Part II: Rare partonic components of the proton

The inner life of protons

brief recap of the introductory material



All-order structure: QCD factorisation theorems

Parton Distributions

Proton energy divided among constituents: **quarks** and **gluons**



Parton Distributions

g(x,Q)

Energy of hard-scattering reaction: inverse of resolution length

Probability of finding a gluon inside a proton, carrying a fraction *x* of the proton momentum, when probed with energy *Q*

x: fraction of proton momentum carried by gluon

Dependence on *x* fixed by **non-perturbative QCD dynamics**: extract from experimental data

e.g.
$$g(x, Q_0^2) = A_g x^{\alpha_g} (1-x)^{\beta_g}$$
 introduce a model, extract its parameters from data

$$\int_0^1 dx \, x \left(\sum_{i=1}^{n_f} \left[q_i((x, Q^2) + \bar{q}_i(x, Q^2)] + g(x, Q^2) \right] = 1 \qquad \begin{array}{c} \text{momentum sum rule} \\ \text{(energy conservation)} \end{array} \right)$$

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Dependence on **Q** fixed by perturbative QCD dynamics: computed up to $\mathcal{O}(\alpha_s^4)$

$$\frac{\partial}{\partial \ln Q^2} q_i(x, Q^2) = \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}, \alpha_s(Q^2)\right) q_j(z, Q^2)$$

DGLAP evolution equations

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The Global QCD analysis paradigm

QCD factorisation theorems: PDF universality

$$\sigma_{\mu p \to \mu X} = \widetilde{\sigma}_{u\gamma^* \to u} \otimes u(x) \implies \sigma_{p p \to W} = \widetilde{\sigma}_{u\bar{d} \to W} \otimes u(x) \otimes \bar{d}(x)$$



Determine PDFs from deepinelastic scattering...

... and use them to compute predictions for **proton-proton collisions**

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A proton structure snapshop



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the heavy quark content of the proton

The many faces of the proton

a QCD bound state of quarks and gluons

 $m_p \simeq 1 \text{ GeV}$





$$m_g = 0$$

gluons and valence quarks are the best understood components of the proton

The many faces of the proton

a QCD bound state of quarks and gluons

 $m_p \simeq 1 \text{ GeV}$



How strange is the proton?

marked differences in the strange PDFs from **recent global analyses**, both for central values and for the size of its uncertainties



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the strange PDF can be constrained by different processes, both collider and fixed-target



sensitivity via correlation of W^+ , W^+ , Z distributions ATLAS 7 TeV data: preference for $R_s=1$









How strange is the proton?

Reappraisal of the proton strangeness based combination of all relevant experimental inputs

Process	Dataset	$n_{ m dat}$	$\chi^2_{ m base}$	$\chi^2_{ m pr}$	$\chi^2_{ m str}$
$\nu \text{DIS} (\mu \mu)$		76/76/95/91/95	0.76	0.71	0.53
	NuTeV [9]	76/76/76/76/76	0.76	0.71	0.53
	NOMAD [10]	//19/15/19	[9.3]	[8.8]	0.55
W, Z (incl.)		391/418/418/418/418	1.45	1.40	1.40
	ATLAS $[12]$	34/61/61/61/61	1.96	1.65	1.67
$W{+}c$		-/37/37/37/37	[0.73]	0.68	0.60
	CMS [17, 18]	-/15/15/15/15	[1.04]	0.98	0.96
	ATLAS $[16]$	-/22/22/22/22	[0.52]	0.48	0.42
W + jets	ATLAS $[15]$	-/32/32/32/32	[1.58]	1.18	1.18
Total		3981/4077/4096/4092/4096	1.18	1.17	1.17





- Satisfactory simultaneous description of all datasets
- No evidence for tension between datasets or groups of processes
- Sizeable constraints from NOMAD neutrino DIS data, consistent with collider data
- Strong preference for a moderately suppressed
 strangeness

$$R_S(x = 0.023, Q = 1.6 \text{ GeV}) = 0.71 \pm 0.10$$

Faura et al 20

A charming proton

say you want to evaluate the **charm DIS structure function**. You have three options

Fixed-flavor scheme: no charm PDF, charm mass effects accounted for exactly



A charming proton

say you want to evaluate the **charm DIS structure function**. You have three options

Zero-mass scheme: charm PDF treated on the same footing as all other quark flavours



the charm PDF is deterministically generated from the gluon (and light quark) PDFs

A charming proton

say you want to evaluate the **charm DIS structure function**. You have three options

General-mass VFN scheme: charm PDF treated on the same footing as all other quark flavours, massive effects included in coefficient functions

$$F_2^c(x, Q^2) \propto \sum_{i=g, u, d, s, c} C_i^{(\text{GM})}(\alpha_s, Q^2/m_c^2) \otimes f_i^{(n_f+1)}$$

Systematically improvable, reliable for all values of Q² from threshold to collider scales



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perturbative, intrinsic, and fitted charm

Let's work in the following in the GM-VFN scheme. The charm PDF above threshold is constructed from the $n_f=3$ PDFs via **matching** in three possible ways:

Perturbative charm: the charm PDF vanishes below threshold, then above threshold ($\mu_c \approx m_c$) the charm PDF is deterministically **generated from the gluon** (and light quark) PDFs

$$f_c^{(n_f)} = 0 \quad \rightarrow \quad f_c^{(n_f+1)} \propto \alpha_s \ln \frac{Q^2}{m_c^2} \left(P_{qg} \otimes f_g^{(n_f+1)} \right) + \mathcal{O} \left(\alpha_s^2 \right) \underset{about \text{ the charm PDF here}}{\text{not much interesting to say}}$$

Intrinsic charm: a model for the charm PDF at the initial evolution scale (below or at threshold) is assumed. Then the charm PDF is this intrinsic component plus the perturbative component

$$f_c^{(n_f)}(x, Q_0) = Ax^2 \left[6x(1+x)\ln x + (1-x)(1+10x+x^2) \right]$$

BHPS model (scale independent)

the model parameters (e.g. normalisation) are extracted from comparison with data

Fitted charm: no assumptions on possible intrinsic component are made. The charm is parametrised above threshold in exactly the same was as all other quark PDFs

$$f_c^{(n_f+1)}(x, Q_0) = x^{-\alpha_c}(1-x)^{\beta_c} NN(x) \qquad \text{NNPDF approach}$$

n.b. the GM-VFN structure functions need to be modified for a non-zero charm PDF in the n_f=3 scheme

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in the following, we assume that the charm PDF is either **fitted** or **intrinsic** (perturbative charm cannot be constrained, since it is generated by DGLAP evolution)

Charm production in DIS



Only measurements of **F**₂^c at large-*x* are sensitive EMC data from the 80s available ... NNPDF 16





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Indirect sensitivity, but high-precision measurements available LHCb data in forward region specially powerful







How charming is the proton?



$$K_{c}(Q) \equiv \frac{\int_{0}^{1} dx \ (c+\bar{c})}{\int_{0}^{1} dx \ (u^{+}+d^{+}+s^{+})}$$

momentum fraction carried by charm in units of that of the light quarks

- ✓ perturbative charm results are sensitive to choice of value of charm mass
- Current data favour a non-zero charm
 - **component** in the n_f=3 scheme

How charming is the proton?



momentum fraction carried by charm in units of that of the light quarks

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Intrinsic bottom?

Our considerations about the charm PDF (perturbative vs fitted vs intrinsic) apply equally well to the bottom PDF, though here on expects deviations from the perturbative picture to be quite suppressed

In **all PDF analysis to date**, the bottom PDF is always **generated dynamically via DGLAP** from light quarks and gluons

$$f_b^{(n_f+1)} \propto \alpha_s \ln \frac{Q^2}{m_b^2} \left(P_{qg} \otimes f_g^{(n_f+1)} \right) + \mathcal{O}\left(\alpha_s^2\right)$$

Assume that there is a "**non-perturbative**" component to the bottom PDF. How we could constrain it?

same as for strange and charm: look for processes directly or indirectly sensitive to **bottom quarks in the initial state of the reaction**

Constraining bottom

Bottom production in DIS



much smaller rates than for charm..... *e.g.* at HERA charm can be up to 25% while bottom is 1%

Constraining bottom





requires **matched scheme** to account for bottom quark mass corrections

Constraining bottom



Impact of parametrised bottom



It has been recently shown, for the case of **Higgs production in bottom fusion**, how a matched scheme for processes involving initial-state bottom quarks simplifies the calculation provided a fitted bottom PDF is introduced

Forte, Giani, Napoletano 19





photons and leptons as partons

Let there be light: the photon PDF

Free proton contains not only quark and gluons as constituents: also **photons!**

Required for consistent implementation of QED/weak corrections at the LHC



The luminous proton

Naively, QED and weak corrections appear to be not relevant for LHC physics, since the electroweak couplings are suppressed as compared to the QCD one. However, one notes that:

MNLO QCD and **NLO electroweak** corrections are of the same order of magnitude

$$\alpha_s^2(M_Z) \sim \frac{1}{70}$$
, $\alpha_{\rm EM}(M_Z^2) \sim \frac{1}{130}$

☑ In the presence of QED effects, new **photon-initiated processes** become available:



Drell-Yan (quark-antiquark annihilation)

Photon-induced "Drell-Yan"

The **DGLAP evolution** with QED effects mixes quarks and gluons with photons

How can we determine the photon content of the proton?

Juan Rojo

The photon PDF

First attempts: models assuming that the photon PDF is radiated off the light quarks



up quark PDF

down quark PDF

splitting function

A model-independent determination (same footing as light quarks) was performed by NNPDF





The photon PDF

In 2016 it was demonstrated that the photon PDF is a **derived quantity** expressed in terms of the (well-known) inclusive DIS structure functions

$$\begin{aligned} x\gamma(x,\mu^2) &= \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \bigg\{ \int_{\frac{x^2m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \bigg[\bigg(zp_{\gamma q}(z) + \frac{2x^2m_p^2}{Q^2} \bigg) F_2 \bigg(\frac{x}{z}, Q^2 \bigg) - z^2 F_L \bigg(\frac{x}{z}, Q^2 \bigg) \bigg] \\ &- \alpha^2(\mu^2) z^2 F_2 \bigg(\frac{x}{z}, \mu^2 \bigg) \bigg\}. \end{aligned}$$
Manohar et al 16,17



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Let there be light

The **photonic content of nucleons** is now determined with high precision, and combined with EW effects for the most accurate QCD+EW productions for LHC cross-sections



Leptons in the proton

The same considerations that motivated the inclusion of the photon PDF indicate that, at some level, **leptons can also be treated as partonic components** (since they mix with the photon via the QED DGLAP evolution equations)

Lepton PDFs
$$\ell^-(x,Q_0) = \ell^+(x,Q_0) = \frac{\alpha(Q_0)}{4\pi} \ln\left(\frac{Q_0^2}{m_\ell^2}\right) \int_x^1 \frac{dy}{y} P_{\ell\gamma}^{(0)}\left(\frac{x}{y}\right) \gamma(y,Q_0)$$
 Photon PDF

DGLAP kernel

The lepton PDF is (QED) perturbatively generated from photon PDF

the above relation however is not accurate since it misses $O(\alpha)$ corrections from lepton evolution

The presence of lepton PDFs opens new channels for processes of phenomenological relevance



Juan Rojo

Leptons in the proton

An initial study assumed that the **lepton PDFs vanish at the proton mass**, $Q_0 \simeq 1 \text{ GeV}$ then generated perturbatively from the photon via DGLAP evolution $\ell^-(x, Q_0) = \ell^+(x, Q_0) = 0$



Large uncertainties in the lepton PDF inherited from NNPDF2.3QED photon

Juan Rojo

Leptons in the proton

Earlier this year the LUXqed formalism was extended to leptons, showing that lepton PDFs can be expressed in terms of DIS structure functions and QED splitting functions

$$\begin{aligned} x_{\ell}f_{\ell}(x_{\ell},\mu_{F}^{2}) &= \left(\frac{1}{2\pi}\right)^{2} \int_{x_{\ell}}^{1} \frac{\mathrm{d}x}{x} z_{\ell} \int_{x}^{1} \frac{\mathrm{d}z}{z} \int_{\frac{m_{F}^{2}x^{2}}{1-z}}^{\frac{\mu_{F}^{2}}{1-z}} \frac{\mathrm{d}Q^{2}}{Q^{2}} \alpha^{2}(Q^{2}) \\ &\left\{ \begin{array}{l} P_{\ell\gamma}(z_{\ell}) \log \frac{\mu_{F}^{2}}{(1-z_{\ell})z_{\ell} \left(Q^{2} + \frac{m_{\ell}^{2}}{z_{\ell}(1-z_{\ell})}\right)}{\left(P_{2}\left(zP_{\gamma q}(z) + \frac{2m_{p}^{2}x^{2}}{Q^{2}}\right) - F_{L}z^{2}\right] \right. \\ &+ F_{2}\left[4(z-2)^{2}z_{\ell}(1-z_{\ell}) - (1+4z_{\ell}(1-z_{\ell}))zP_{\gamma q}(z)\right] \\ &\left. + F_{L}z^{2}P_{\ell\gamma}(z_{\ell}) - \frac{2m_{p}^{2}x^{2}}{Q^{2}}F_{2} - \left(F_{2}\frac{2m_{p}^{2}x^{2}}{Q^{2}} - z^{2}F_{L}\right)4z_{\ell}(1-z_{\ell}) \\ &+ \frac{m_{\ell}^{2}F_{2}}{m_{\ell}^{2} + Q^{2}z_{\ell}(1-z_{\ell})}\left[zP_{\gamma q}(z) - 8z_{\ell}(1-z_{\ell})\left(1-z - \frac{m_{p}^{2}x^{2}}{Q^{2}}\right) + \frac{2m_{p}^{2}x^{2}}{Q^{2}}\right] \\ &- \frac{m_{\ell}^{2}F_{L}z^{2}}{m_{\ell}^{2} + Q^{2}z_{\ell}(1-z_{\ell})}\left[2 - P_{\ell\gamma}(z_{\ell})\right] \right\}. \end{aligned}$$

this implies that, like in the photon, we can evaluate the lepton PDFs with high precision

The LHC as a lepton-quark collider

The presence of lepton PDFs as partons offers remarkable new opportunities to search for BSM physics, *e.g.* in this case the **resonant production of leptoquarks** become possible



Another nice illustration of the unexpected applications that proton studies provide!

The proton at TeV scales: From Higgs to top quarks

The proton with TeV resolution

In the same way that we treat charm and bottom quarks as massless partons, if we go at **high enough energies** do we also need to treat the top quark as massless and include a **top PDF**?

In the same way that we have to account for photon and lepton PDFs in the presence of QED effects, if we go **at high enough energies** do we also need to account for weak effects in the DGLAP evolution equations and introduce **PDFs for the weak gauge and Higgs bosons**?

these questions are now academic, but might become relevant at a 100 TeV pp collider



The top quark as parton

Consider the production of a heavy scalar particle at 100 TeV. Two calculation methods:



Top as massive particle: gg => ttH



Top as parton: tt => H

The top quark as parton

Consider the production of a heavy scalar particle at 100 TeV. Two calculation methods:



Top as parton: tt => H

SM Parton Distributions

Assuming that we can neglect all fermion and boson masses (including Higgs) at **very high energies**, one can write the full set of SM evolution equations

$$q\frac{\partial}{\partial q}f_i(x,q) = \sum_I \frac{\alpha_I(q)}{\pi} \left[P_{i,I}^V(q) f_i(x,q) + \sum_j C_{ij,I} \int_x^{z_{\max}^{ij,I}(q)} \mathrm{d}z P_{ij,I}^R(z) f_j(x/z,q) \right]$$

sum over QCD and EW couplings

Ciafaloni, Comelli 05 Bauer, Webber 17

and ends up with 52 ``SM PDFs", where e.g. right and left handed quarks evolve separately

$\{\mathbf{T}, \mathrm{CP}\}$	fields
$\{0, +\}$	$2n_g \times q_R, n_g \times \ell_R, n_g \times q_L, n_g \times \ell_L, g, W, B, H$
$\{0, -\}$	$2n_g imes q_R, n_g imes \ell_R, n_g imes q_L, n_g imes \ell_L, H$
$\{1, +\}$	$n_g imes q_L, n_g imes \ell_L, BW, H$ $n_g = #$ generations
$\{1, -\}$	$n_g imes q_L, n_g imes \ell_L, W, H$
$\{2, +\}$	W

These PDFs can be perturbatively generated from the usual **quark**, **gluon and photon PDFs** at some matching scale around 100 GeV

SM Parton Distributions



If a 100 TeV collider is built, electroweak PDFs might play an important role for many processes

Summary and outlook

The accurate determination of the **quark and gluon structure of the proton** is an essential ingredient for **LHC phenomenology** and **beyond**

- Recent progress in our understanding of the strange and charm content of the proton, but still several important open questions. What about intrinsic bottom?
- QED effects and the photon PDF are now a standard ingredient of global PDF determinations, with photon- and lepton-initiated processes now computable with high precision
- At very high energies, one needs to account for the **full gauge structure of the SM** in the evolution equations: Higgs and gauge boson PDFs, quark PDF polarisation

The fascinating study of the proton structure never stops surprising us, stay tuned!