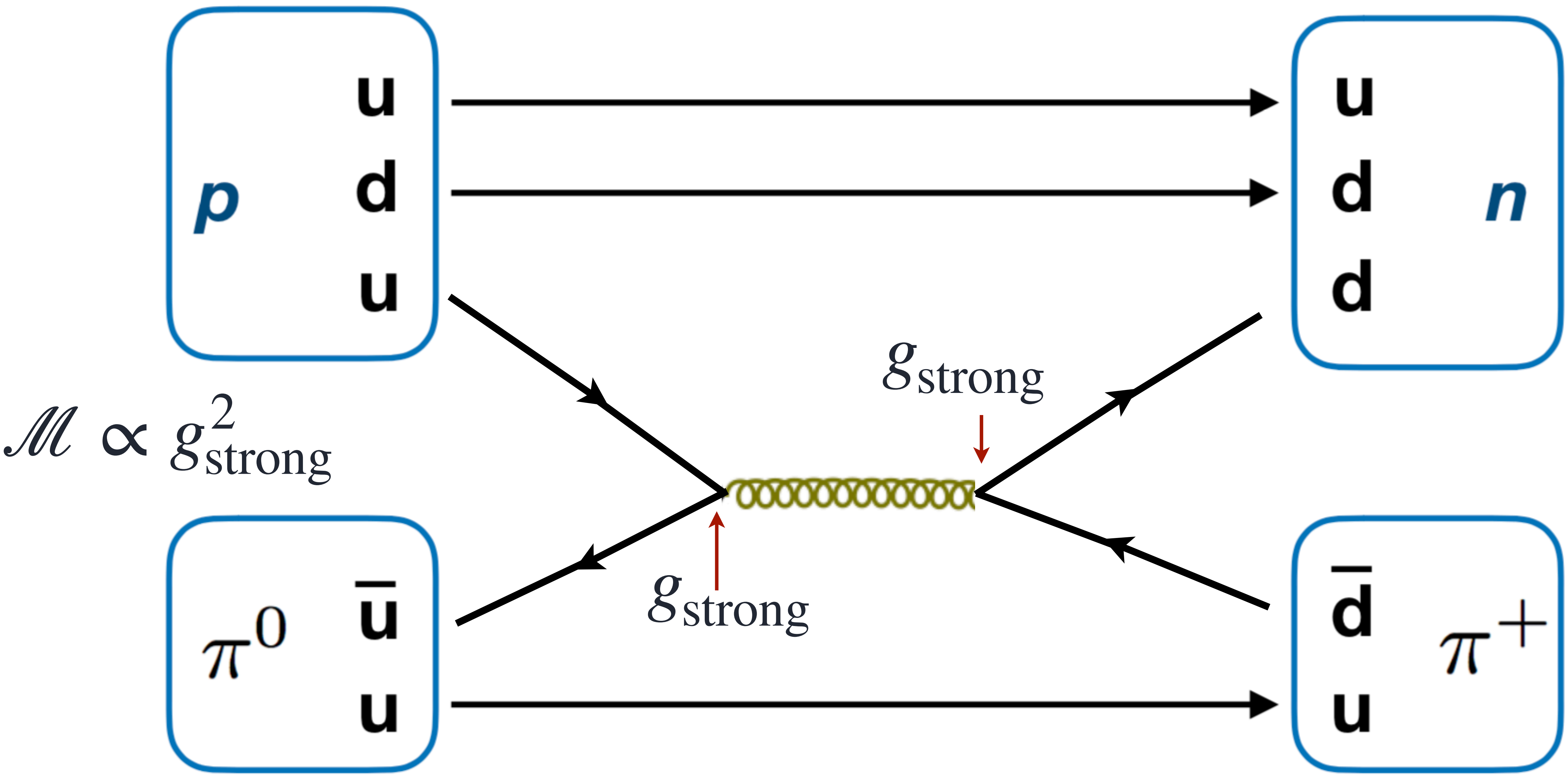
- Determine whether a given scattering process is **possible** or **impossible**

- Determine which of two processes has a **higher likelihood of taking place** based on **power counting** of coupling constants

Why are Feynman diagrams useful?

- Determine whether a given scattering process is **possible** or **impossible**

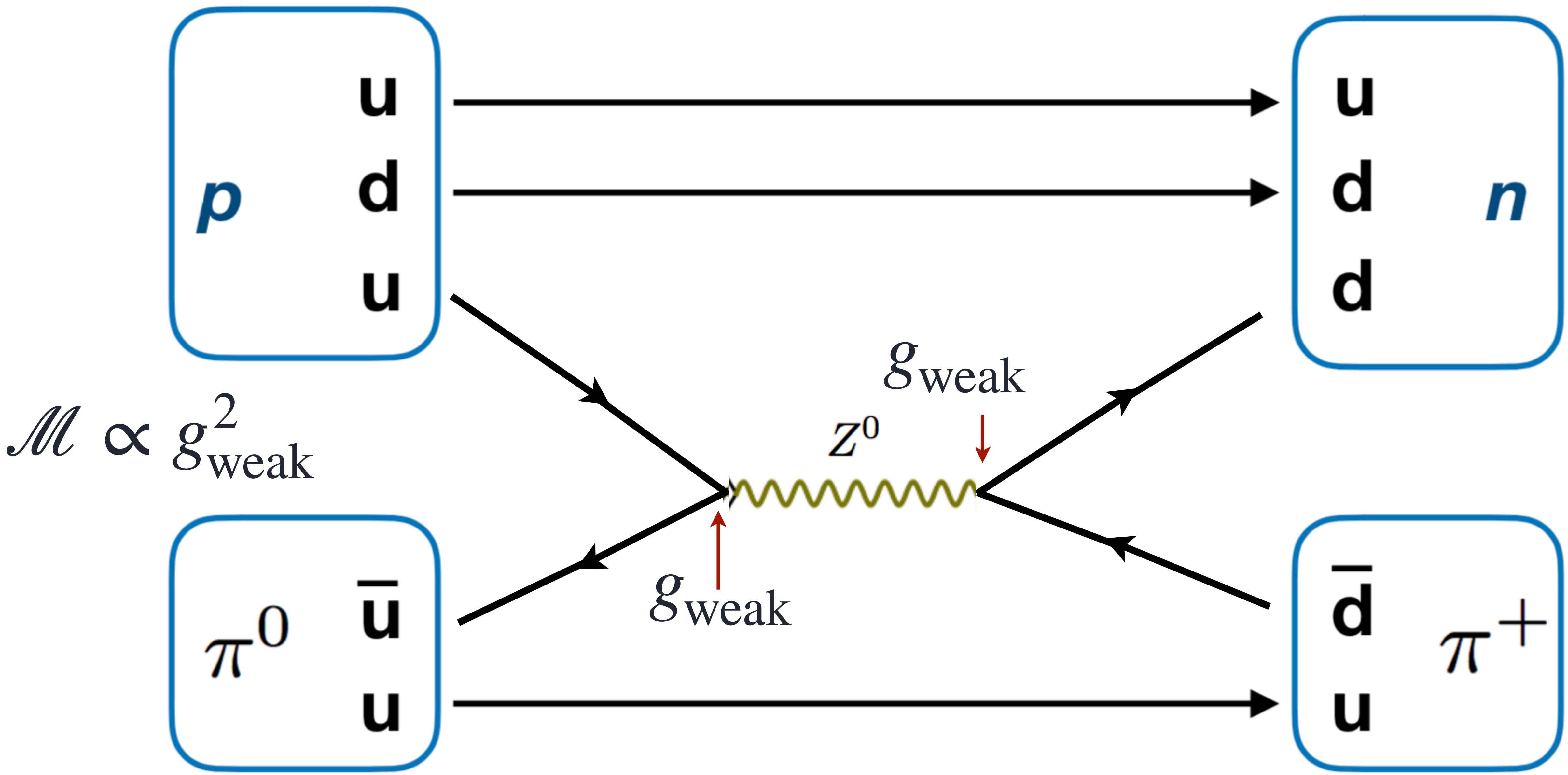
- Determine which of two processes has a **higher likelihood of taking place** based on **power counting** of coupling constants



Why are Feynman diagrams useful?

- ☑ Determine whether a given scattering process is **possible** or **impossible**

- ☑ Determine which of two processes has a **higher likelihood of taking place** based on **power counting** of coupling constants



Why are Feynman diagrams useful?

- Determine whether a given scattering process is **possible** or **impossible**

- Determine which of two processes has a **higher likelihood of taking place** based on **power counting** of coupling constants

The scattering amplitude mediated by a **Z boson** has amplitude

$$\mathcal{M} \propto g_{\text{weak}}^2$$

The scattering amplitude mediated by a **gluon** has amplitude

$$\mathcal{M} \propto g_{\text{strong}}^2$$

Since $g_{strong} \gg g_{weak}$ the reaction mediated by a gluon is **much more likely!**

Why are Feynman diagrams useful?

- Determine whether a given scattering process is **possible** or **impossible**

- Determine which of two processes has a **higher likelihood of taking place** based on **power counting** of coupling constants

- Exploit similarities between **apparently different scattering reactions**

Why are Feynman diagrams useful?

$$D^0 \to K^- + \pi^+ \qquad D^0 \to K^- + \mu^+ + \nu_\mu$$

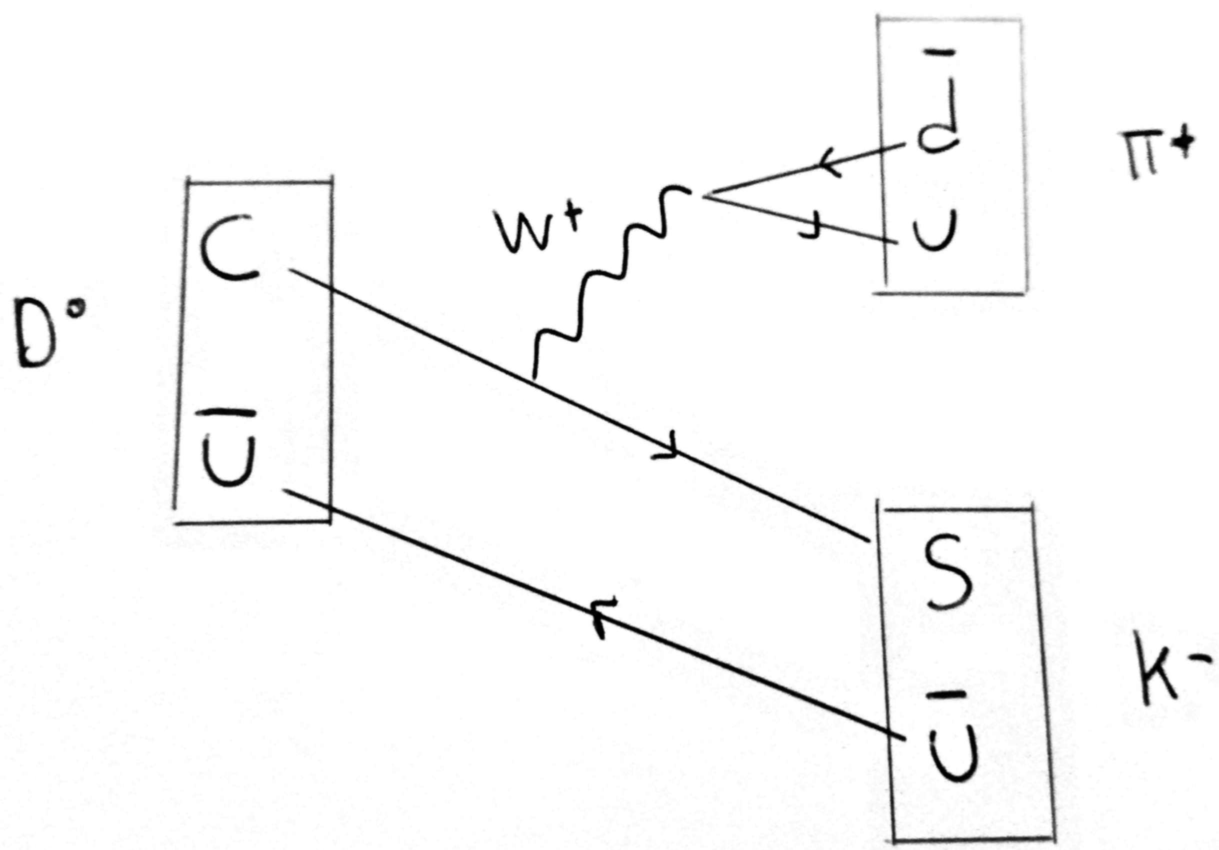
Seem to be very different reactions …

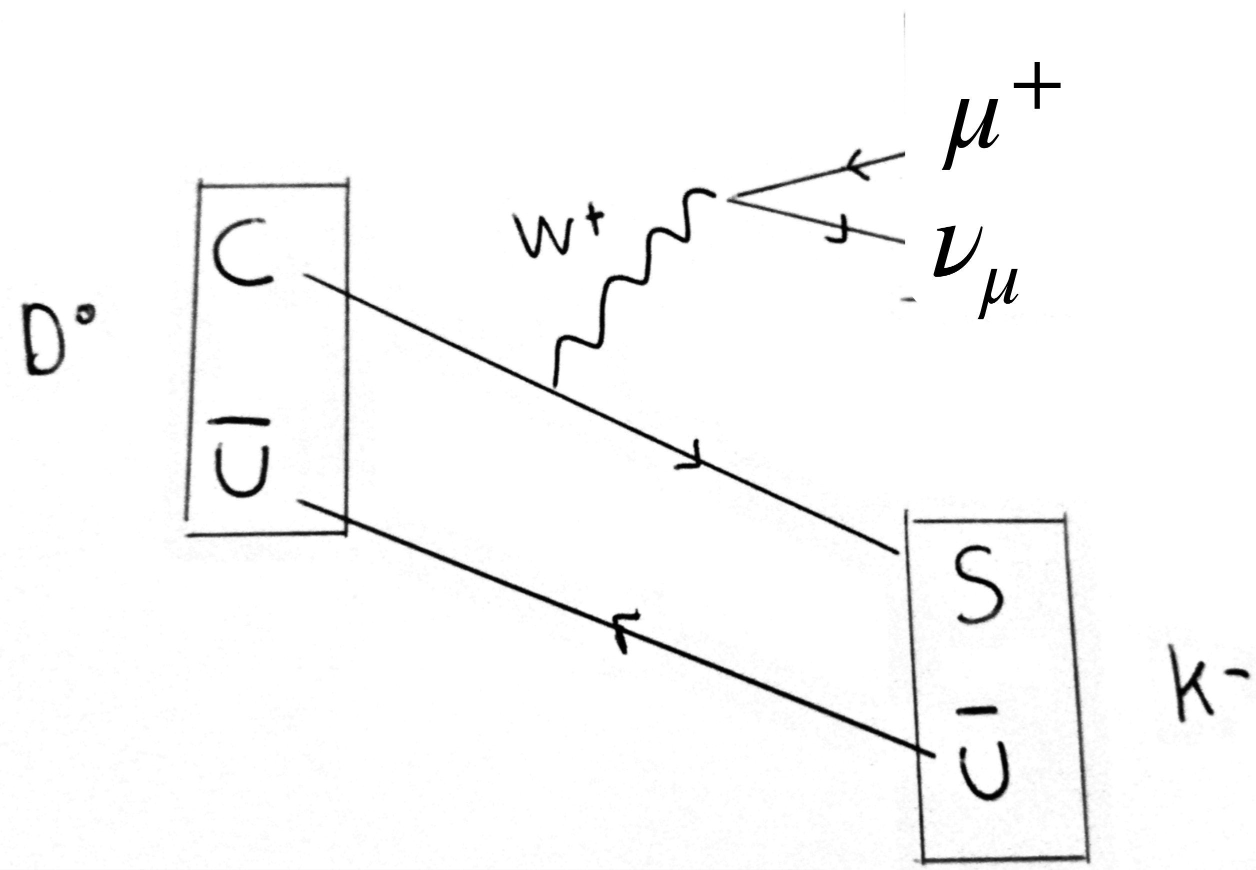
Why are Feynman diagrams useful?

$$D^0 \to K^- + \pi^+ \qquad D^0 \to K^- + \mu^+ + \nu_\mu$$

Seem to be very different reactions …

until we express them in terms of **Feynman diagrams!**





The two processes take place at **very similar rates**

Why are Feynman diagrams useful?

- Determine whether a given scattering process is **possible** or **impossible**

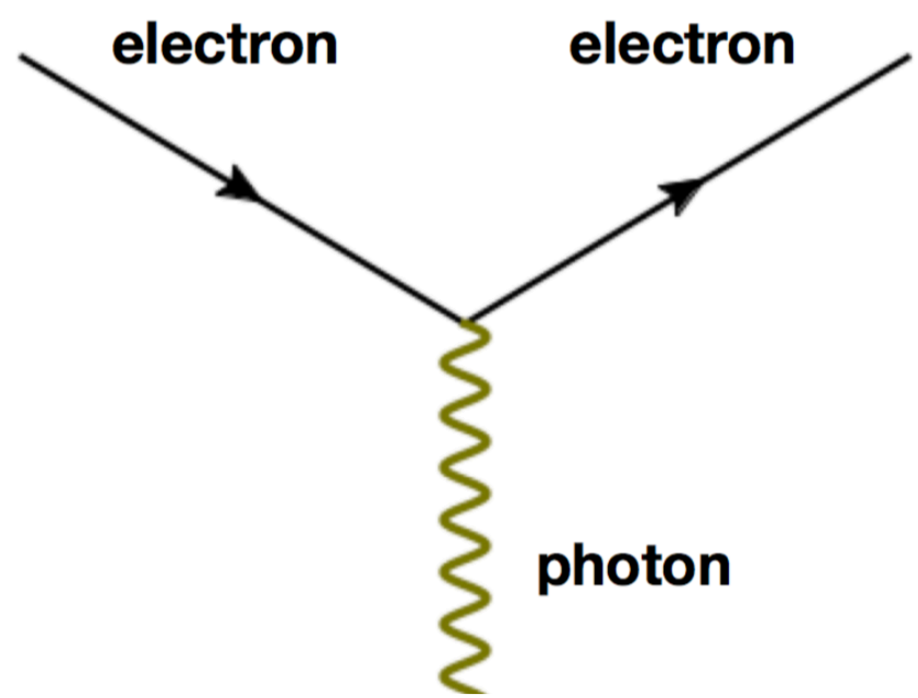
- Determine which of two processes has a **higher likelihood of taking place** based on **power counting** of coupling constants

- Exploit similarities between **apparently different scattering reactions**

Quantum Electrodynamics

Quantum Electromagnetism

In QED there is a unique **interaction vertex:**

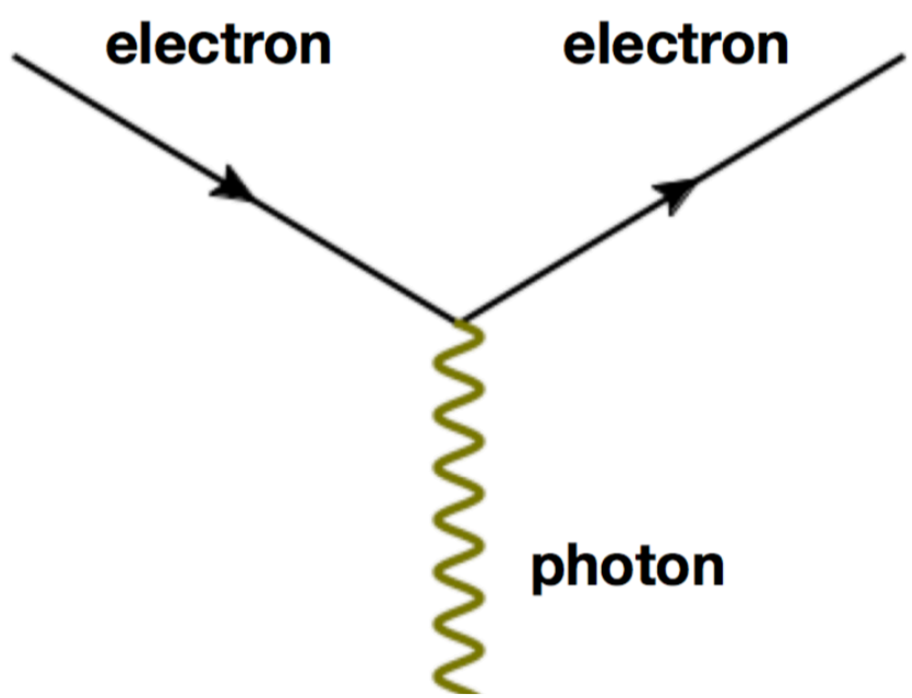


This fact implies the following **important properties** about the electromagnetic interaction:

☑ Electric charge is always conserved because the photon **does not carry electric charge**

Quantum Electromagnetism

In QED there is a unique **interaction vertex:**

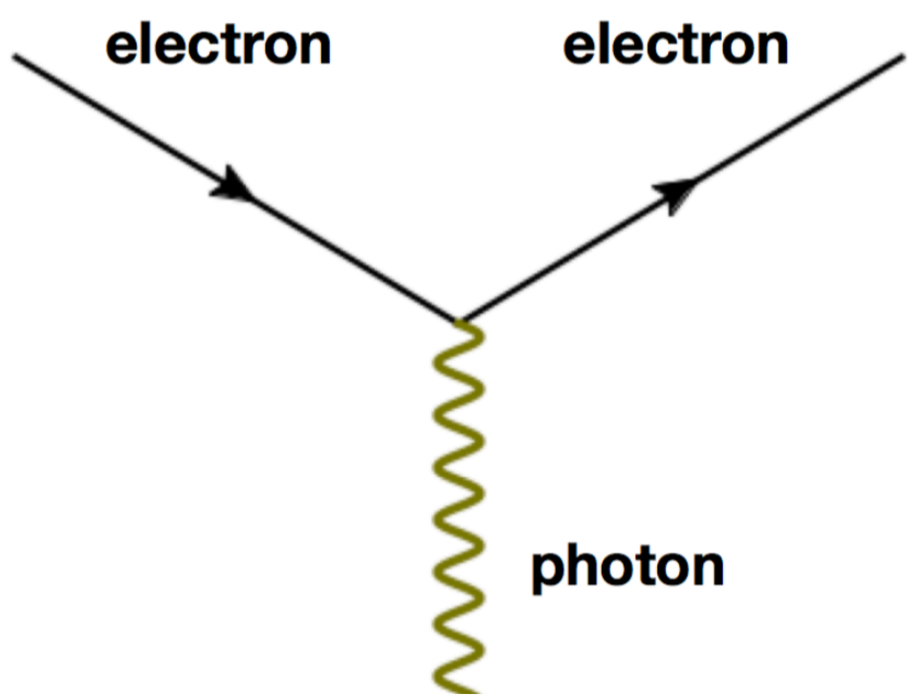


This fact implies the following **important properties** about the electromagnetic interaction:

- ☑ Electric charge is always conserved because the photon **does not carry electric charge**
- ☑ Being electrically neutral, the photon **cannot interact with itself**

Quantum Electromagnetism

In QED there is a unique **interaction vertex:**

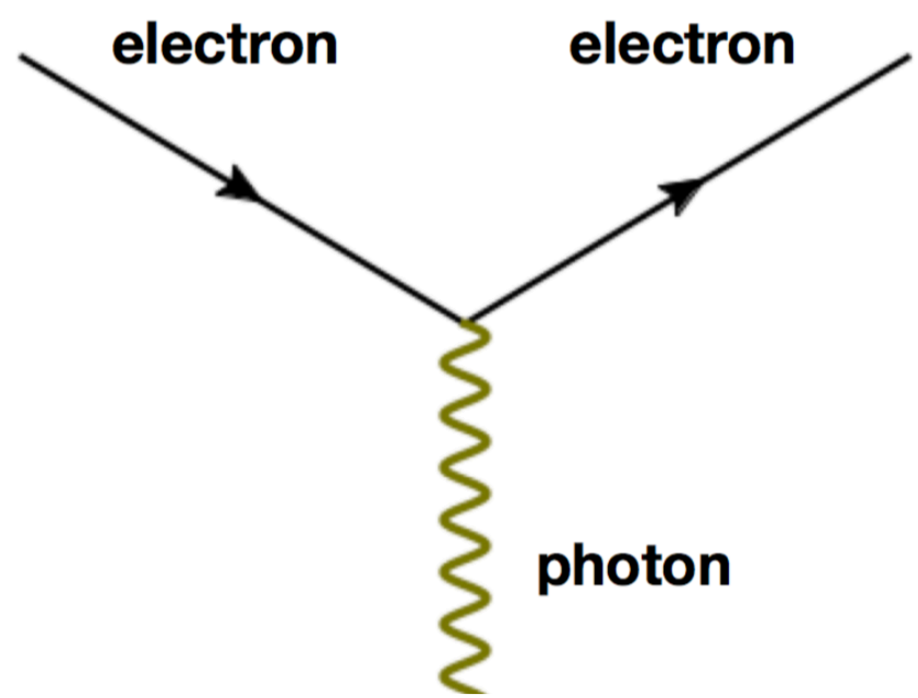


This fact implies the following **important properties** about the electromagnetic interaction:

- ☑ Electric charge is always conserved because the photon **does not carry electric charge**
- ☑ Being electrically neutral, the photon **cannot interact with itself**
- ☑ **Flavour** is conserved by QED interactions: automatic conservation of leptonic and baryonic numbers, as well as strangeness, charmness, and bottomness

Quantum Electromagnetism

In QED there is a unique **interaction vertex:**



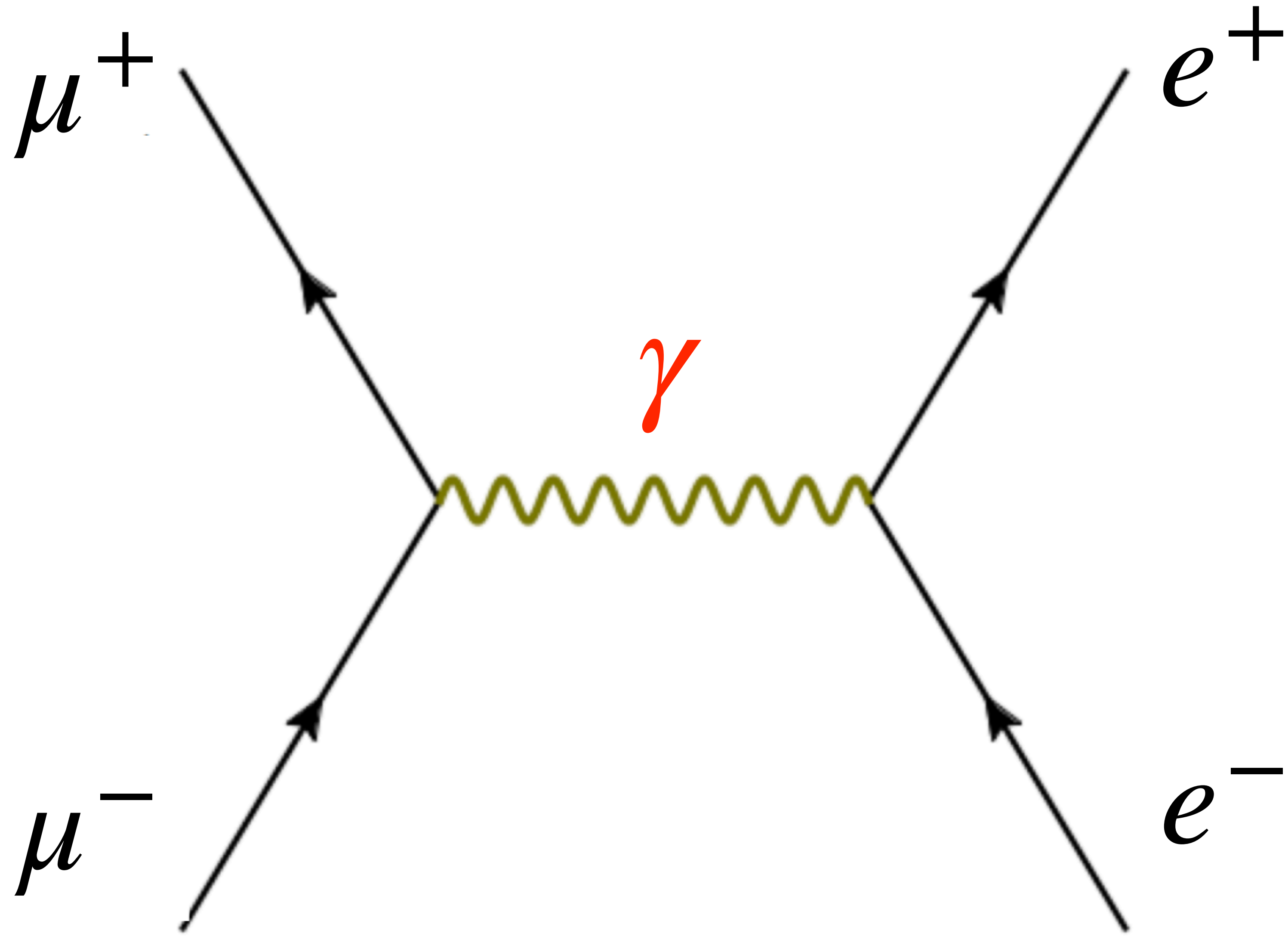
This fact implies the following **important properties** about the electromagnetic interaction:

- ☑ Electric charge is always conserved because the photon **does not carry electric charge**
- ☑ Being electrically neutral, the photon **cannot interact with itself**
- ☑ **Flavour** is conserved by QED interactions: automatic conservation of leptonic and baryonic numbers, as well as strangeness, charmness, and bottomness
- ☑ Since the photon is **exactly massless,** electromagnetism is a **long-range force**

Quantum Electromagnetism

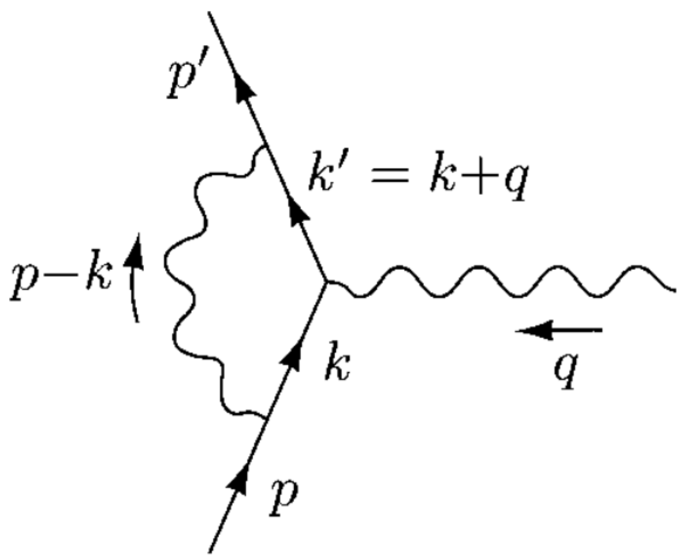
Quantum Electrodynamics can mediate interactions between any particles with **electric charge**

$$\mu^+ + \mu^- \rightarrow e^+ + e^-$$



Quantum Electromagnetism

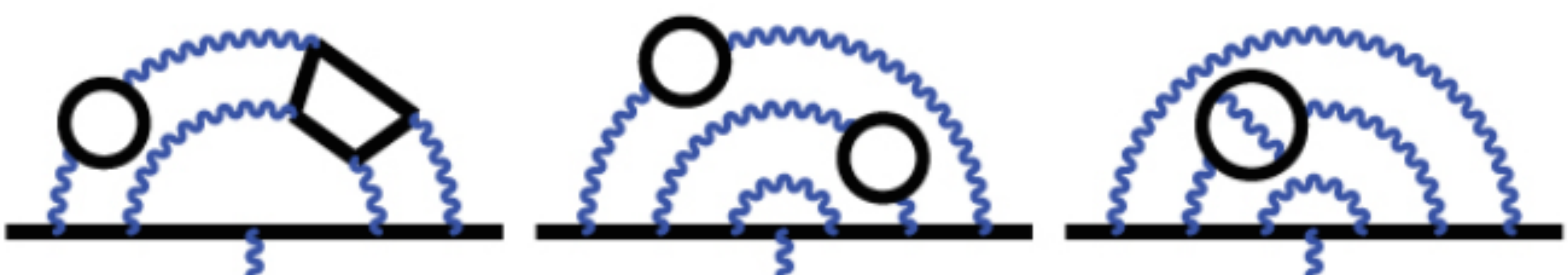
Quantum Electrodynamics used to compute **the most precise prediction** ever produced with any physical theory



The **anomalous magnetic moment** of the electron

$$a_{\text{exp}} = 0{,}00115965218073 \pm 0{,}00000000000028$$

$$a_{\text{teo}} = 0{,}00115965218178 \pm 0{,}00000000000077$$



Quantum Chromodynamics

Strong force vs electromagnetism

It is useful to enumerate the **properties of the strong interaction** by comparing them with those of the electromagnetic interactions

Electromagnetism

- A **single type of electric charge** exists: the only thing that varies is its sign and magnitude
- Electromagnetism is transmitted by **photons,** which are **massless** and **charge-neutral**
- The **strength of the electromagnetic interaction** is always small: electromagnetism looks the same at all energies/distances

Strong interactions

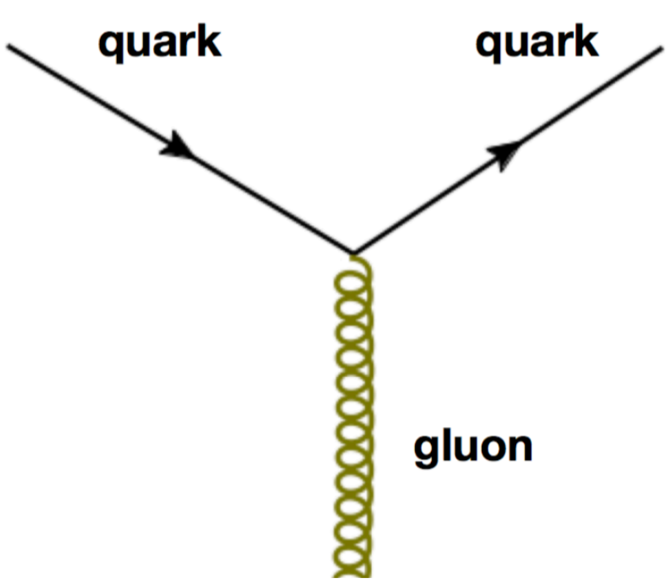
- **Three different types of colour charge** exist: ***blue, green, red,*** with their own sign and magnitude
- The strong interaction is transmitted by **gluons,** which are **massless** but **charged under color**
- The **strength of the strong interaction** varies with the energy/distance: very different behaviour depending on energy/distance

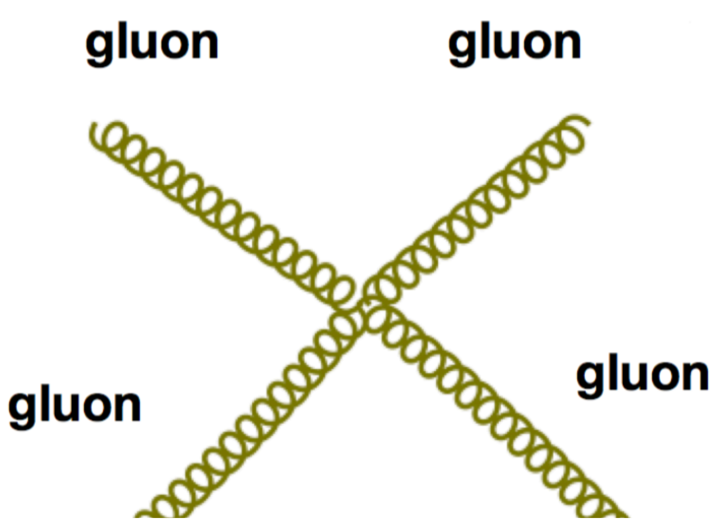
Quantum Chromodynamics

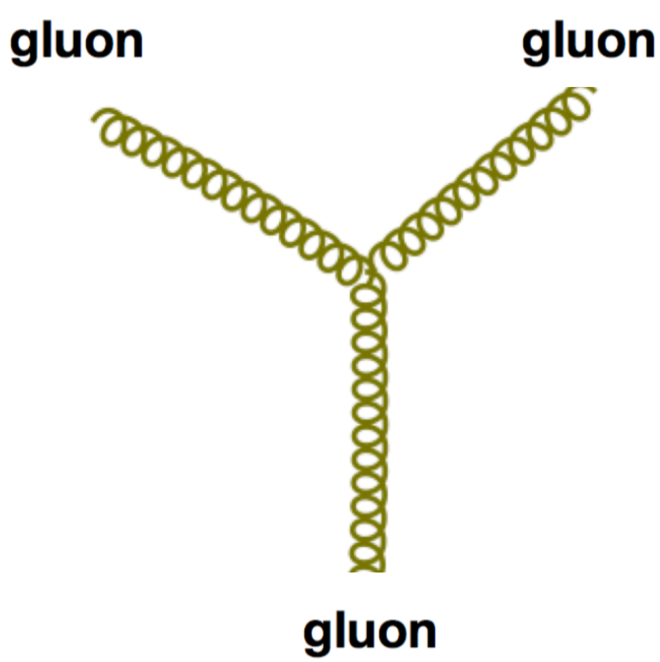
Let us summarise what we have learned about the **quantum theory of the strong interactions**: Quantum Chromodynamics (QCD)

- **Flavour** is always **conserved** by strong interactions: automatic conservation of leptonic and baryonic numbers, as well as strangeness, charmness, and bottomness

This is a consequence of the fact that the only possible **interaction vertices** are:







And that gluons do not carry **flavour quantum numbers**

Quantum Chromodynamics

Let us summarise what we have learned about the **quantum theory of the strong interactions**: Quantum Chromodynamics (QCD)

- ☑ **Flavour** is always **conserved** by strong interactions: automatic conservation of leptonic and baryonic numbers, as well as strangeness, charmness, and bottomness

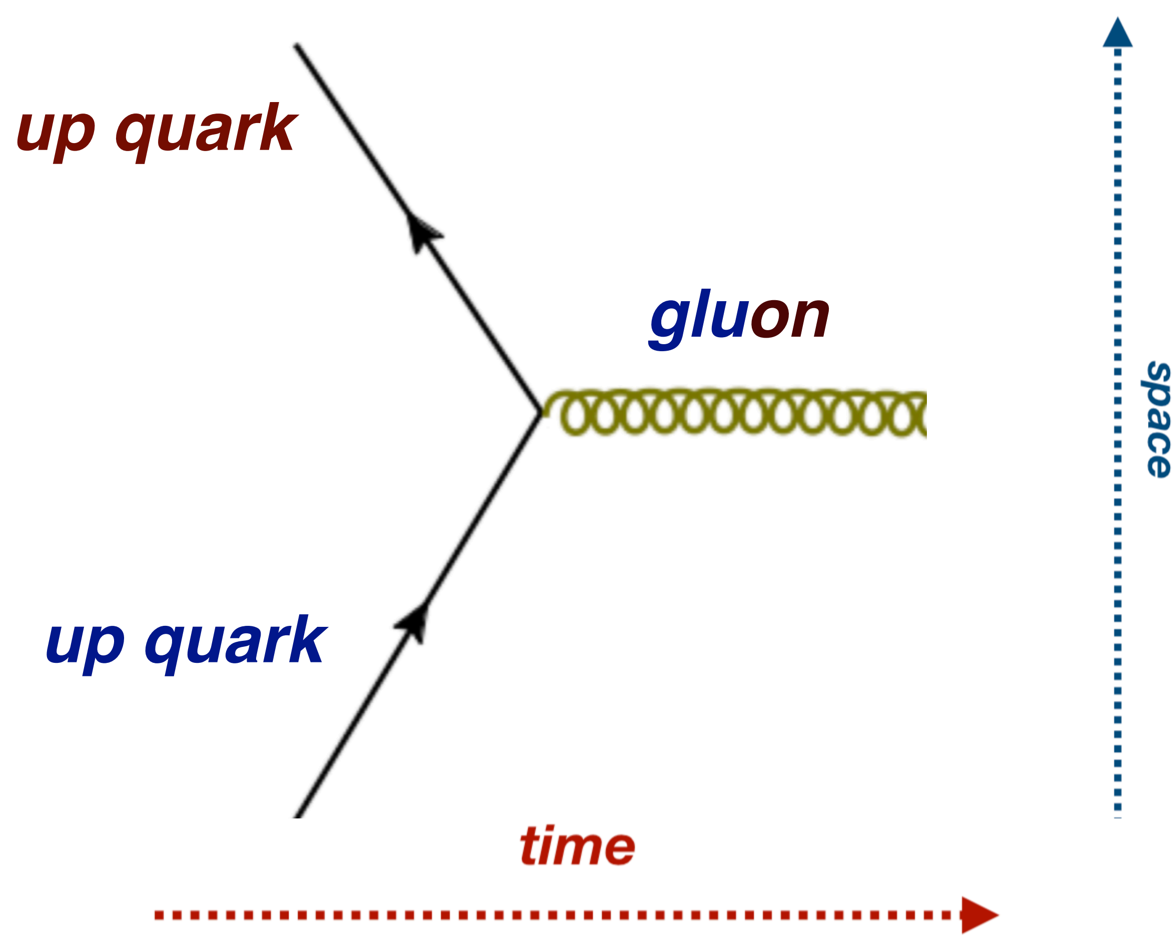
- ☑ Gluons are **charged under color** so they can interact with themselves. They are however **electrically neutral** to they don't affect the electric charge in strongly interacting processes

Quantum Chromodynamics

Let us summarise what we have learned about the **quantum theory of the strong interactions**: Quantum Chromodynamics (QCD)

- ☑ **Flavour** is always **conserved** by strong interactions: automatic conservation of leptonic and baryonic numbers, as well as strangeness, charmness, and bottomness

- ☑ Gluons are **charged under color** so they can interact with themselves. They are however **electrically neutral** to they don't affect the electric charge in strongly interacting processes



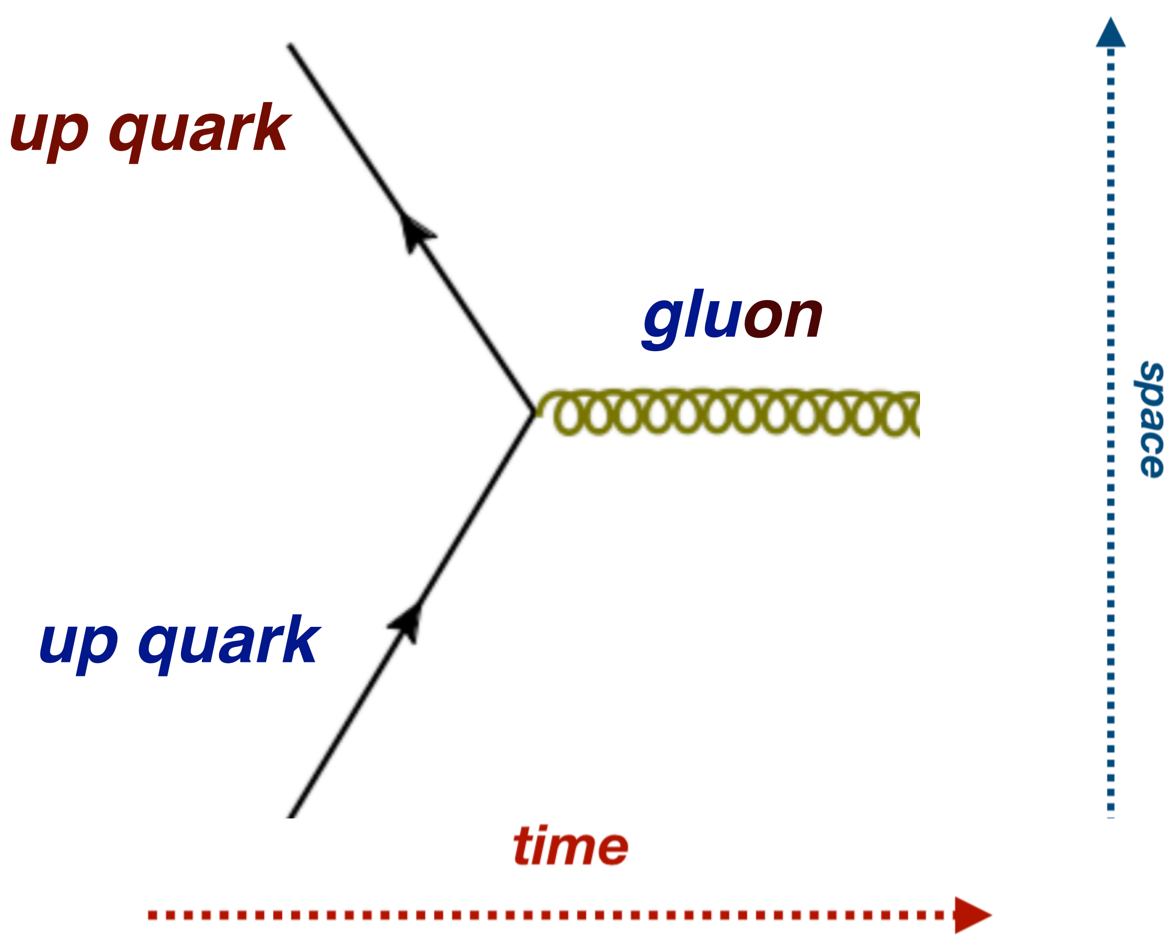
Is this reaction possible?

Quantum Chromodynamics

Let us summarise what we have learned about the **quantum theory of the strong interactions**: Quantum Chromodynamics (QCD)

- **Flavour** is always **conserved** by strong interactions: automatic conservation of leptonic and baryonic numbers, as well as strangeness, charmness, and bottomness

- Gluons are **charged under color** so they can interact with themselves. They are however **electrically neutral** to they don't affect the electric charge in strongly interacting processes



It is not possible since:

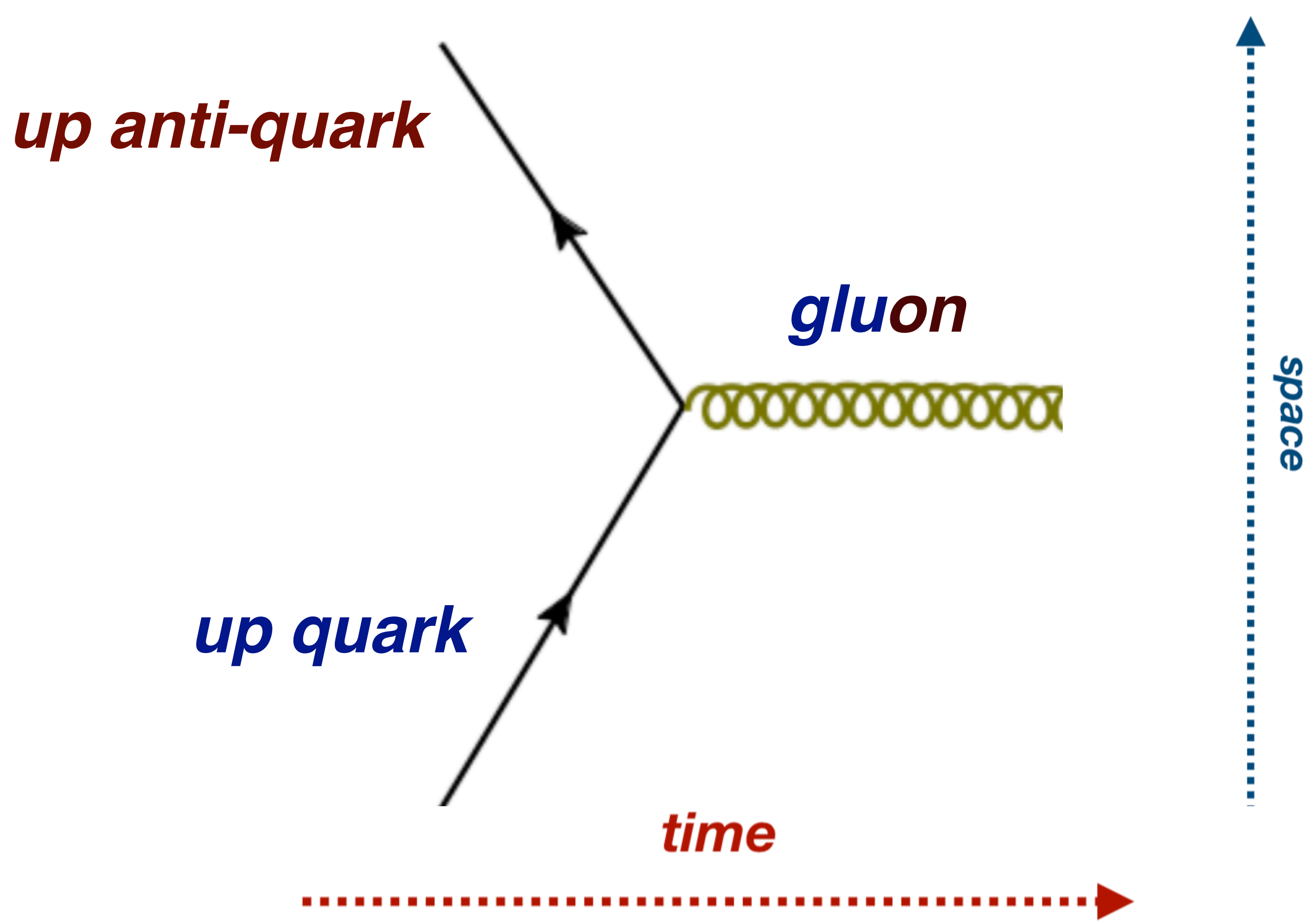
$$Q_{\rm in} = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \neq Q_{\rm fin} = 0$$

Quantum Chromodynamics

Let us summarise what we have learned about the **quantum theory of the strong interactions**: Quantum Chromodynamics (QCD)

- **Flavour** is always **conserved** by strong interactions: automatic conservation of leptonic and baryonic numbers, as well as strangeness, charmness, and bottomness

- Gluons are **charged under color** so they can interact with themselves. They are however **electrically neutral** to they don't affect the electric charge in strongly interacting processes



This reaction is instead possible:

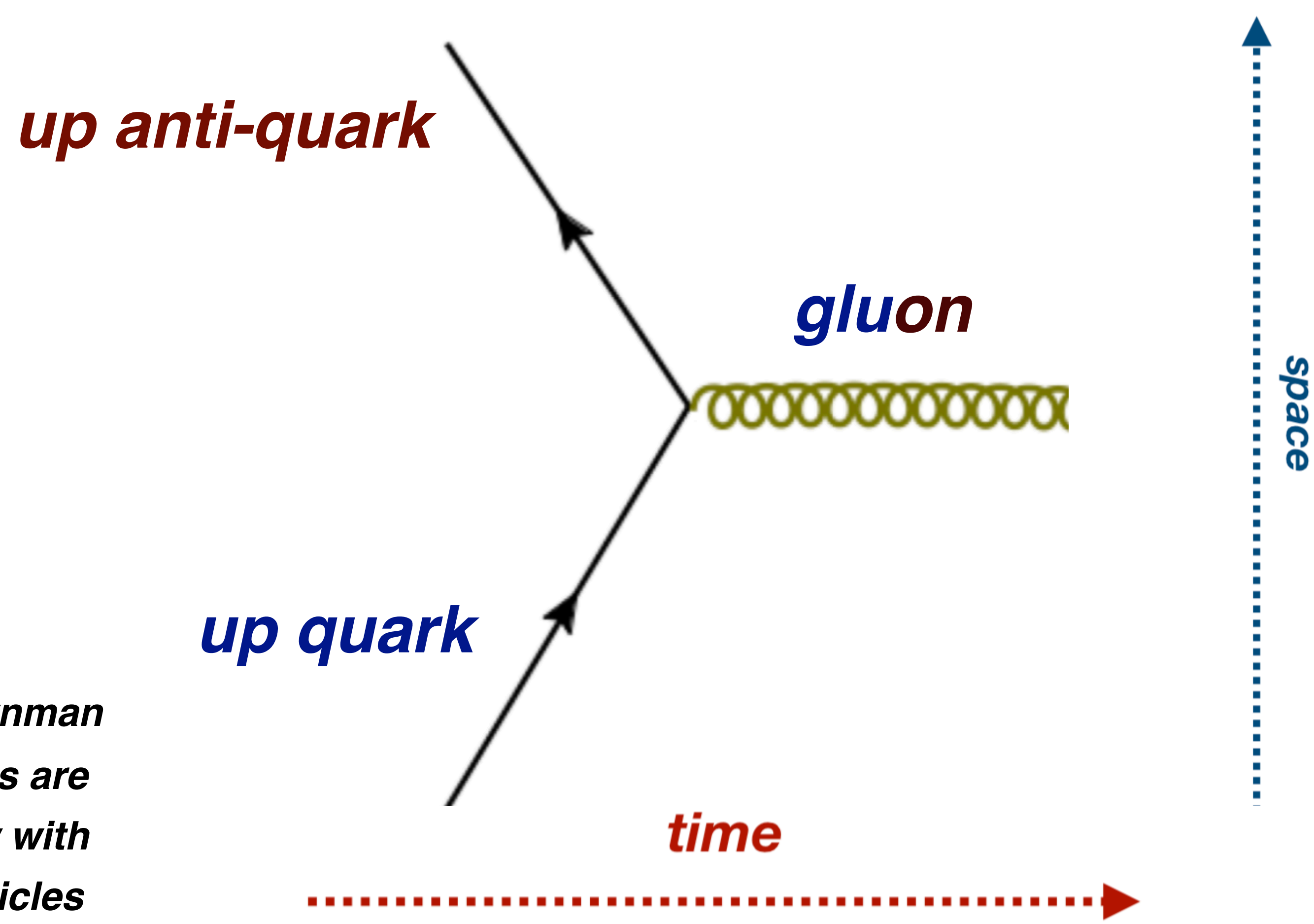
$$Q_{\rm in} = \frac{2}{3} - \frac{2}{3} = 0 = Q_{\rm fin}$$

Quantum Chromodynamics

Let us summarise what we have learned about the **quantum theory of the strong interactions**: Quantum Chromodynamics (QCD)

- **Flavour** is always **conserved** by strong interactions: automatic conservation of leptonic and baryonic numbers, as well as strangeness, charmness, and bottomness

- Gluons are **charged under color** so they can interact with themselves. They are however **electrically neutral** to they don't affect the electric charge in strongly interacting processes



This reaction is instead possible:

$$Q_{\rm in} = \frac{2}{3} - \frac{2}{3} = 0 = Q_{\rm fin}$$

by convention, in Feynman diagrams antiparticles are indicated by an arrow with opposite sign as particles

Quantum Chromodynamics

Let us summarise what we have learned about the **quantum theory of the strong interactions**: Quantum Chromodynamics (QCD)

- ☑ **Flavour** is always **conserved** by strong interactions: automatic conservation of leptonic and baryonic numbers, as well as strangeness, charmness, and bottomness

- ☑ Gluons are **charged under color** so they can interact with themselves. They are however **electrically neutral** to they don't affect the electric charge in strongly interacting processes

- ☑ The **strength** of the strong force is not constant: it is more at low energies / large distances (leading to quark confinement into hadrons) but less at high energies / low distances (where it behaves like electromagnetism)

Quantum Chromodynamics

Let us summarise what we have learned about the **quantum theory of the strong interactions**: Quantum Chromodynamics (QCD)

- ☑ **Flavour** is always **conserved** by strong interactions: automatic conservation of leptonic and baryonic numbers, as well as strangeness, charmness, and bottomness

- ☑ Gluons are **charged under color** so they can interact with themselves. They are however **electrically neutral** to they don't affect the electric charge in strongly interacting processes

- ☑ The **strength** of the strong force is not constant: it is more at low energies / large distances (leading to quark confinement into hadrons) but less at high energies / low distances (where it behaves like electromagnetism)

- ☑ While quarks have **fractional electric charge** and baryon number, only hadrons with **integer electric charge** and baryon number are physically allowed

Scattering reactions in QCD

Let's try to understand some **strong-interacting scattering processes** in terms of QCD

$$\pi^0 + p \to n + \pi^+$$

Write the corresponding Feynman diagram using **only quarks and gluons**

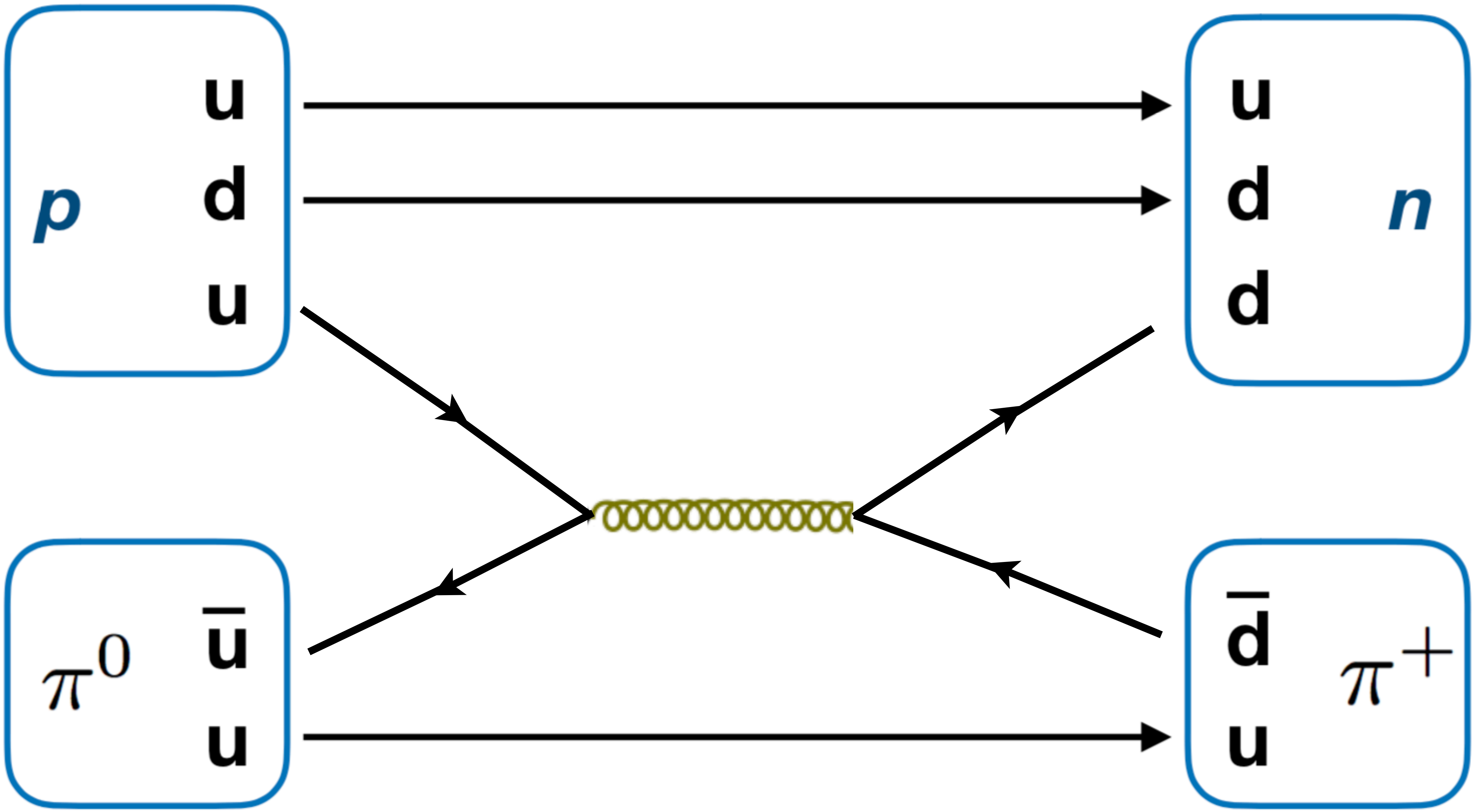
$$\pi^0 = (u\,\bar{u}) \qquad\qquad \pi^+ = \left(u\,\bar{d}\right)$$

Note also how ***Q, B, S, C, …*** are conserved in this reaction

Scattering reactions in QCD

Let's try to understand some **strong-interacting scattering processes** in terms of QCD

$$\pi^0 + p \to n + \pi^+$$



Scattering reactions in QCD

Let's try to understand some **strong-interacting scattering processes** in terms of QCD

$$\rho^+ \to \pi^+ + \pi^0$$

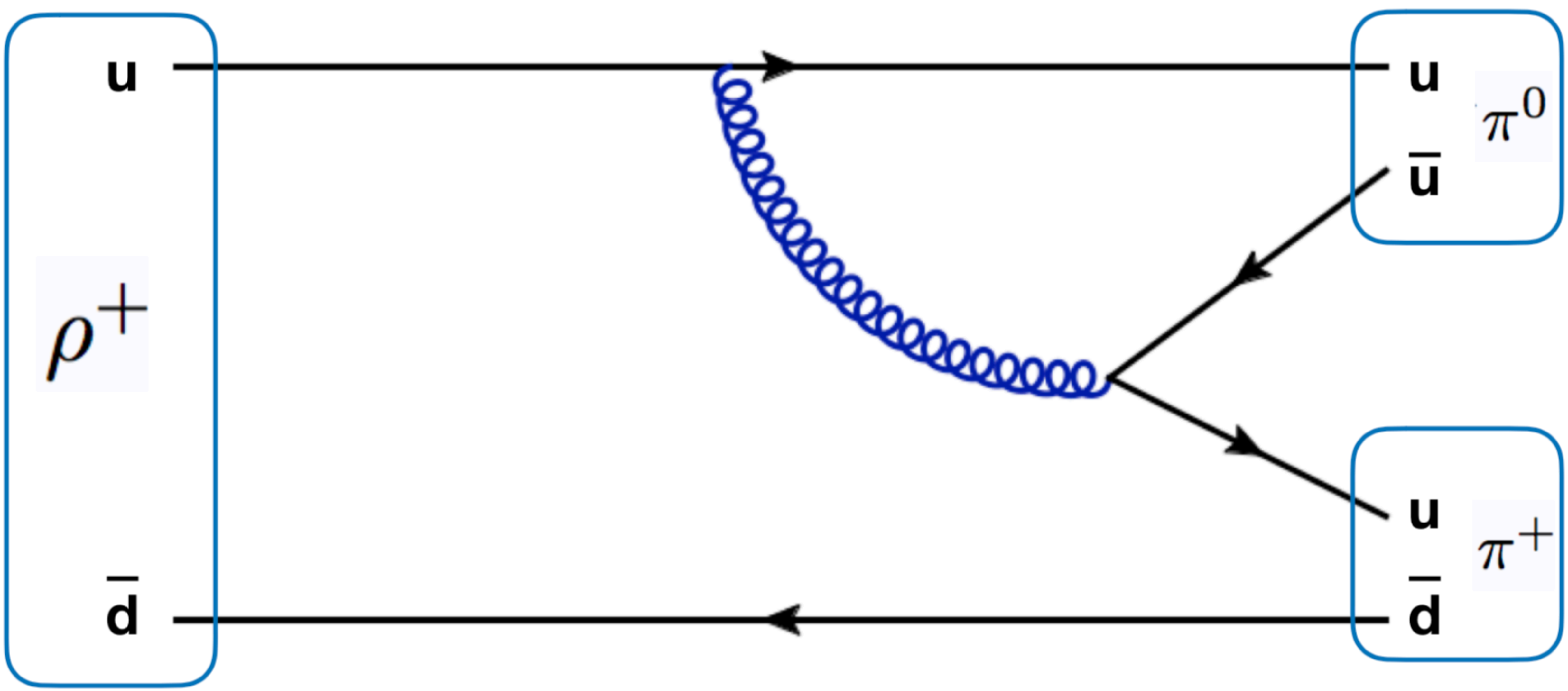
Write the corresponding Feynman diagram using **only quarks and gluons**

$$\pi^0 = (u\,\bar{u}) \qquad\qquad \pi^+ = \left(u\,\bar{d}\right)$$

Scattering reactions in QCD

Let's try to understand some **strong-interacting scattering processes** in terms of QCD

$$\rho^+ \to \pi^+ + \pi^0$$



Note also how ***Q, B, S, C, …*** are conserved in this reaction

Electroweak Theory

Weak force vs electromagnetism

It is useful to enumerate the **properties of the weak interaction** by comparing them with those of the electromagnetic interactions

Electromagnetism

- ☑ A **single type of electric charge** exists: the only thing that varies is its sign and magnitude
- ☑ Electromagnetism is transmitted by **photons,** which are **massless** and **charge-neutral**
- ☑ The **strength of the electromagnetic interaction** is always small: electromagnetism looks the same at all energies/distances

Weak interactions

- ☑ All particles in the SM are carry a **weak charge**, and the specific values **depend on the matter particle**
- ☑ The strong interaction is transmitted by the ***W*** and ***Z*** bosons, which are **massive** and charged under the weak force
- ☑ The weak interaction is always weak and confined to small scales (large value of $m_{W,Z}$)

$$\text{range} : \Delta r \sim m^{-1} \quad \begin{cases} \Delta r \simeq \infty \quad (\text{EM}, \text{QCD}) \\ \Delta r \simeq 10^{-18}\,\text{m} \quad (\text{weak}) \end{cases}$$

The weak boson *W*

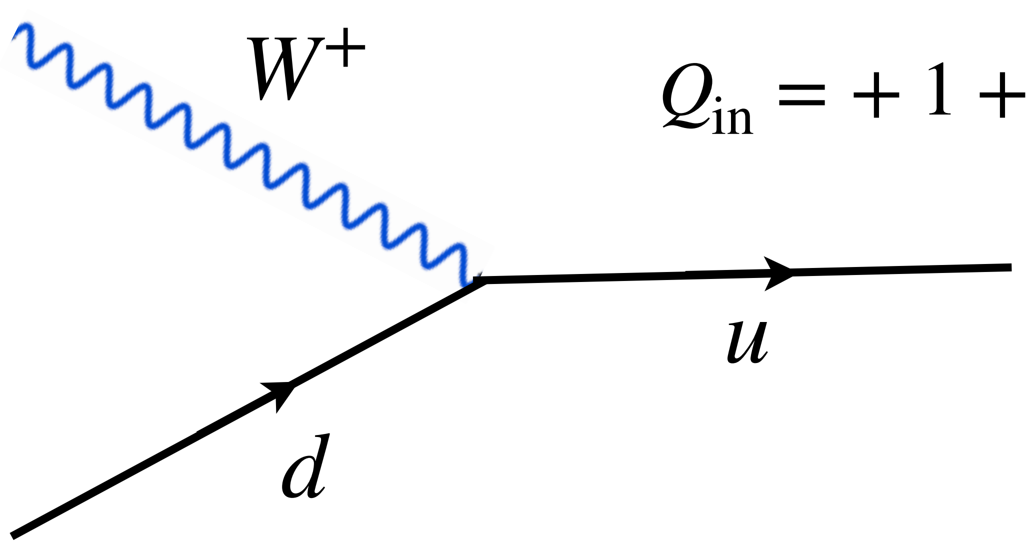
The weak interactions are mediated by three massive bosons: $\boldsymbol{W^+}$, $\boldsymbol{W^+}$, $\boldsymbol{Z^0}$

The main properties of the ***W* bosons** are:

- As opposed to the **massless** gluons and photons, the W boson is **very massive,** around 80 times the proton mass

- As in the case of the gluons (but not the photons), the *W* boson is charged under both **electric** and **weak** charges, and therefore can interact with **itself**

- When interacting with **quarks,** the *W* boson will change its charge by **one unit** and therefore also **its flavour** (including possibly across generations)



$$Q_{\text{in}} = +1 + (-1/3) = +2/3 = Q_{\text{fin}}$$

The weak boson *W*

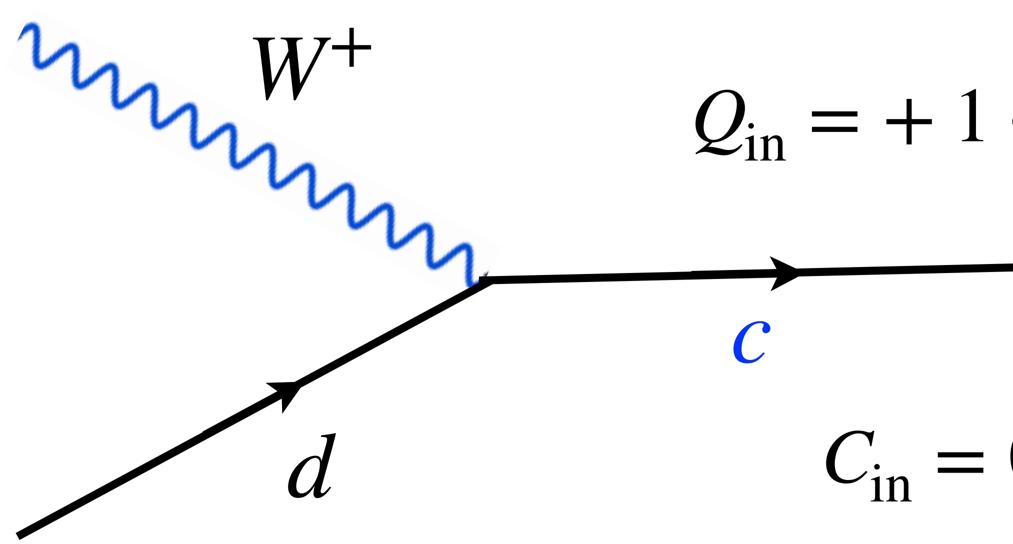
The weak interactions are mediated by three massive bosons: ***W^+*, *W^+*, *Z^0***

The main properties of the ***W* bosons** are:

- ☑ As opposed to the **massless** gluons and photons, the W boson is **very massive,** around 80 times the proton mass

- ☑ As in the case of the gluons (but not the photons), the *W* boson is charged under both **electric** and **weak** charges, and therefore can interact with **itself**

- ☑ When interacting with **quarks,** the *W* boson will change its charge by **one unit** and therefore also **its flavour** (including possibly across generations)



$$Q_{\rm in} = +1 + (-1/3) = +2/3 = Q_{\rm fin}$$

$$C_{\rm in} = 0 \neq C_{\rm fin} = +1$$

The weak boson *W*

The weak interactions are mediated by three massive bosons: $\boldsymbol{W^+}$, $\boldsymbol{W^+}$, $\boldsymbol{Z^0}$

The main properties of the ***W* bosons** are:

- ☑ As opposed to the **massless** gluons and photons, the W boson is **very massive,** around 80 times the proton mass
- ☑ As in the case of the gluons (but not the photons), the *W* boson is charged under both **electric** and **weak** charges, and therefore can interact with **itself**
- ☑ When interacting with **quarks,** the *W* boson will change its charge by **one unit** and therefore also **its flavour** (including possibly across generations)
- ☑ In weak interaction processes **mediated by the *W* boson,** the flavour quantum numbers (strangeness, charmness, botomness) are **not conserved** quantities

The weak boson *W*

Taking into account these properties, some of the physically allowed reactions involving **quarks** and ***W* bosons** will be:

$$u + W^- \to d\,, \quad u + W^- \to s\,, \quad d + W^+ \to u\,, \quad s + W^+ \to u\,,$$

$$\bar{u} + W^+ \to \bar{d}\,, \quad \bar{u} + W^+ \to \bar{s}\,, \quad \bar{d} + W^- \to \bar{u}\,, \quad \bar{s} + W^- \to \bar{u}\,,$$

$$W^+ \to u + \bar{d}\,, \quad W^+ \to u + \bar{s}\,, \quad W^- \to d + \bar{u}\,, \quad W^- \to s + \bar{u}\,,$$

- ☑ **Electric charge** is always conserved

- ☑ You can always **replace** a given quark by the corresponding quark of a **different generation**: for example a down antiquark by a strange antiquark

- ☑ If a given reaction is allowed, the corresponding reaction involving the **antiparticles** is also physically allowed

$$\bar{u} + W^+ \to \bar{s} \quad \Rightarrow \quad u + W^- \to s$$

The weak boson *W*

Taking into account these properties, some of the physically allowed reactions involving **leptons** and ***W* bosons** will be:

$$e^+ + W^- \to \bar{\nu}_e \,, \quad e^- + W^+ \to \nu_e \,, \quad \nu_e + W^+ \to e^- \,, \quad \bar{\nu}_e + W^+ \to e^+$$

$$W^+ \to e^+ + \nu_e \,, \quad W^- \to e^- + \bar{\nu}_e \,, \quad e^+ + \nu_e \to W^+ \,, \quad e^- + \bar{\nu}_e \to W^-$$

The weak boson *W*

Taking into account these properties, some of the physically allowed reactions involving **leptons** and ***W* bosons** will be:

$$e^+ + W^- \to \bar{\nu}_e \,, \quad e^- + W^+ \to \nu_e \,, \quad \nu_e + W^+ \to e^- \,, \quad \bar{\nu}_e + W^+ \to e^+$$

$$W^+ \to e^+ + \nu_e \,, \quad W^- \to e^- + \bar{\nu}_e \,, \quad e^+ + \nu_e \to W^+ \,, \quad e^- + \bar{\nu}_e \to W^-$$

☑ **Electric charge** is always conserved

The weak boson *W*

Taking into account these properties, some of the physically allowed reactions involving **leptons** and ***W* bosons** will be:

$$e^+ + W^- \to \bar{\nu}_e\,, \quad e^- + W^+ \to \nu_e\,, \quad \nu_e + W^+ \to e^-\,, \quad \bar{\nu}_e + W^+ \to e^+$$

$$W^+ \to e^+ + \nu_e\,, \quad W^- \to e^- + \bar{\nu}_e\,, \quad e^+ + \nu_e \to W^+\,, \quad e^- + \bar{\nu}_e \to W^-$$

- **Electric charge** is always conserved

- Each interaction vertex involves a **charged** and a **neutral lepton** that belong to the **same lepton generation**

The weak boson *W*

Taking into account these properties, some of the physically allowed reactions involving **leptons** and ***W* bosons** will be:

$$e^+ + W^- \to \bar{\nu}_e \,, \quad e^- + W^+ \to \nu_e \,, \quad \nu_e + W^+ \to e^- \,, \quad \bar{\nu}_e + W^+ \to e^+$$

$$W^+ \to e^+ + \nu_e \,, \quad W^- \to e^- + \bar{\nu}_e \,, \quad e^+ + \nu_e \to W^+ \,, \quad e^- + \bar{\nu}_e \to W^-$$

- **Electric charge** is always conserved

- Each interaction vertex involves a **charged** and a **neutral lepton** that belong to the **same lepton generation**

- You can always **replace** the two leptons of a given generation for the corresponding two leptons of **another generation**

$$e^+ + W^- \to \bar{\nu}_e \quad \Rightarrow \quad \tau^+ + W^- \to \bar{\nu}_\tau$$

The weak boson *W*

Taking into account these properties, some of the physically allowed reactions involving **leptons** and ***W* bosons** will be:

$$e^+ + W^- \to \bar{\nu}_e\,, \quad e^- + W^+ \to \nu_e\,, \quad \nu_e + W^+ \to e^-\,, \quad \bar{\nu}_e + W^+ \to e^+$$

$$W^+ \to e^+ + \nu_e\,, \quad W^- \to e^- + \bar{\nu}_e\,, \quad e^+ + \nu_e \to W^+\,, \quad e^- + \bar{\nu}_e \to W^-$$

- **Electric charge** is always conserved
- Each interaction vertex involves a **charged** and a **neutral lepton** that belong to the **same lepton generation**
- You can always **replace** the two leptons of a given generation for the corresponding two leptons of **another generation**
- The **individual leptonic quantum numbers** are always conserved in weak reactions

The weak boson W

Draw the Feynman diagram for the following process

$$\pi^+ \to \mu^+ + \nu_\mu \qquad \pi^+ = \left(u\,\bar{d}\right)$$

The weak boson *W*

Draw the Feynman diagram for the following process

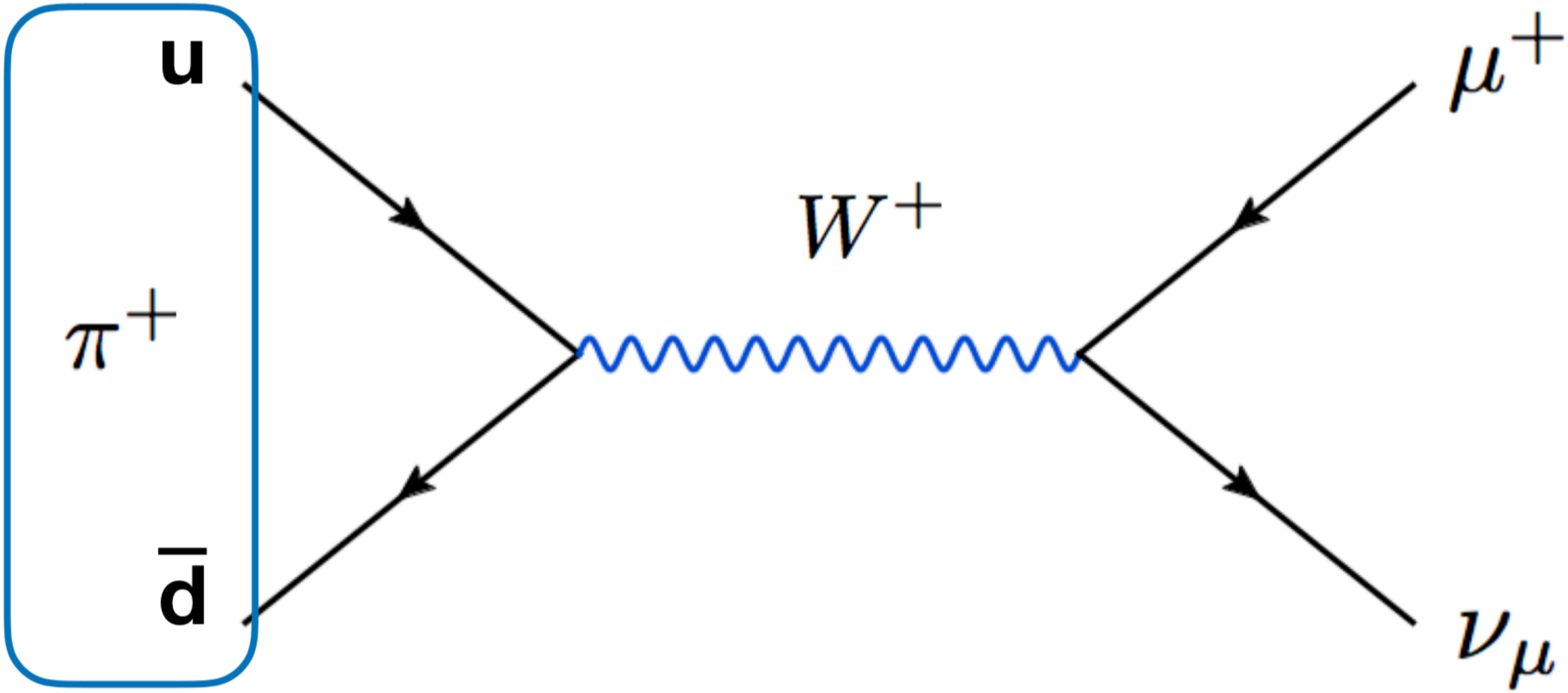
$$\pi^+ \to \mu^+ + \nu_\mu \qquad \pi^+ = \left(u\,\bar{d}\right)$$

- We have a neutrino in the final state: the **weak interaction** must be involved
- Quarks and leptons only interact indirectly via either photons or *W, Z* bosons
- Since the electric charge is ***Q=+-1,*** then a positively charged ***W boson*** is involved
- We know what **vertices are allowed** involving quarks or leptons and a *W* boson

The weak boson *W*

Draw the Feynman diagram for the following process

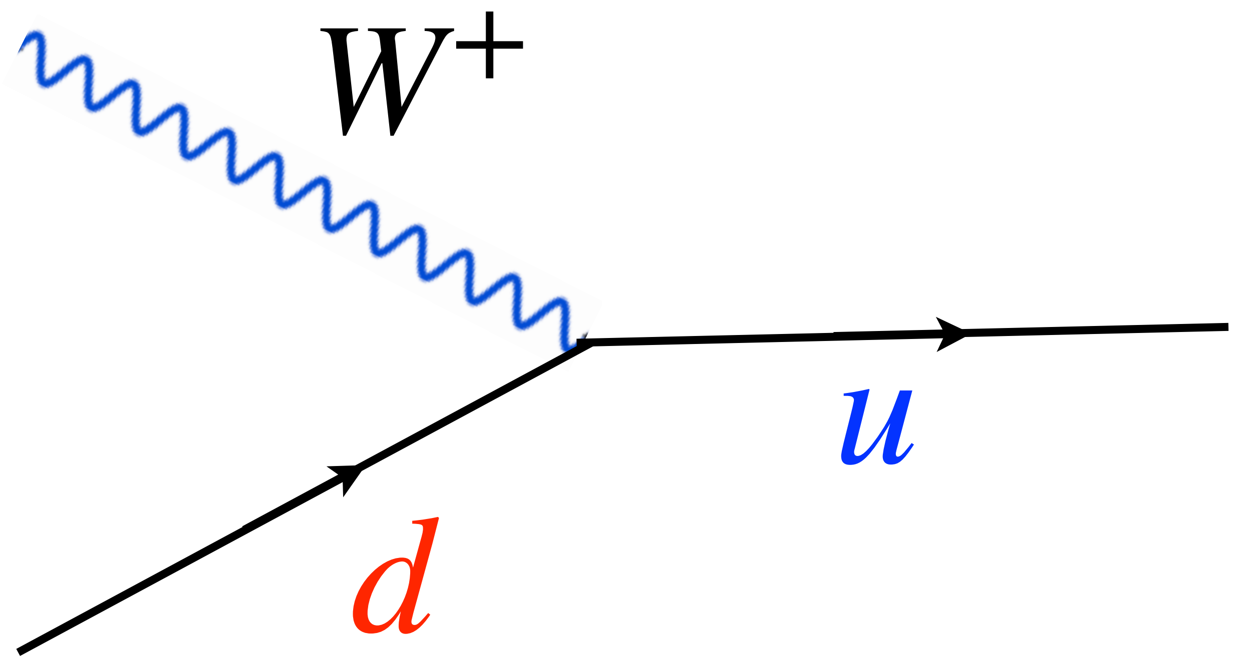
$$\pi^+ \rightarrow \mu^+ + \nu_\mu \qquad \pi^+ = \left(u\,\bar{d}\right)$$



You can check that all relevant quantum numbers are **conserved:** *L, B, Q, …*

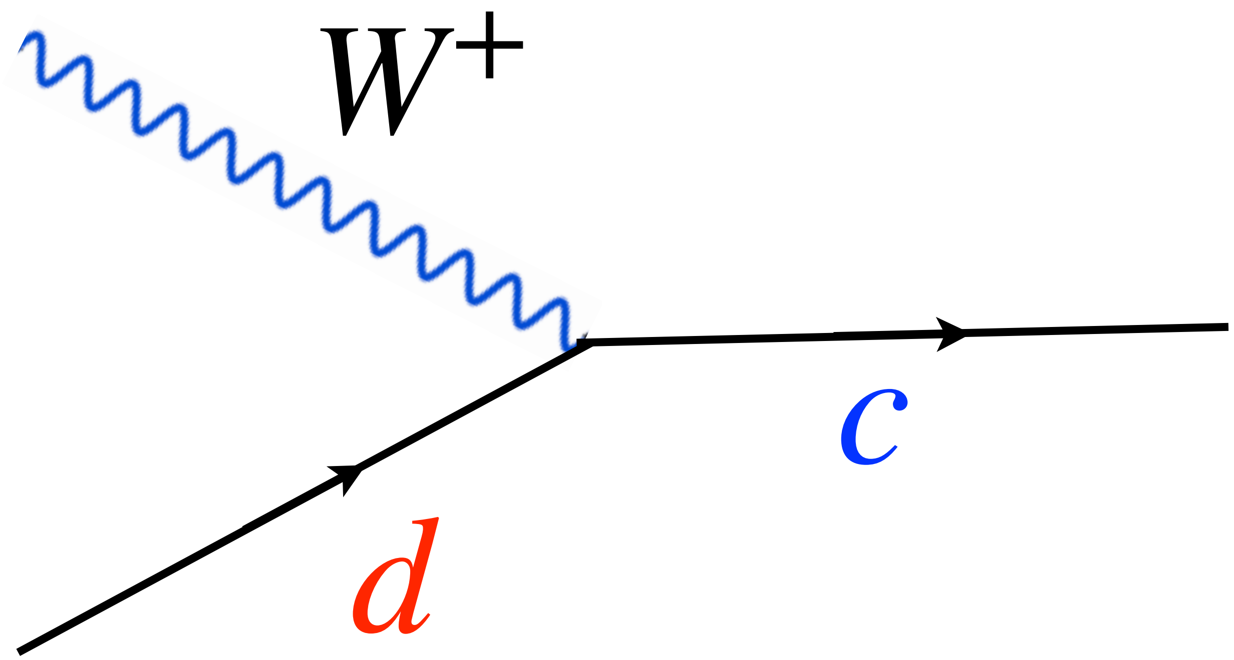
Weak coupling between generations

We have seen that in processes mediated by the weak gauge boson *W* the **flavour** of the quarks will **change**



Weak coupling between generations

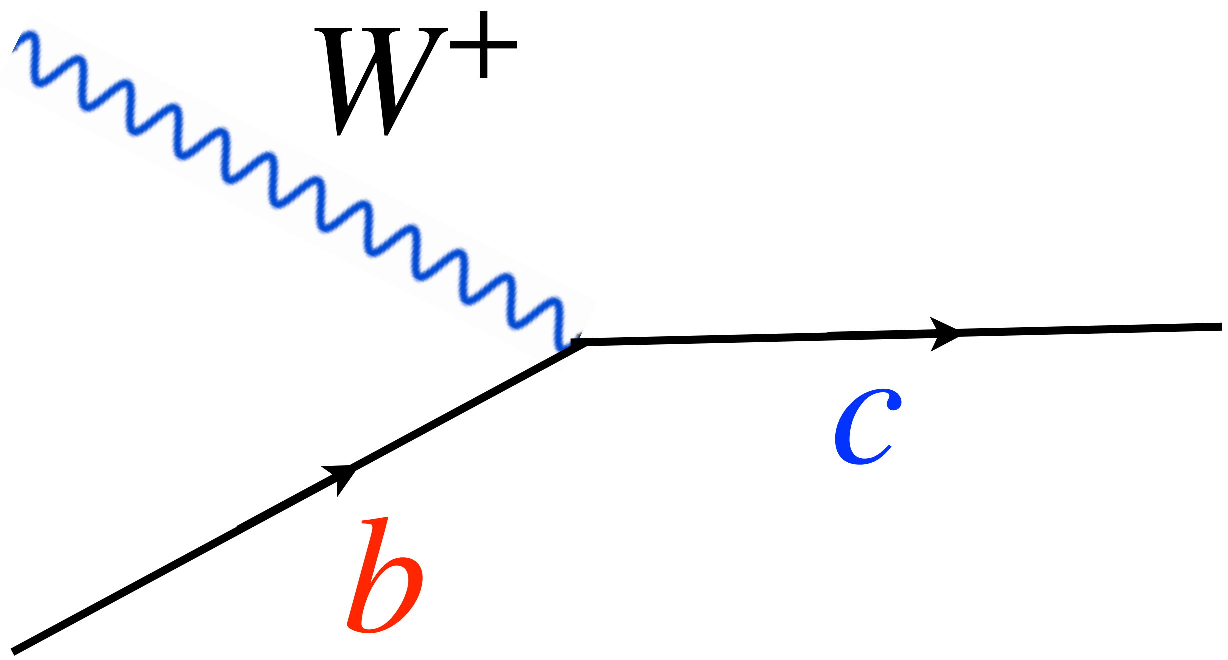
We have seen that in processes mediated by the weak gauge boson W the **flavour** of the quarks will **change**



Moreover we can always replace a given quark by the corresponding quark of a **different generation**

Weak coupling between generations

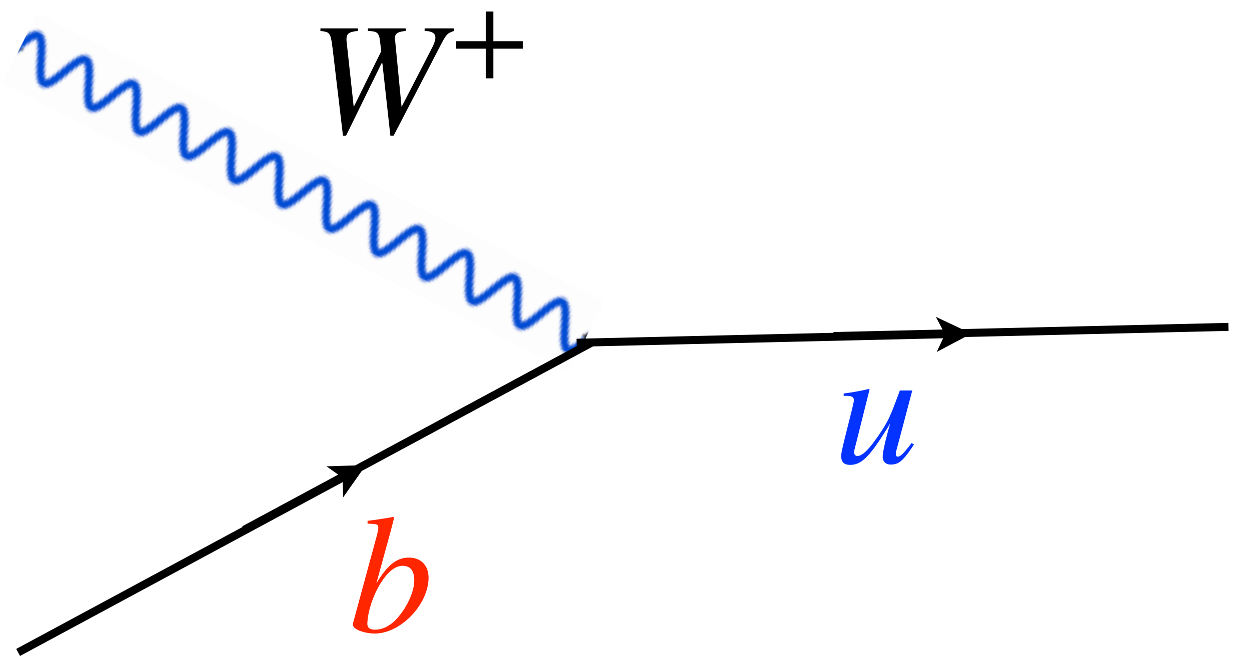
We have seen that in processes mediated by the weak gauge boson *W* the **flavour** of the quarks will **change**



Moreover we can always replace a given quark by the corresponding quark of a **different generation**

Weak coupling between generations

We have seen that in processes mediated by the weak gauge boson *W* the **flavour** of the quarks will **change**

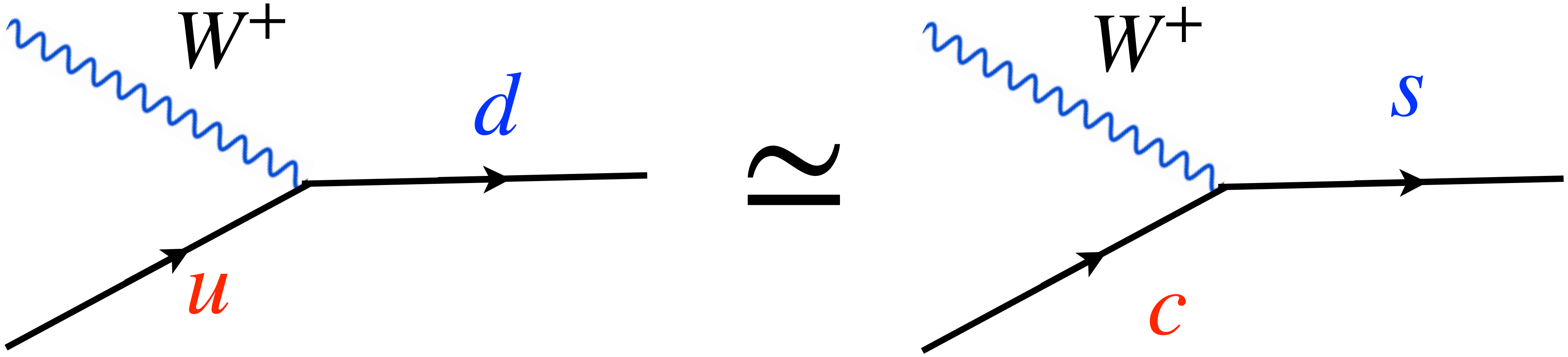


Moreover we can always replace a given quark by the corresponding quark of a **different generation**

The weak interactions mediates **transitions** between **quarks of different generations**

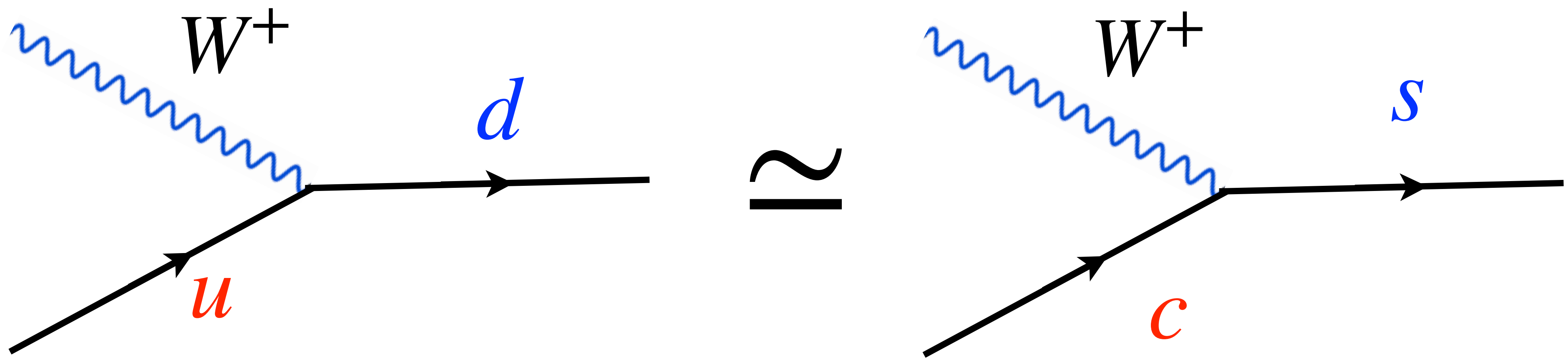
Weak coupling between generations

The **strength of the weak coupling** is similar between **quarks of the same generation**

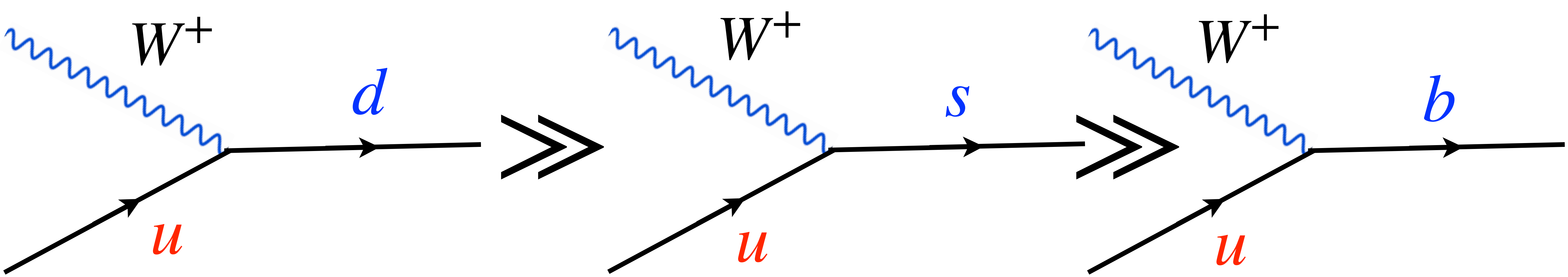


Weak coupling between generations

The **strength of the weak coupling** is similar between **quarks of the same generation**



The **strength of the weak coupling** is smaller between **quarks of different generation**



Weak coupling between **gens 1 and 2** bigger than between **gens 1 and 3**

Heavy hadron decays

This **hierarchy of the weak couplings** between quark generations is particularly important in order to understand the decays of **hadrons** that contain **heavy quarks**

$$B^0 \to D^- + \mu^+ + \nu_\mu \qquad B^0 = \left(d\,\bar{b}\right) \qquad D^- = \left(d\bar{c}\right)$$

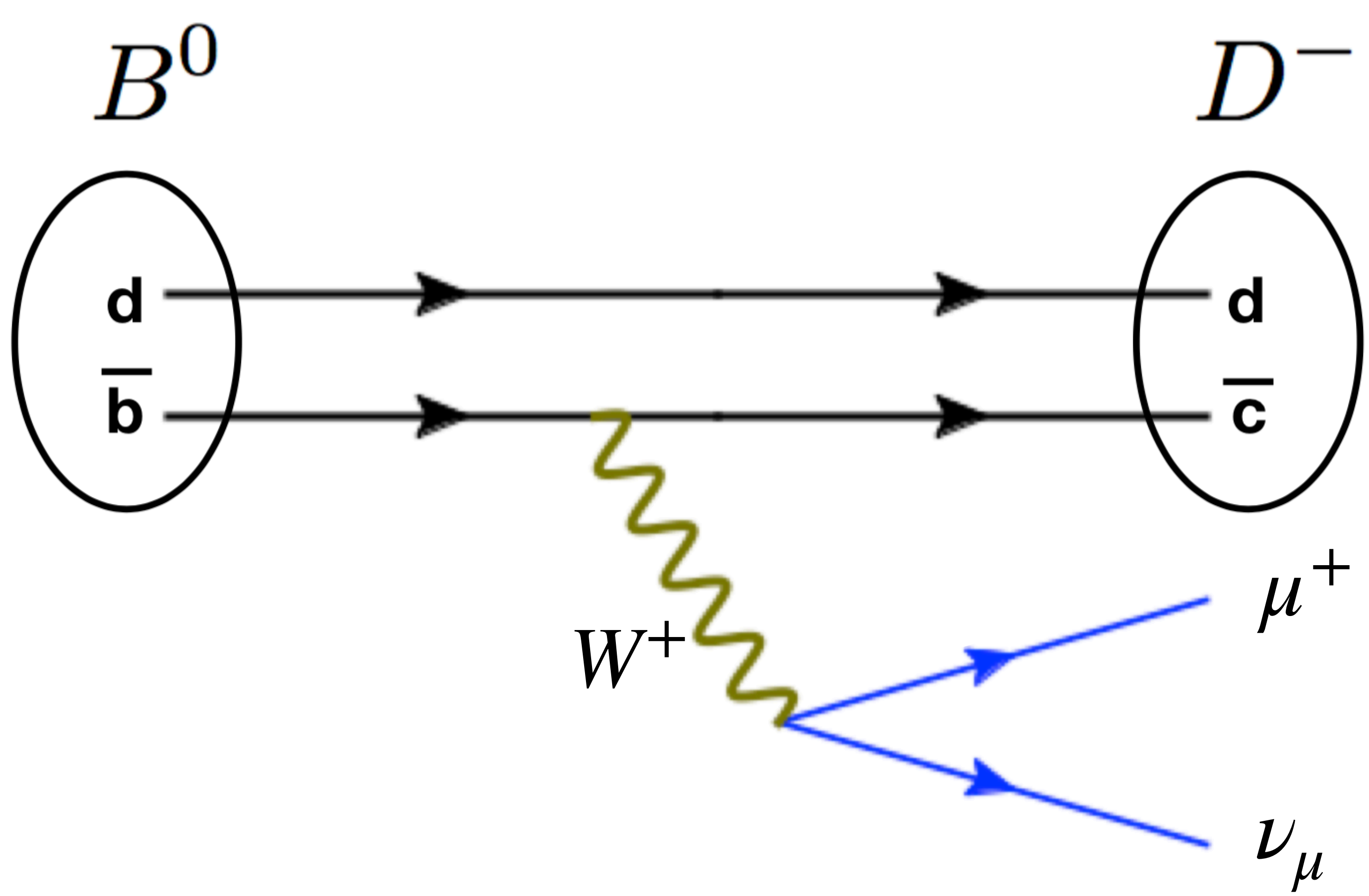
what is the corresponding Feynman diagram?

Heavy hadron decays

This **hierarchy of the weak couplings** between quark generations is particularly important in order to understand the decays of **hadrons** that contain **heavy quarks**

$$B^0 \to D^- + \mu^+ + \nu_\mu \qquad B^0 = \left(d\,\bar{b}\right) \qquad D^- = \left(d\bar{c}\right)$$

what is the corresponding Feynman diagram?



Heavy hadron decays

This **hierarchy of the weak couplings** between quark generations is particularly important in order to understand the decays of **hadrons** that contain **heavy quarks**

$$B^0 \to \pi^- + \mu^+ + \nu_\mu \qquad B^0 = \left(d\,\bar{b}\right) \qquad \pi^- = \left(d\bar{u}\right)$$

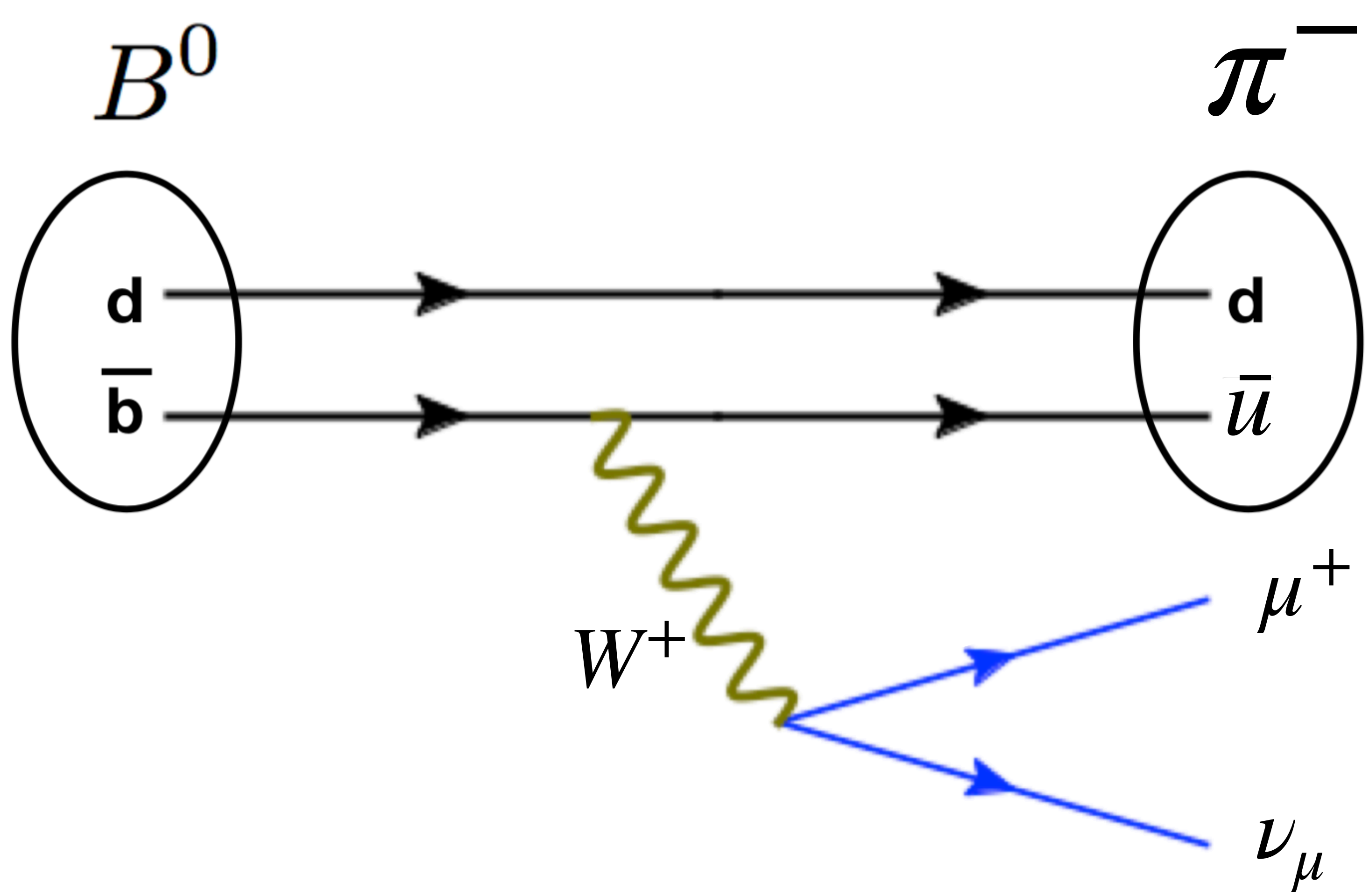
what is the corresponding Feynman diagram?

Heavy hadron decays

This **hierarchy of the weak couplings** between quark generations is particularly important in order to understand the decays of **hadrons** that contain **heavy quarks**

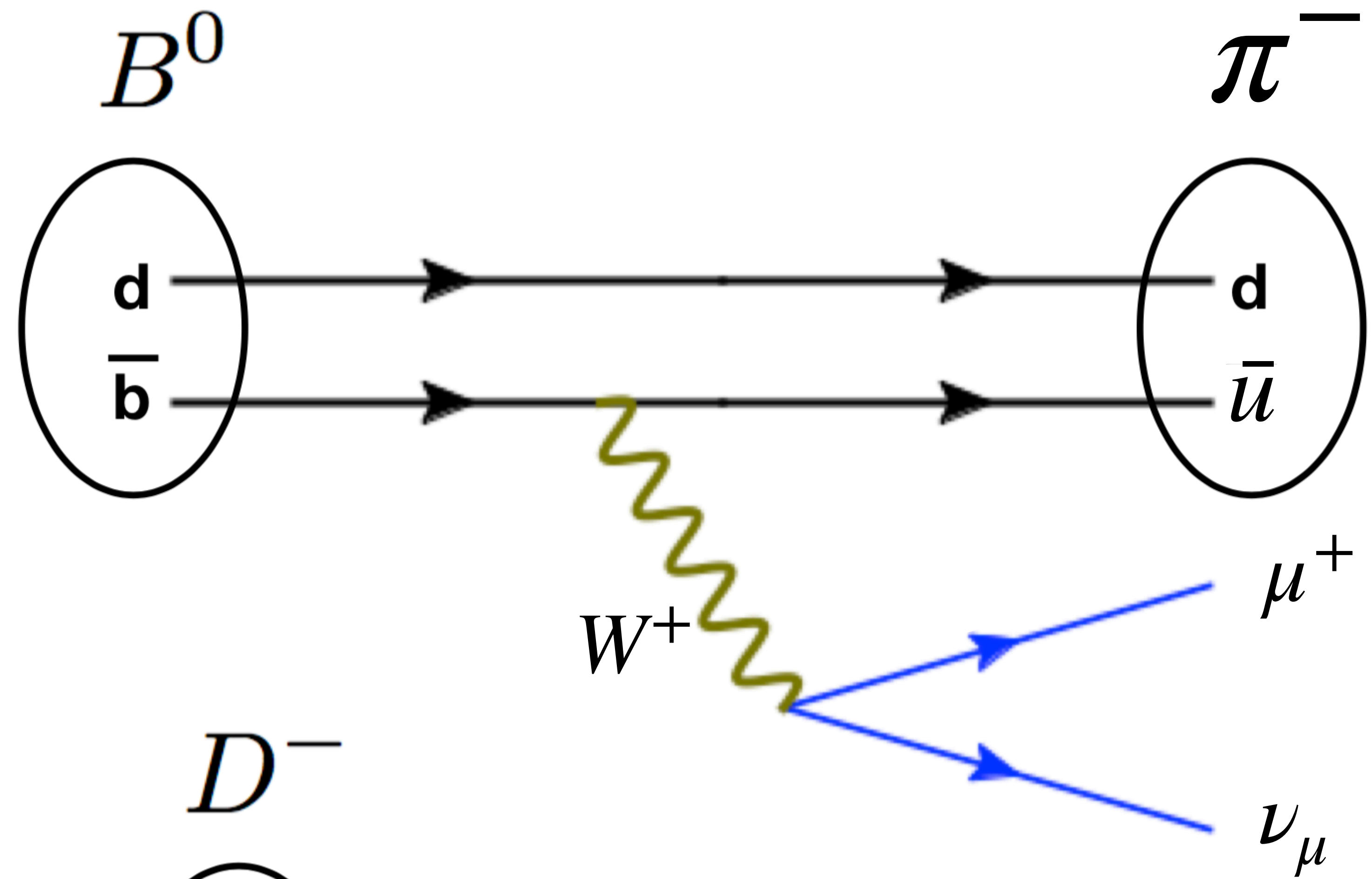
$$B^0 \to \pi^- + \mu^+ + \nu_\mu \qquad B^0 = \left(d\,\bar{b}\right) \qquad \pi^- = \left(d\bar{u}\right)$$

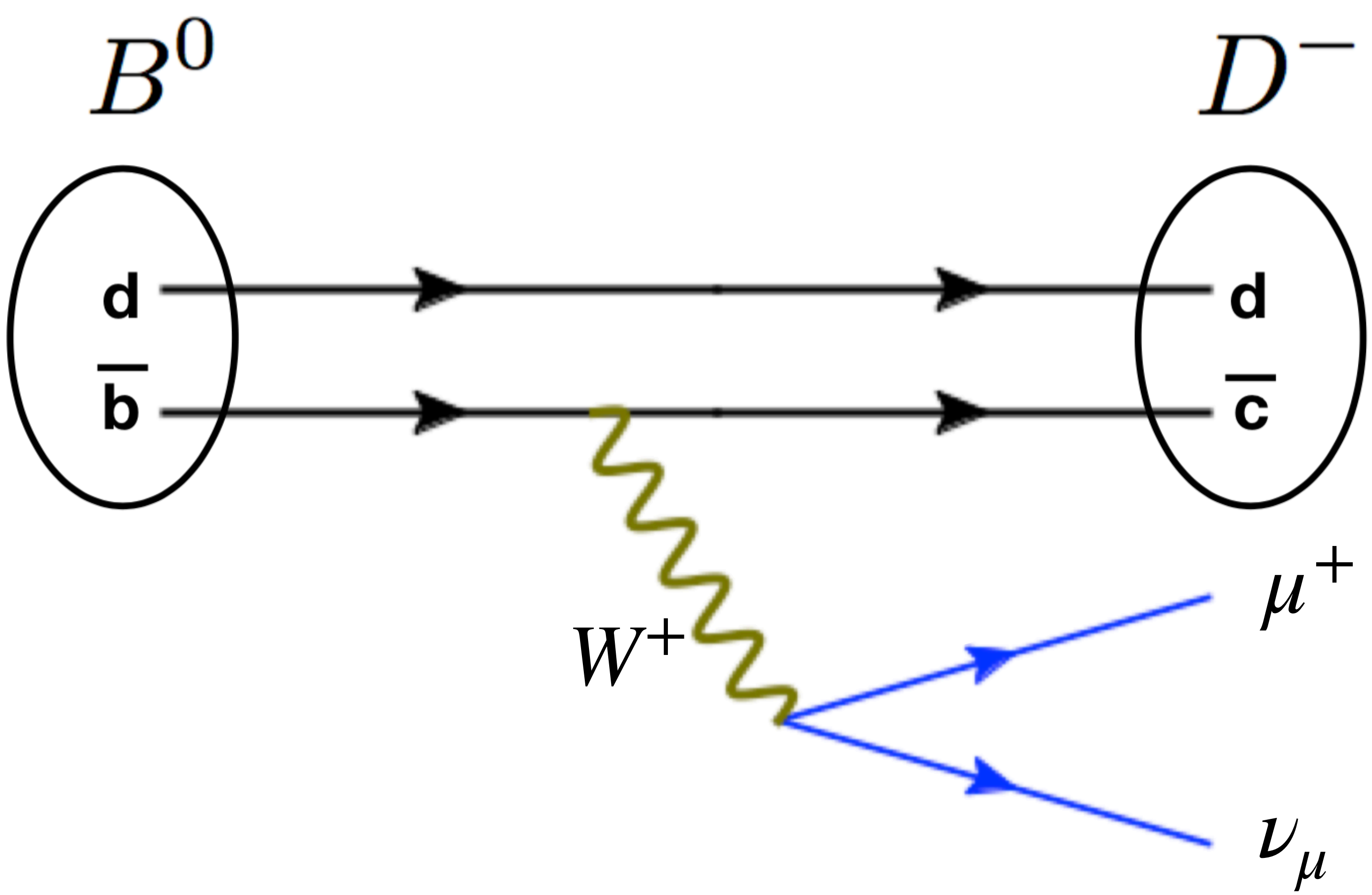
what is the corresponding Feynman diagram?



Heavy hadron decays

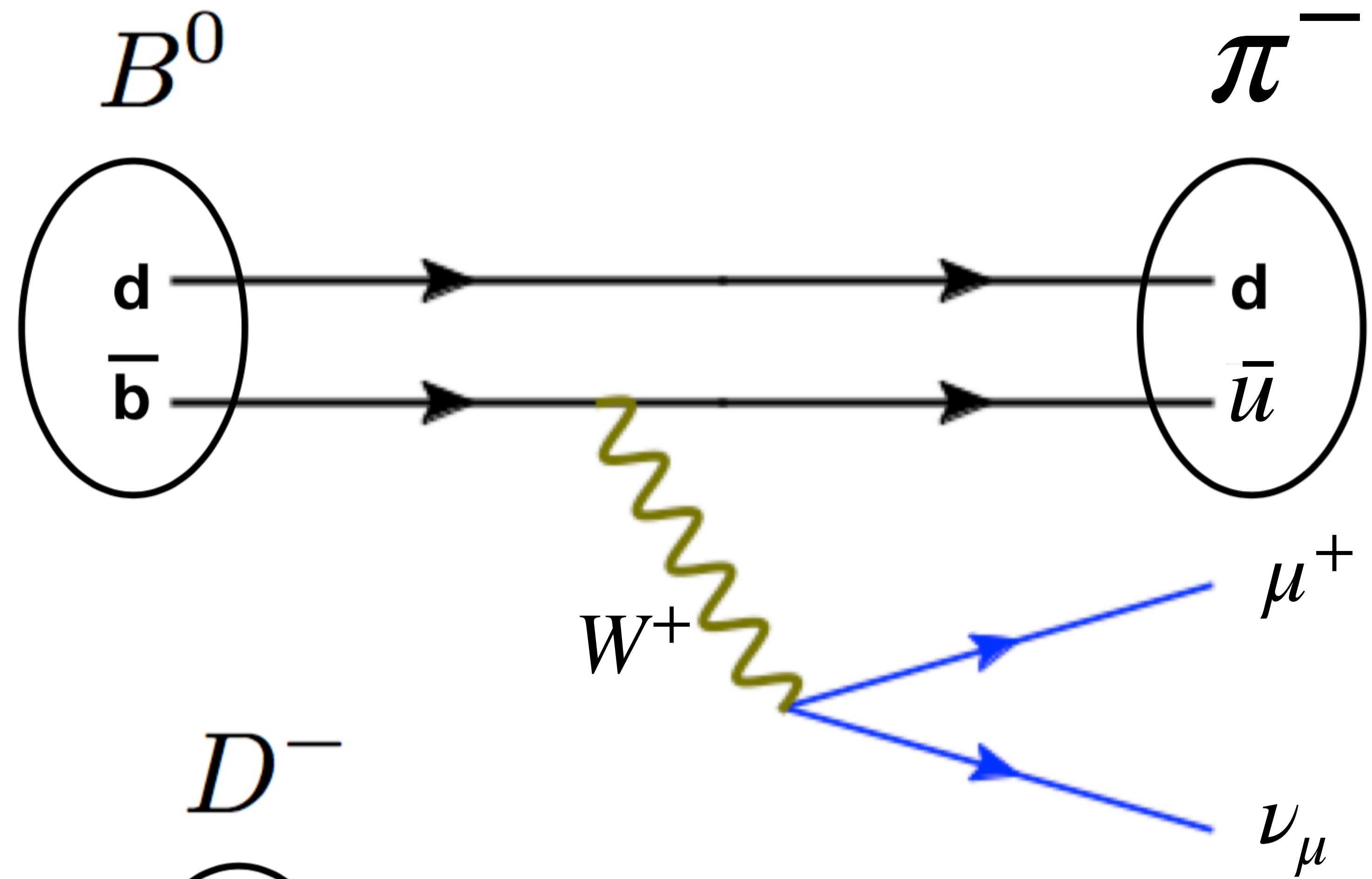
Which of the two reactions is **most likely** to take place?

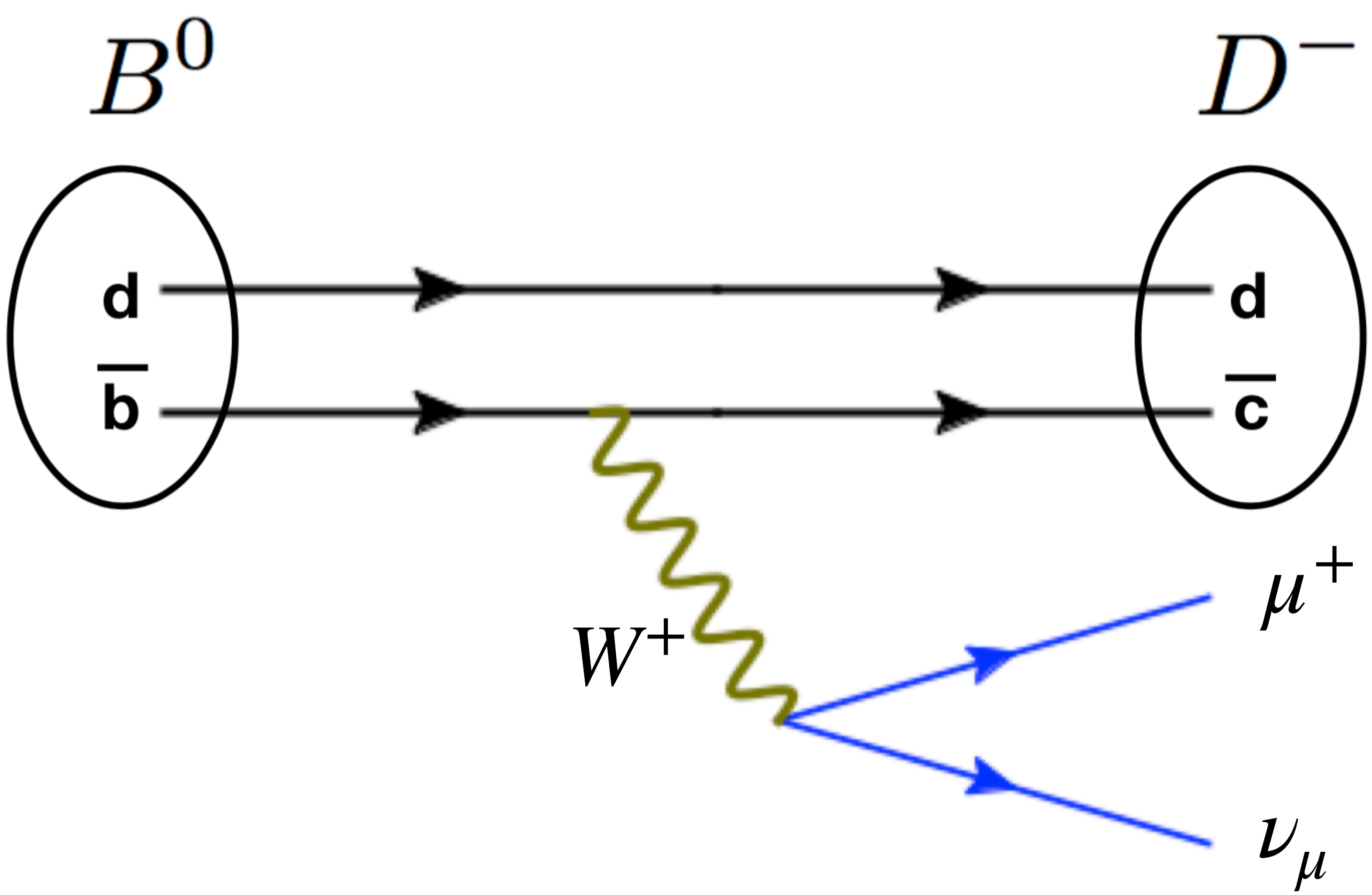




Heavy hadron decays

Which of the two reactions is **most likely** to take place?





The decay into a pion is more suppressed, since the **coupling** ***Wub*** (gens 1 and 3) is **smaller** than the **coupling** ***Wcb*** (gens 1 and 2)

Heavy hadron decays

Note that some reaction processes might look **very different from the outside**, but their similarities become apparent at the **Feynman diagram level**

$$B^0 \to D^- + \mu^+ + \nu_\mu$$

$$B^0 \to D^- + \pi^+$$

How do these two decay models relate to each other?

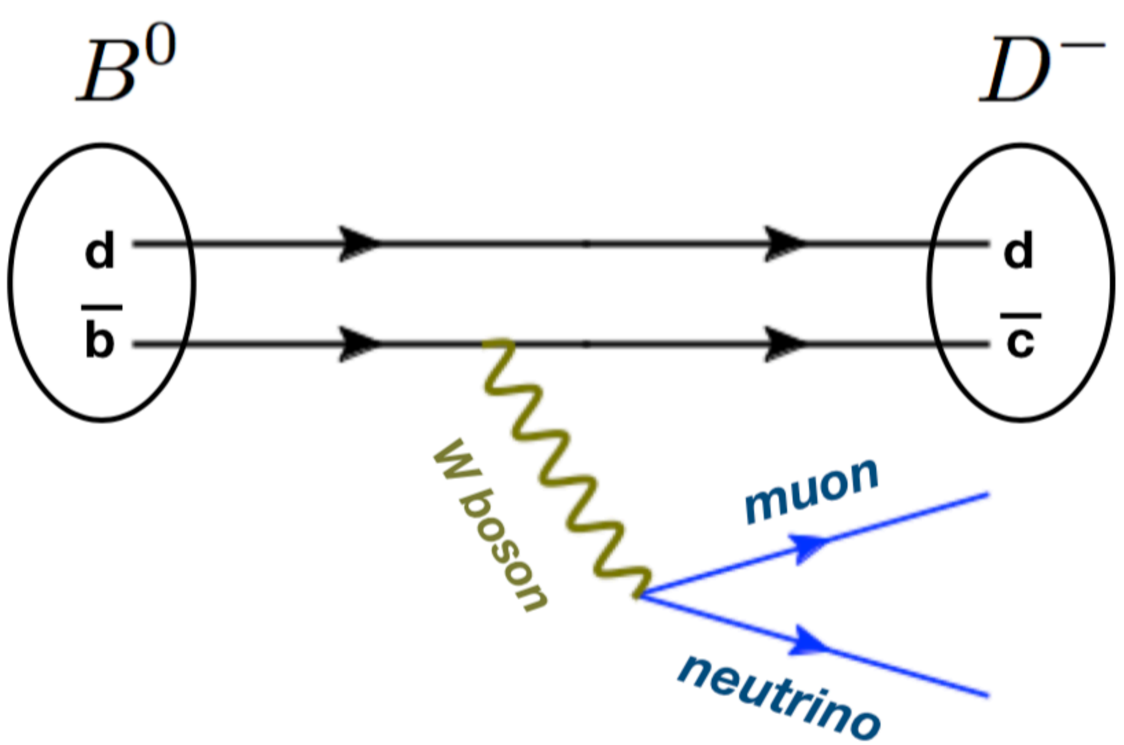
Heavy hadron decays

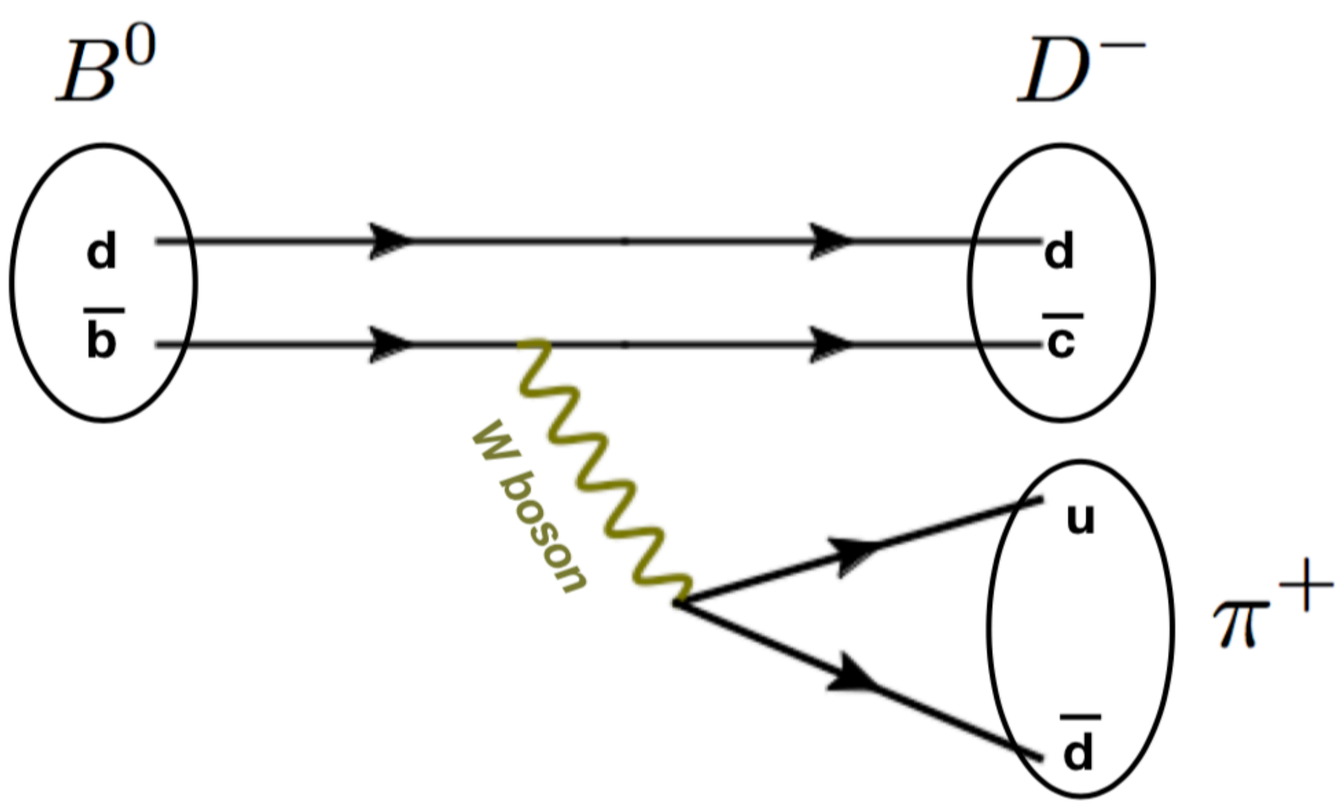
Note that some reaction processes might look **very different from the outside**, but their similarities become apparent at the **Feynman diagram level**

$$B^0 \rightarrow D^- + \mu^+ + \nu_\mu$$

$$B^0 \rightarrow D^- + \pi^+$$

How do these two decay models relate to each other?





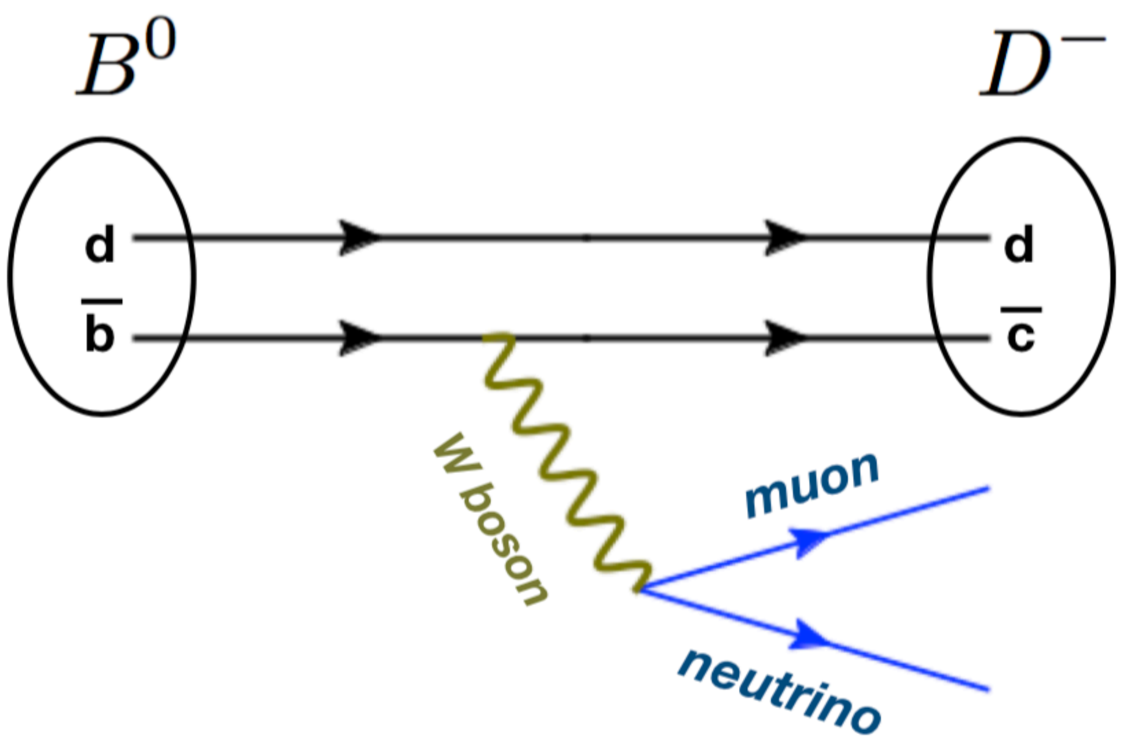
Heavy hadron decays

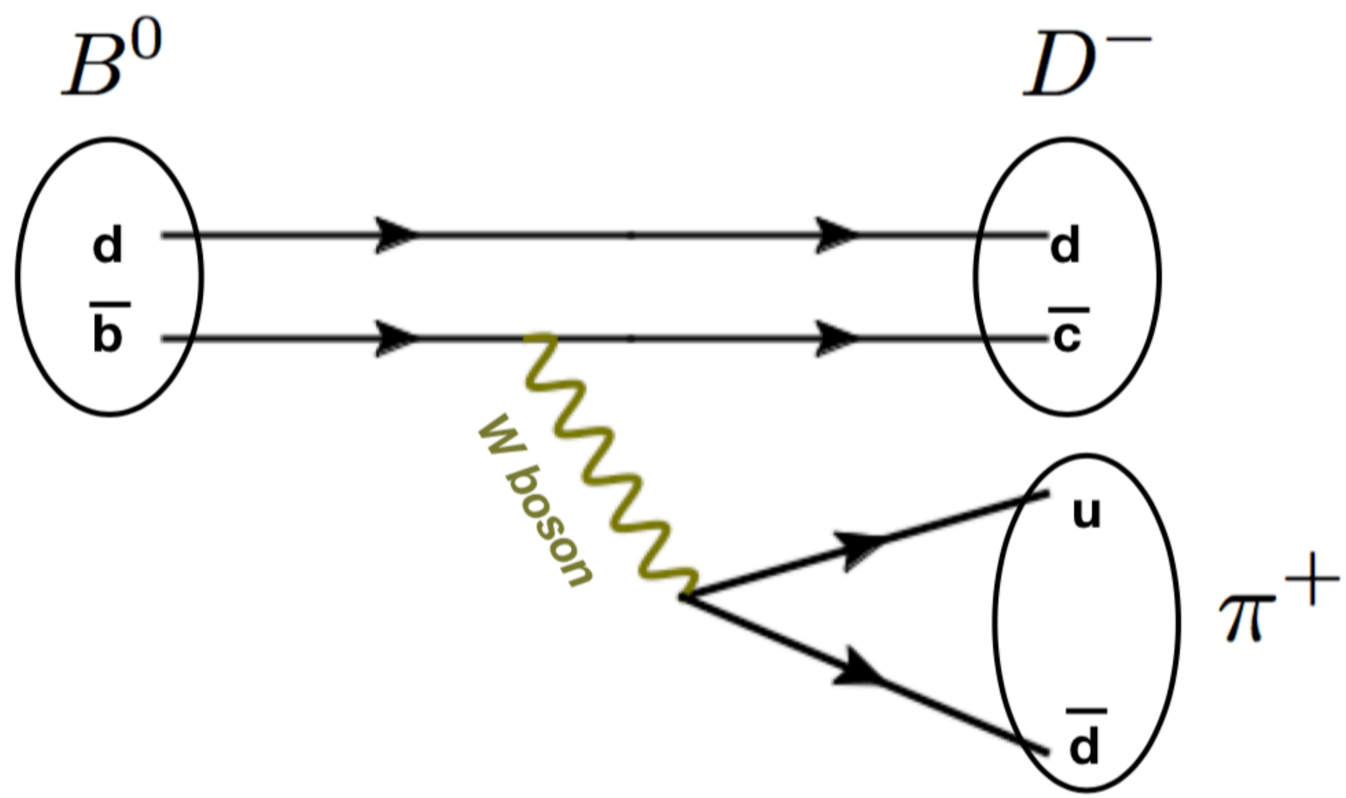
Note that some reaction processes might look **very different from the outside**, but their similarities become apparent at the **Feynman diagram level**

$$B^0 \to D^- + \mu^+ + \nu_\mu$$

$$B^0 \to D^- + \pi^+$$

How do these two decay models relate to each other?





These two processes have a very **similar probability** to happen!

The weak boson Z

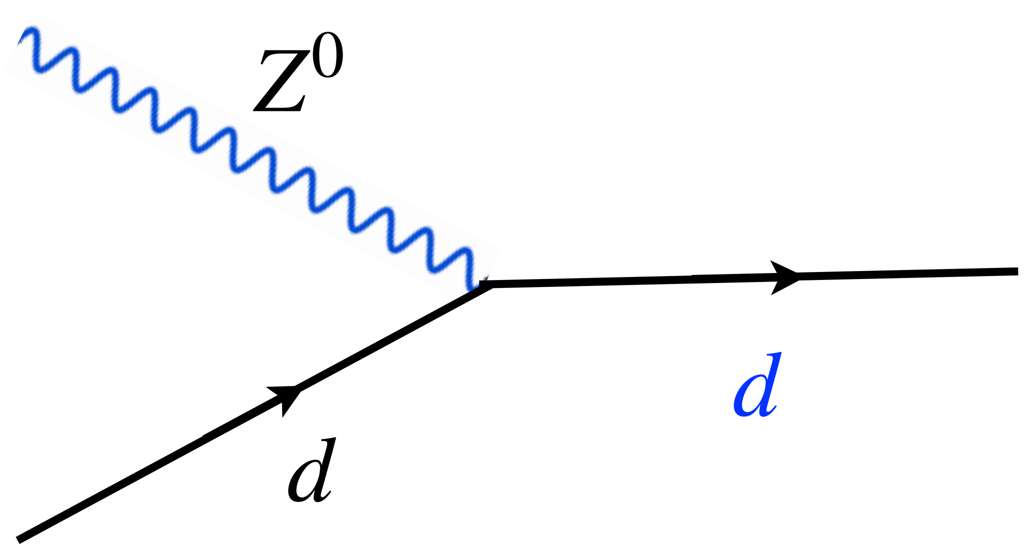
The weak interactions are mediated by three massive bosons: ***W^+, W^+, Z^0***

The main properties of the ***Z* bosons** are:

- As opposed to the **massless** gluons and photons, the **Z** boson is **very massive,** around 91 times the proton mass (similar to *W* boson)

- As in the case of the gluons (but not the photons), the *Z* boson is charged under the **weak** charges, and therefore can interact with **itself.** It is **electrically neutral** so it cannot interact via electromagnetism

- When interacting with **quarks,** the Z boson does **not change the quark flavour**



The weak boson Z

In terms of its interactions, the weak boson Z is a kind of ***``heavy photon”***

In diagrams involving quarks and charged leptons, and where the photon mediates the interaction, one can **replace the photon by a Z boson**

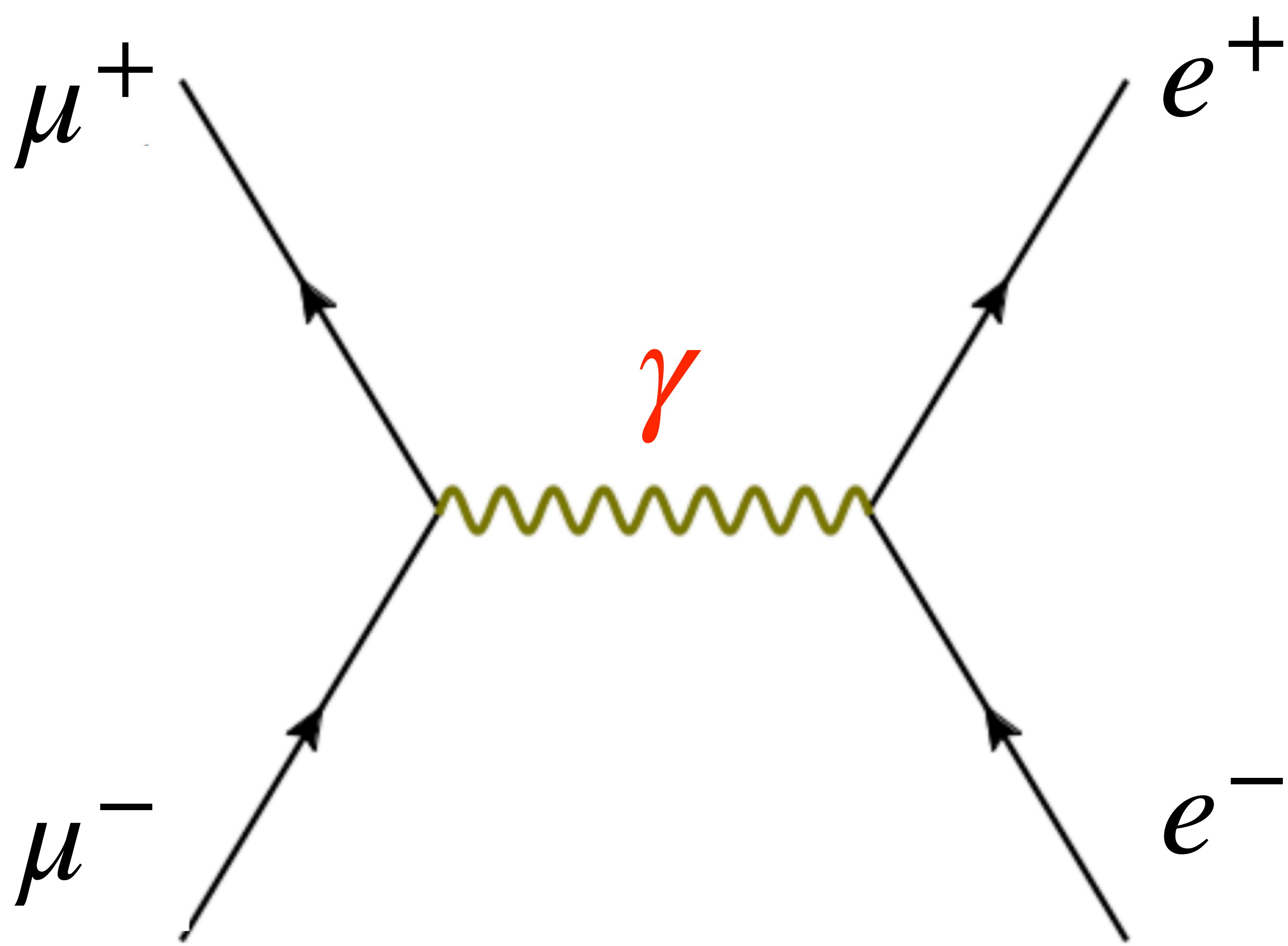
$$\mu^+ + \mu^- \rightarrow e^+ + e^-$$

The weak boson Z

In terms of its interactions, the weak boson Z is a kind of ***``heavy photon"***

In diagrams involving quarks and charged leptons, and where the photon mediates the interaction, one can **replace the photon by a Z boson**

$$\mu^+ + \mu^- \to e^+ + e^-$$

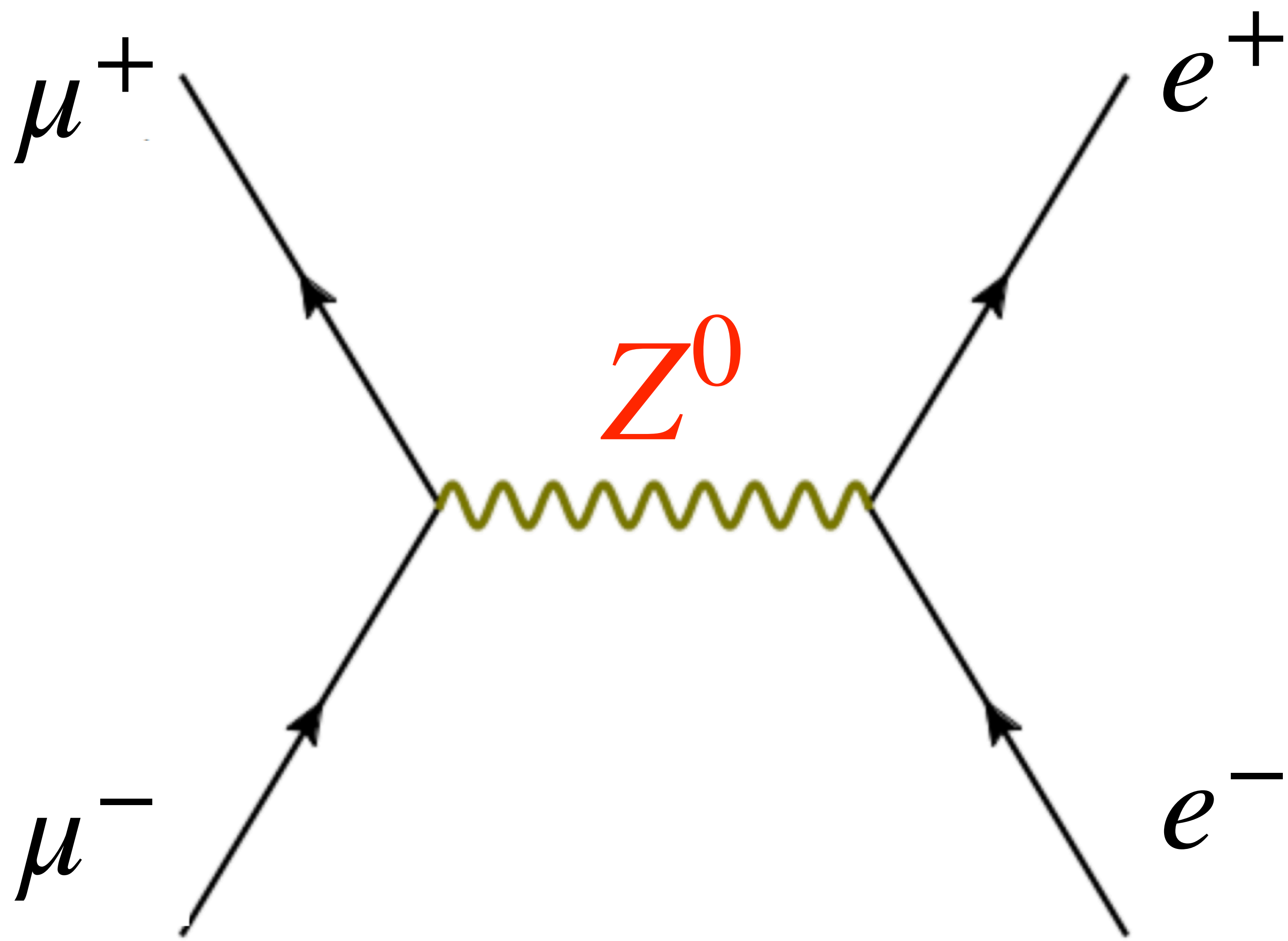


The weak boson Z

In terms of its interactions, the weak boson Z is a kind of ***``heavy photon"***

In diagrams involving quarks and charged leptons, and where the photon mediates the interaction, one can **replace the photon by a Z boson**

$$\mu^+ + \mu^- \rightarrow e^+ + e^-$$

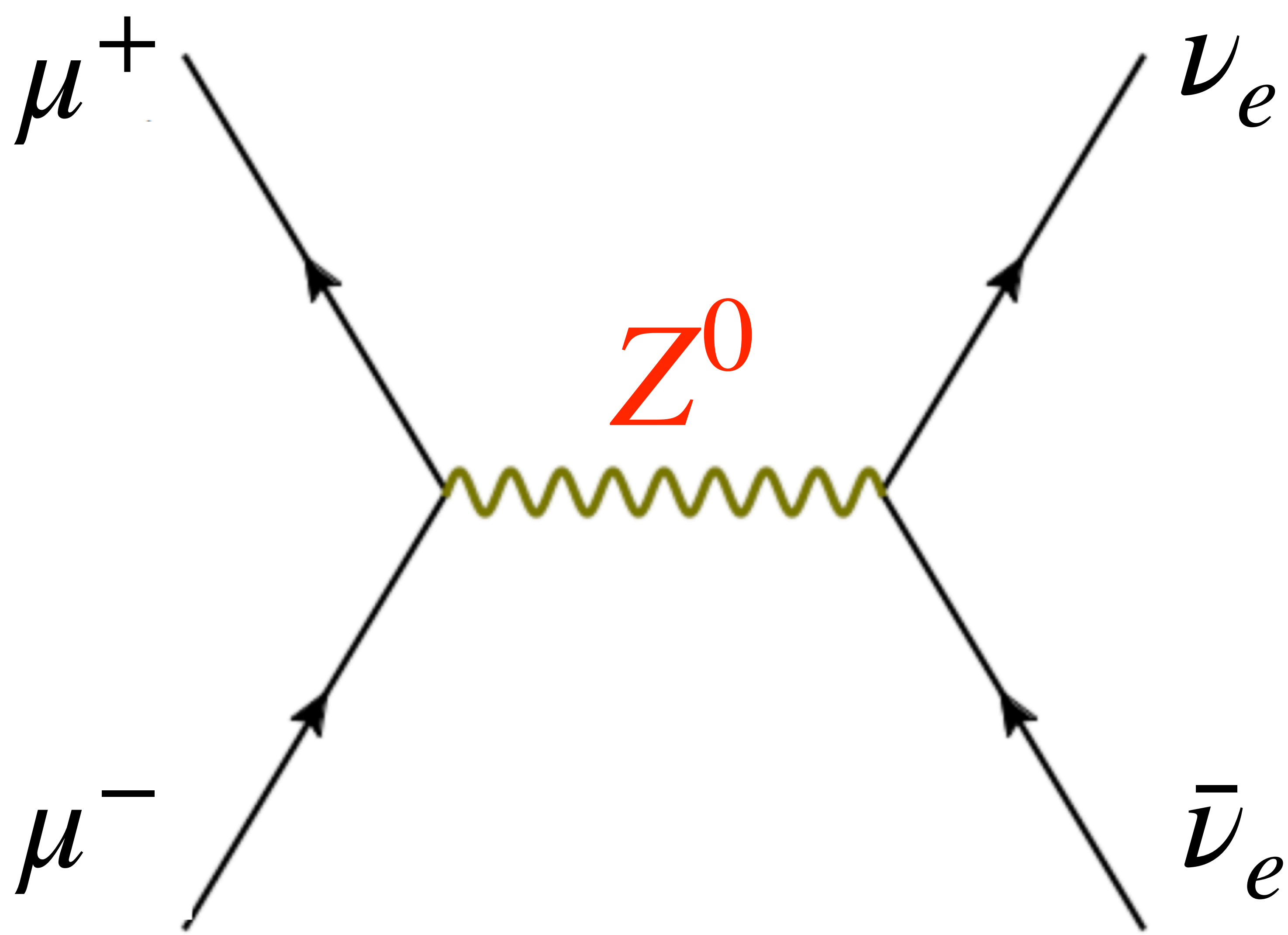


The weak boson Z

In terms of its interactions, the weak boson Z is a kind of ``***heavy photon***"

The Z boson also mediates processes involving **neutrinos**

$$\mu^+ + \mu^- \rightarrow \nu_e + \bar{\nu}_e$$



The weak boson Z

We can now summarise the **weak interaction vertices** involving the Z boson

with quarks

$$u + \bar{u} \to Z^0\,, \quad d + \bar{d} \to Z^0\,, \quad s + \bar{s} \to Z^0\,, \ldots$$

$$u + Z^0 \to u\,, \quad d + Z^0 \to d\,, \quad s + Z^0 \to s\,, \ldots$$

$$Z^0 \to u + \bar{u}\,, \quad Z^0 \to d + \bar{d}\,, \quad Z^0 \to s + \bar{s}\,, \ldots$$

The weak boson Z

We can now summarise the **weak interaction vertices** involving the *Z* boson

with quarks

$$u + \bar{u} \to Z^0, \quad d + \bar{d} \to Z^0, \quad s + \bar{s} \to Z^0, \ldots$$

$$u + Z^0 \to u, \quad d + Z^0 \to d, \quad s + Z^0 \to s, \ldots$$

$$Z^0 \to u + \bar{u}, \quad Z^0 \to d + \bar{d}, \quad Z^0 \to s + \bar{s}, \ldots$$

with leptons

$$e^+ + e^- \to Z^0, \quad \mu^+ + \mu^- \to Z^0, \quad \nu_e + \bar{\nu}_e \to Z^0, \ldots$$

$$e^- + Z^0 \to e^-, \quad \nu_e + Z^0 \to \nu_e, \quad \tau^+ + Z^0 \to \tau^+, \ldots$$

$$Z^0 \to e^- + e^+, \quad Z^0 \to \tau^+ + \tau^-, \quad Z^0 \to \nu_\mu + \bar{\nu}_\mu, \ldots$$

Any allowed reaction when **particles are interchanged by antiparticles** is also **allowed**

The weak interactions

Let us summarise what we have learned about the **weak interactions**

- **Flavour** is **not necessarily conserved** by the weak interactions: strangeness, charmness, and bottomness can vary in reactions mediated by the ***W* bosons** (but not by the *Z* boson)

- The individual leptonic numbers, electric charge, and the baryonic number are **conserved** in reactions mediated by the weak interaction

- The weak interaction is a **short range form** due to the masses of the *W* and *Z* bosons

- The strength of the weak interaction is **larger** between quarks of the **same generation** than between quarks of **different generation**

- From the point of view of the interactions with **leptons** and **charged quarks,** the Z boson behaves as it it was a **heavy photon**

Case study: Scalar QED

Feynman rules in scalar QED

- **Scalar Quantum Electrodynamics** is a theory that contains a **charged scalar particle** (possibly with self-interactions) and a **photon**

- Formally it is a **Quantum Field Theory** defined by the following Lagrangian

scalar mass

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(D_\mu\psi\right)^*\left(D_\mu\psi\right) - m^2\psi^*\psi$$

charged scalar

where we have defined:

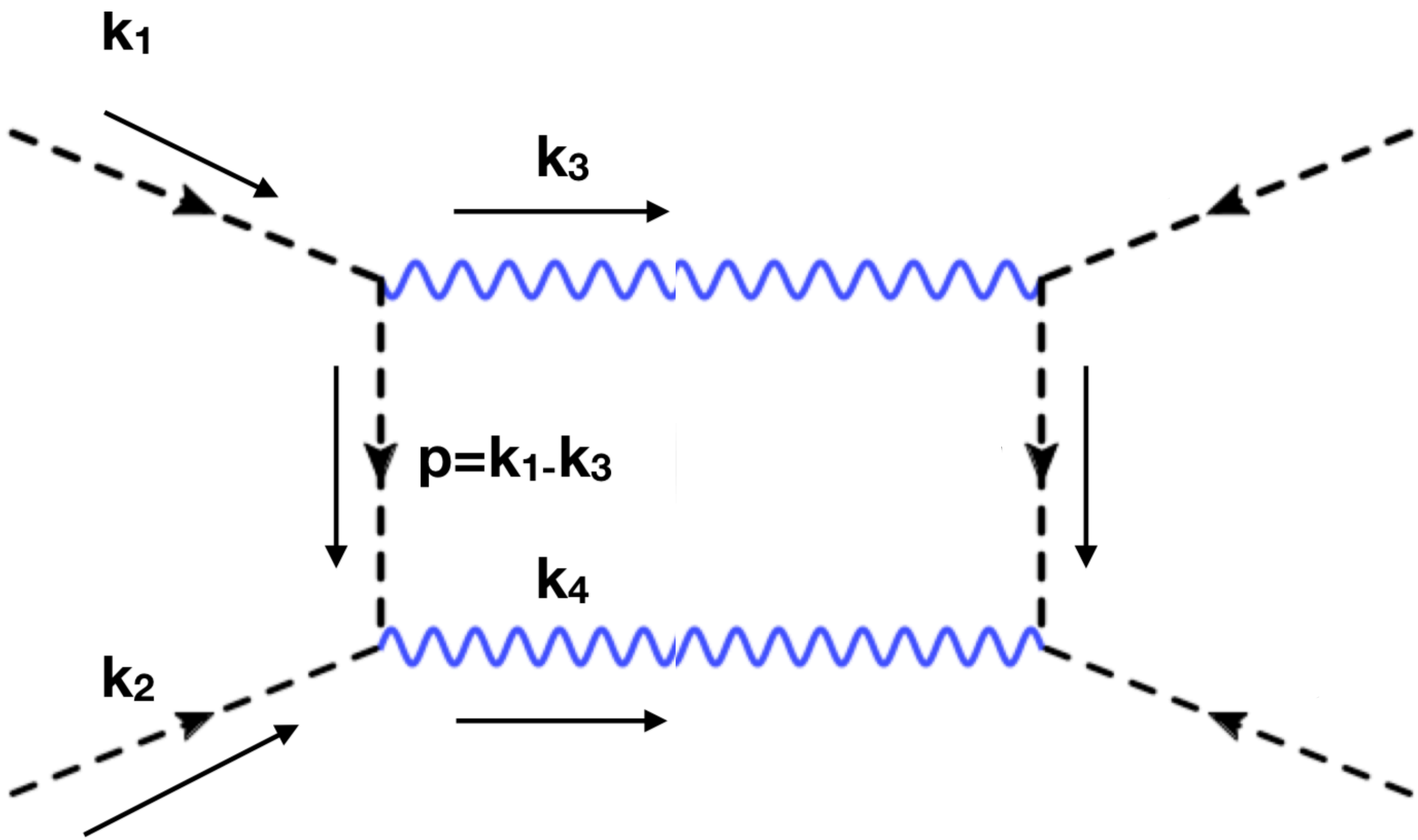
$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu\,, \qquad D_\mu \equiv \partial_\mu + ieA_\mu$$

photon

electric charge

- How can we compute the **probability** that a given scattering reaction in scalar QED takes place?

Feynman rules in scalar QED



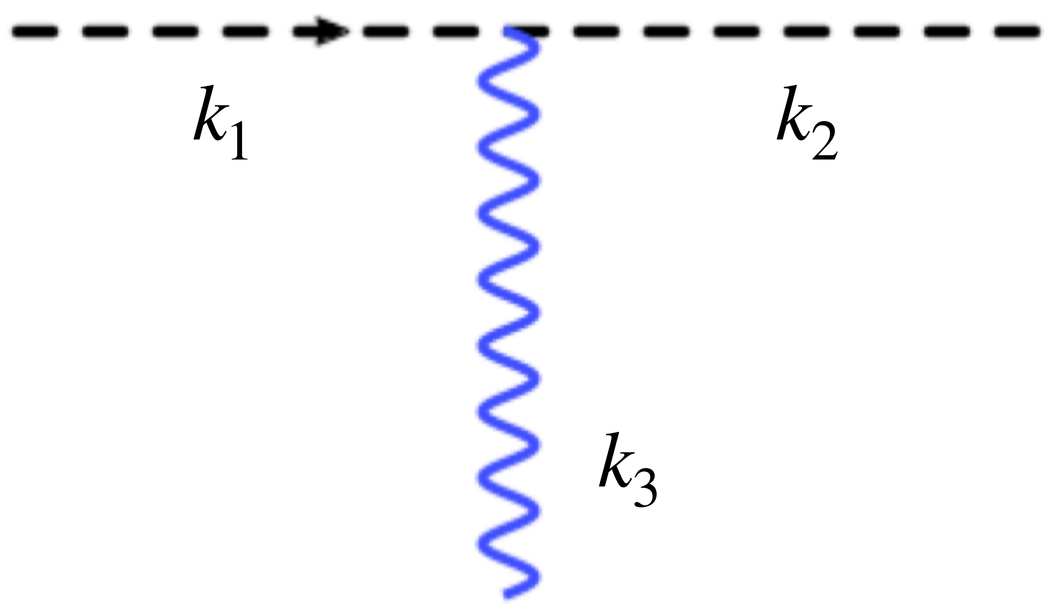
☑ How can we compute the **probability** that a given scattering reaction in scalar QED takes place?

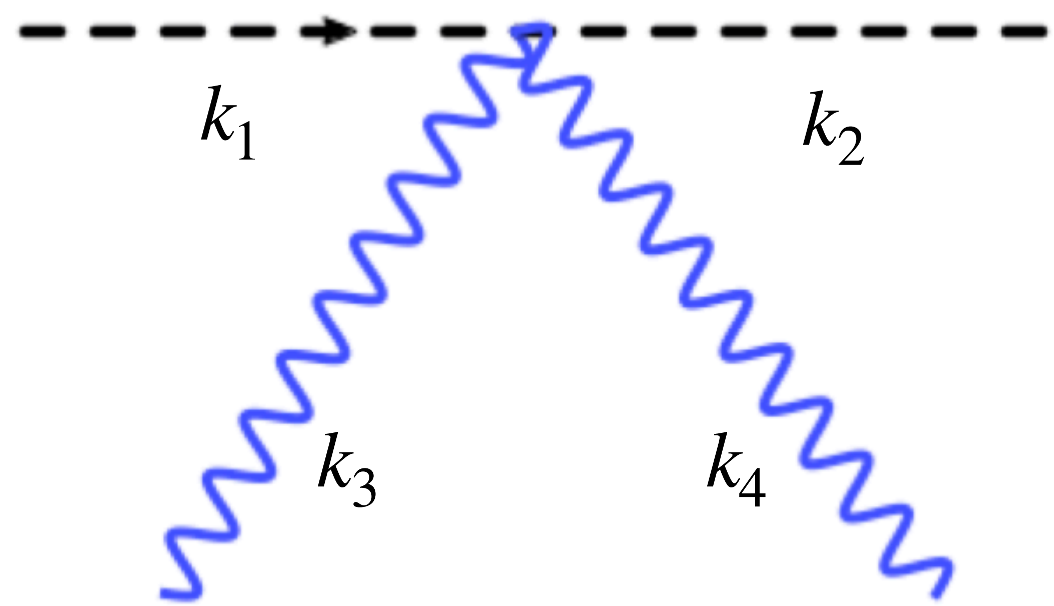
Feynman rules in scalar QED

☑ First we need to construct the **scattering amplitude** following the **Feynman rules** of the theory

each of the interaction terms that appear in the **Lagrangian** of the theory ….

$$ieA^\mu\left(\partial_\mu\psi^*\psi - \psi^*\partial_\mu\psi\right) \qquad\qquad - e^2 A_\mu A^\mu \psi^*\psi$$





$$ie(k_1+k_2)_\mu(2\pi)^4\delta\left(\sum k_i\right) \qquad\qquad -2ie^2\eta_{\mu\nu}(2\pi)^4\delta\left(\sum k_i\right)$$

…. corresponds to specific rules in the **scattering amplitude**

Feynman rules in scalar QED

- First we need to construct the **scattering amplitude** following the **Feynman rules** of the theory

In Feynman diagrams we also have in general propagators (internal lines)

Scalar propagator

k^μ

$$\frac{-i}{k^2 + m^2 - i\epsilon}$$

Photon propagator

k^μ

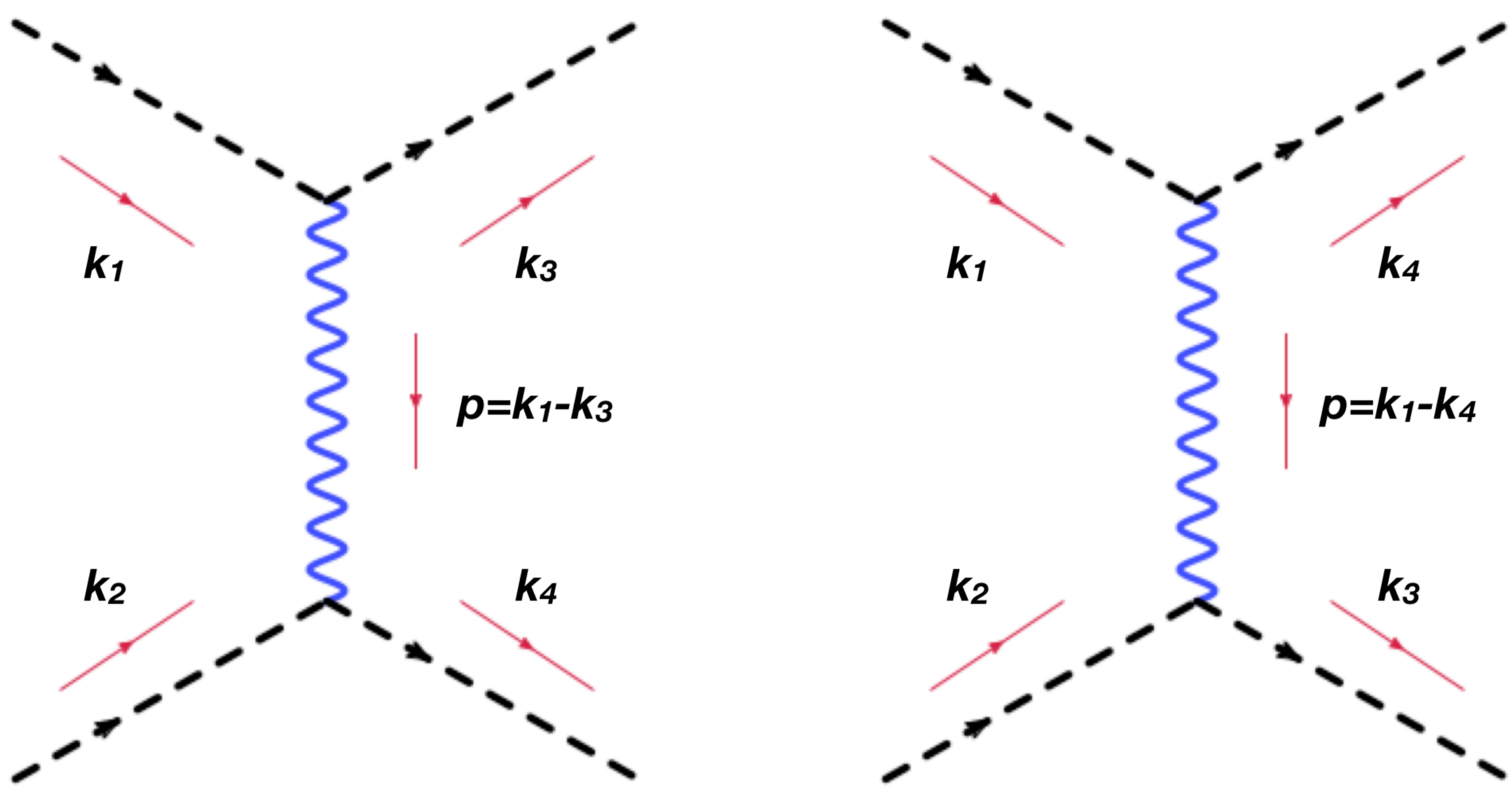
$$\frac{-i}{k^2 + i\epsilon}\left(\eta^{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)$$

…. which also have specific rules in the **scattering amplitude**

Born level (leading order)

- Compute the scattering amplitude for **Moeller scattering** in scalar QED

$$\mathscr{M}\,(e^- + e^- \to e^- + e^-)$$

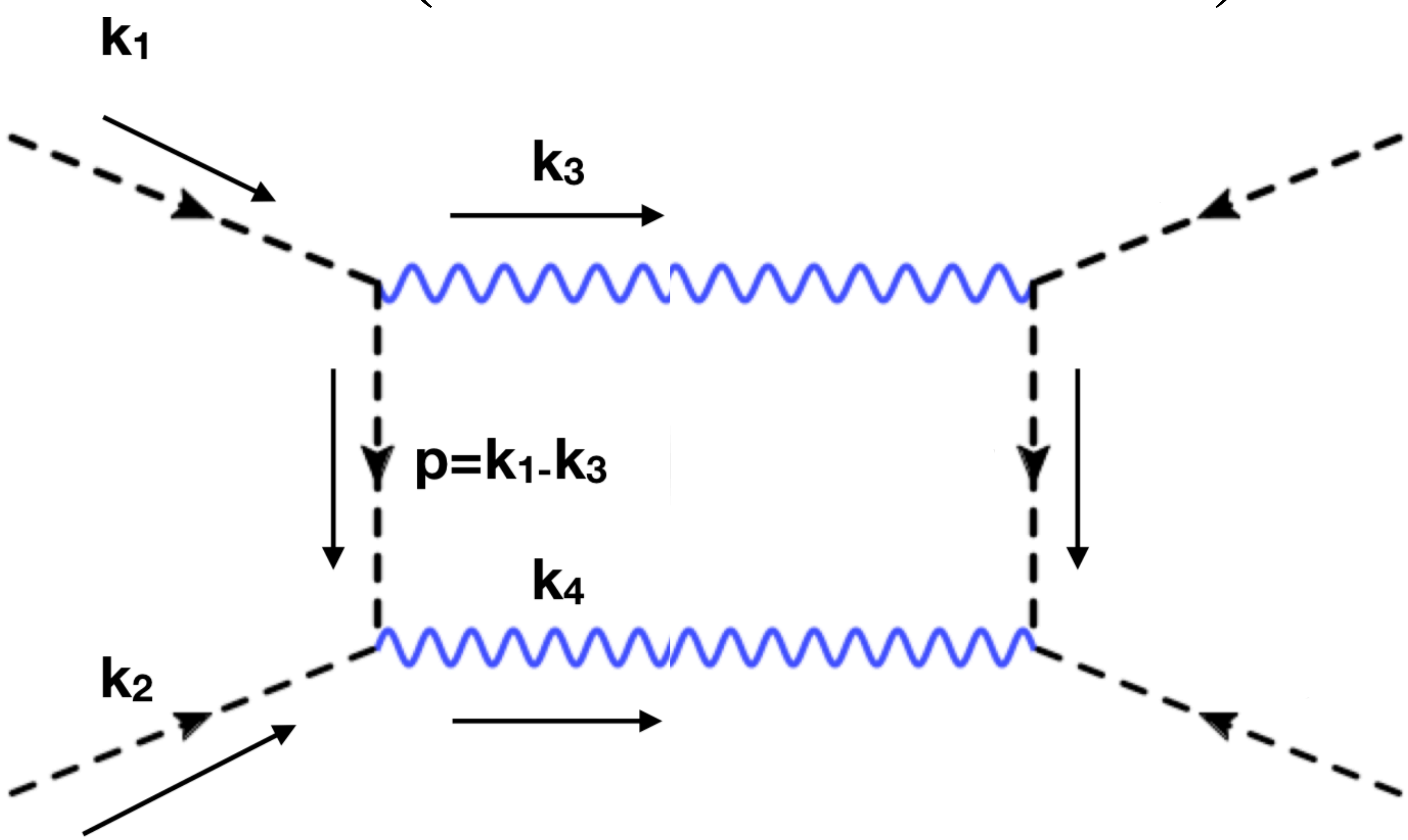


$$\begin{aligned}\mathcal{M} &= \Big[ie(k_1+k_3)_\mu \frac{-i}{p^2}\left(\eta^{\mu\nu} - (1-\xi)\frac{p^\mu p^\nu}{p^2}\right) ie(k_2+k_4)_\nu \\ &+ \; ie(k_1+k_4)_\mu \frac{-i}{\tilde{p}^2}\left(\eta^{\mu\nu} - (1-\xi)\frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2}\right) ie(k_2+k_3)_\nu\Big](2\pi)^4\delta(k_1+k_2-k_3-k_4)\,.\end{aligned}$$

One-loop (next-to-leading order)

☑ Compute the scattering amplitude for **Moeller scattering** in scalar QED **at one loop level**

$$\mathscr{M}\,(e^- + e^- \to e^- + e^-)$$



$$\mathscr{M}_{\rm NLO} \propto e^4 \int \frac{d^4k}{k^4} \propto e^4 \ln \Lambda \to \infty$$

The amplitude at NLO is infinite! What does it mean? We **need to renormalise the theory ….**

Feynman diagrams: summary

- ☑ Feynman diagrams provide a powerful too to understand what goes on in **reactions between elementary particles**

- ☑ To being with, with Feynman diagrams we can determine whether a given scattering process is **possible or impossible**

- ☑ Using them we can also determine which of two processes has a **higher likelihood of taking place** based on power counting of coupling constants

- ☑ Moreover, with Feynman diagrams the similarities between **apparently different scattering reactions** become transparent

- ☑ Most importantly: they allow us to provide **precise predictions for the rates of high-energy scattering reactions**, which are key to find possible **new particles and interactions beyond the Standard Model!**