Parton Distributions with Theoretical Uncertainties

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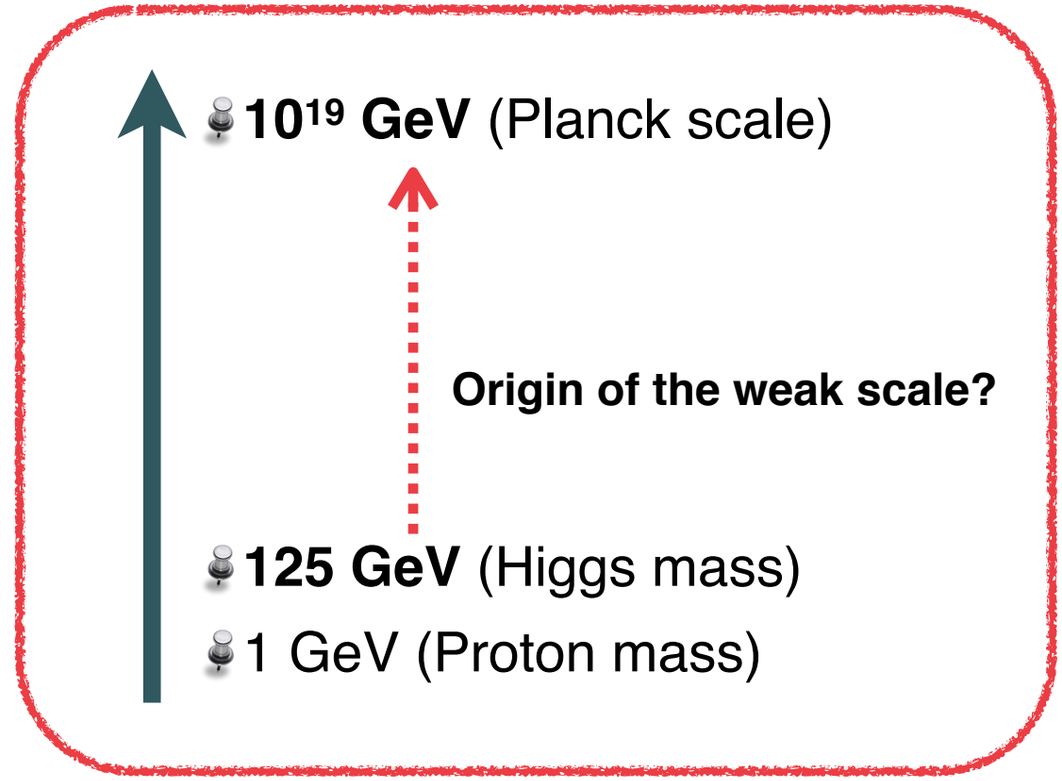
Seminar Fundamentele Wechselwirkungen
University of Freiburg, 09/07/2019

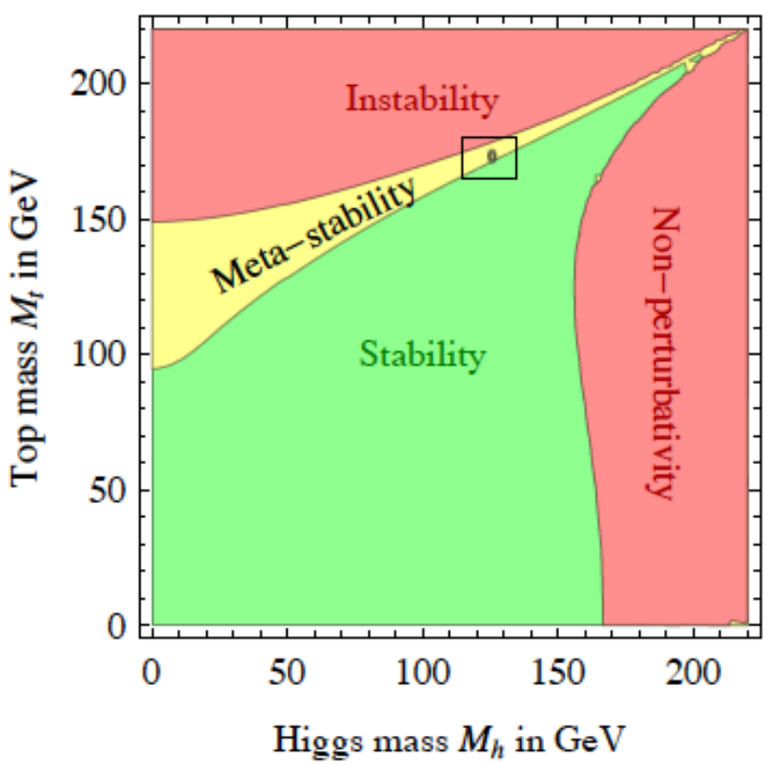
Particle physics in the LHC precision era

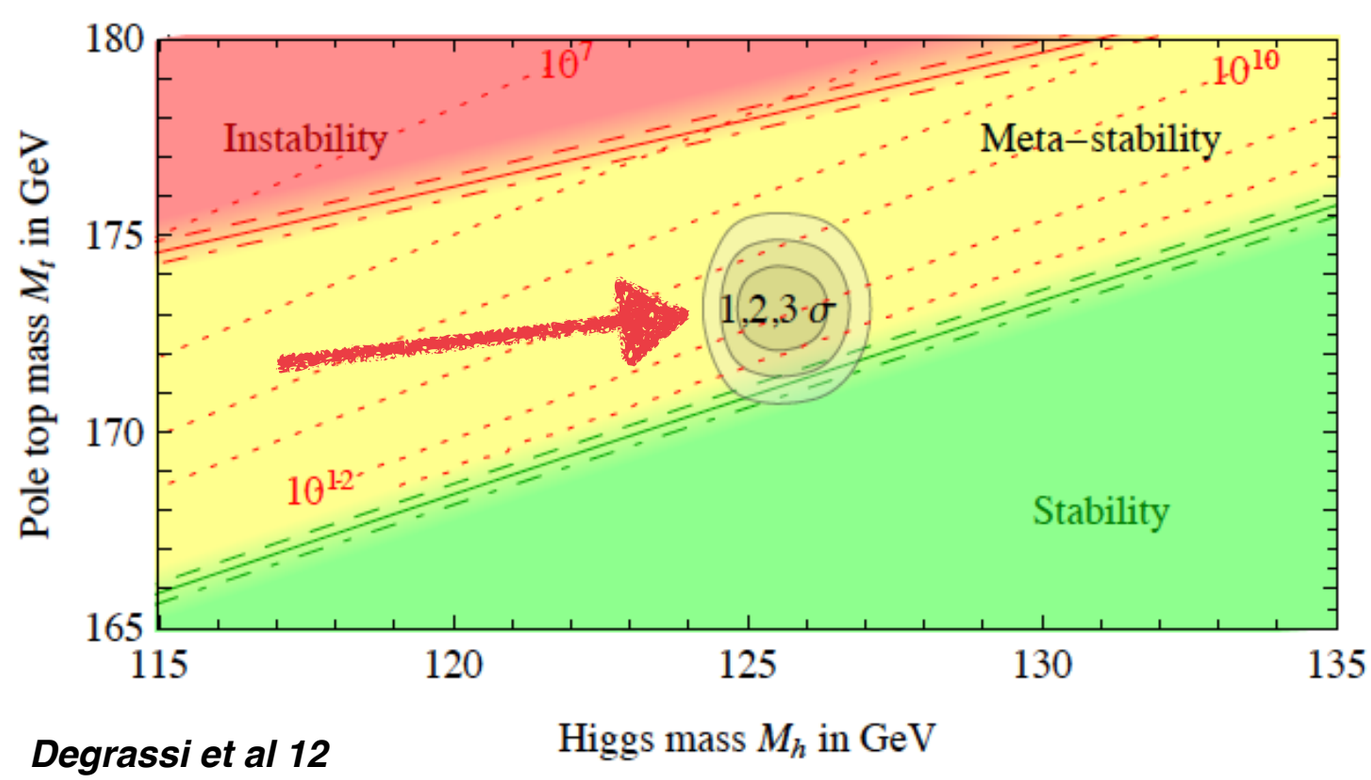
Open questions in particle physics

The Higgs boson

- Huge gap between **weak** and **Plank scales?**
- **Compositeness**? Non-minimal Higgs sector?
- Coupling to **Dark Matter**? Role in cosmological phase transitions?
- Is the **vacuum state of the Universe** stable?







Degrassi et al 12

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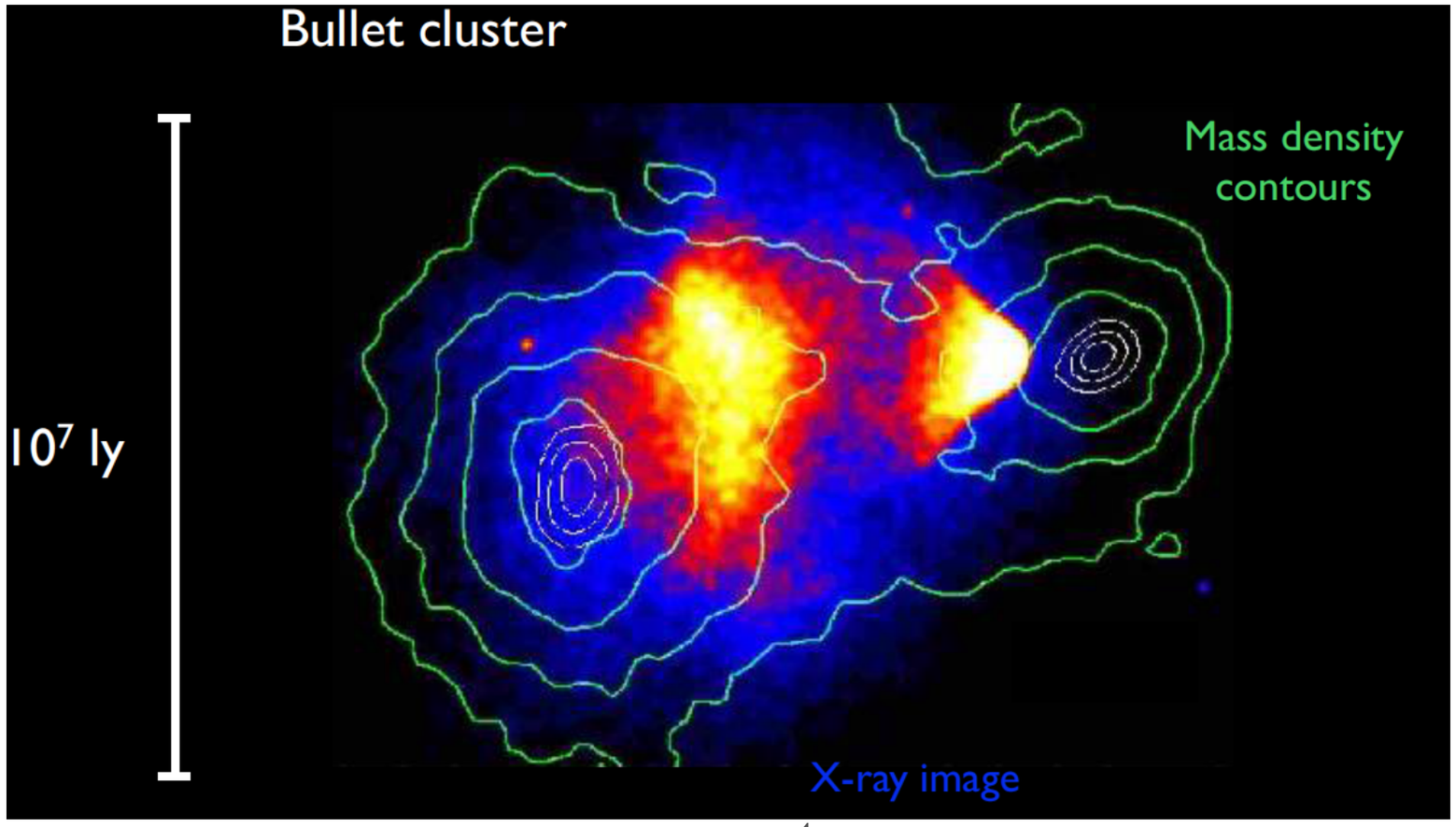
Dark matter

- **Weakly interacting massive particles**? Neutrinos? Ultralight particles (axions)?
- **Interactions** with SM particles? Self-interactions?
- **Structure** of the Dark Sector?



Open questions in particle physics

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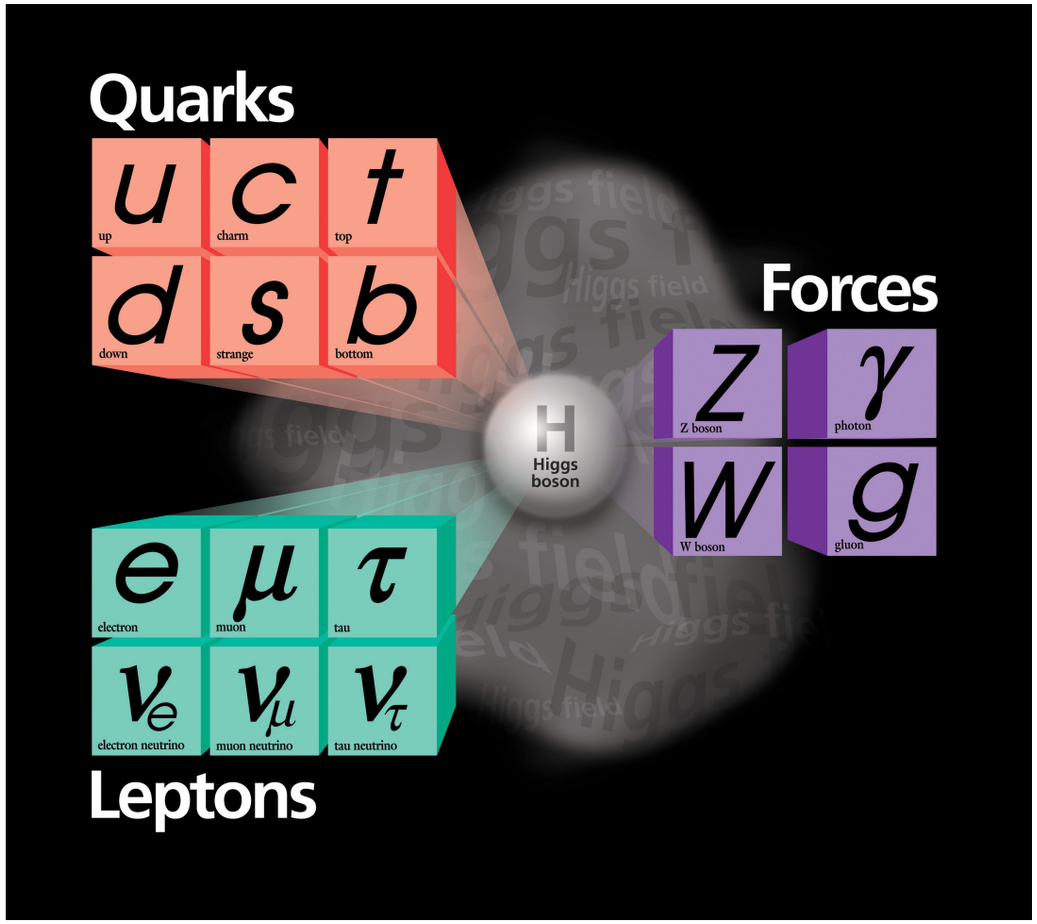
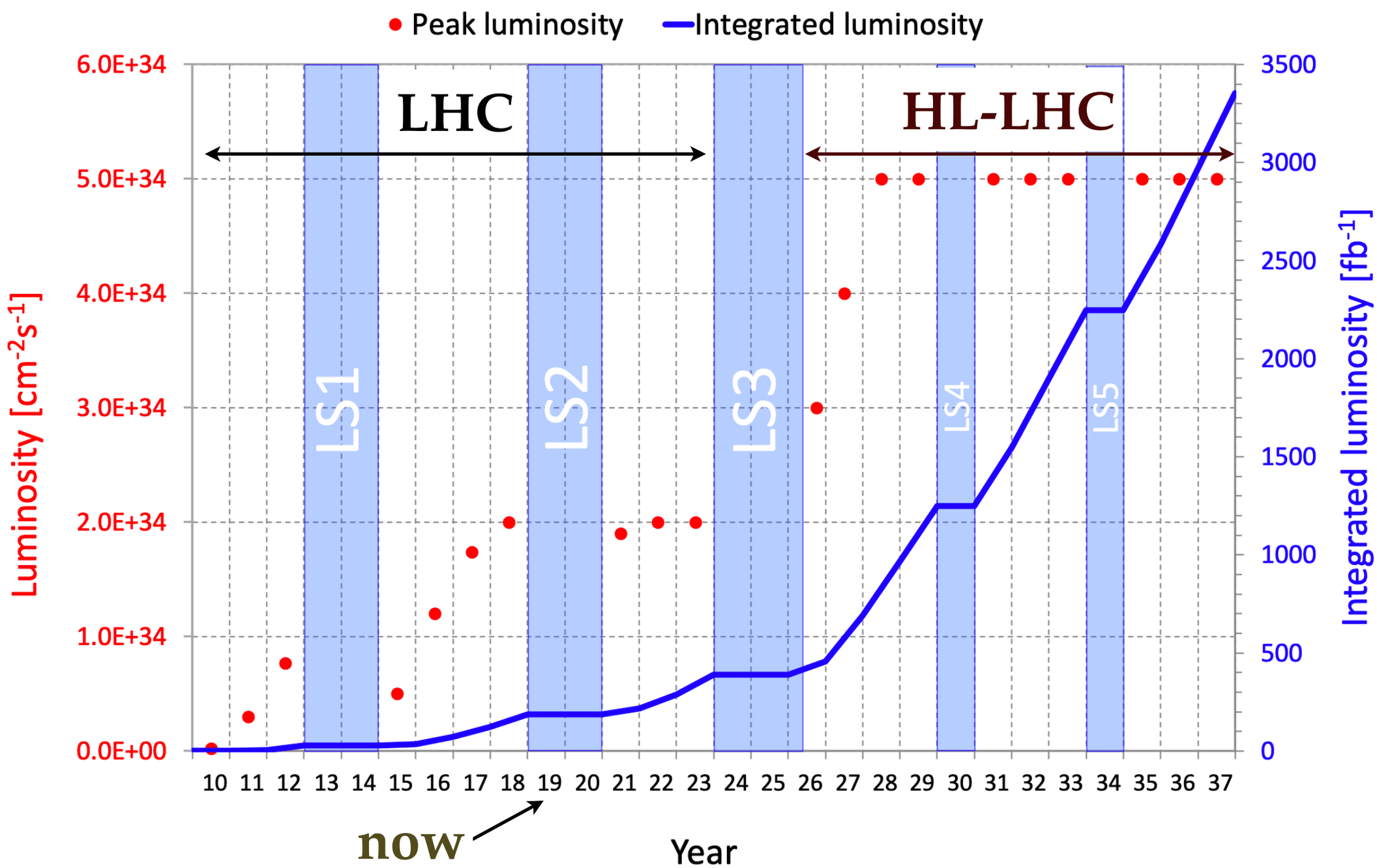
Quarks and leptons

- Why **3 families?** Origin of **masses, mixings**?
- Origin of **Matter-Antimatter asymmetry?**
- Lepton Flavour **Universality?**
- Origin of **neutrino masses?** Are neutrinos Majorana or Dirac?



Open questions in particle physics



Crucial information on these fundamental questions will be provided by the LHC: the **exploration of the high-energy frontier** has just started!

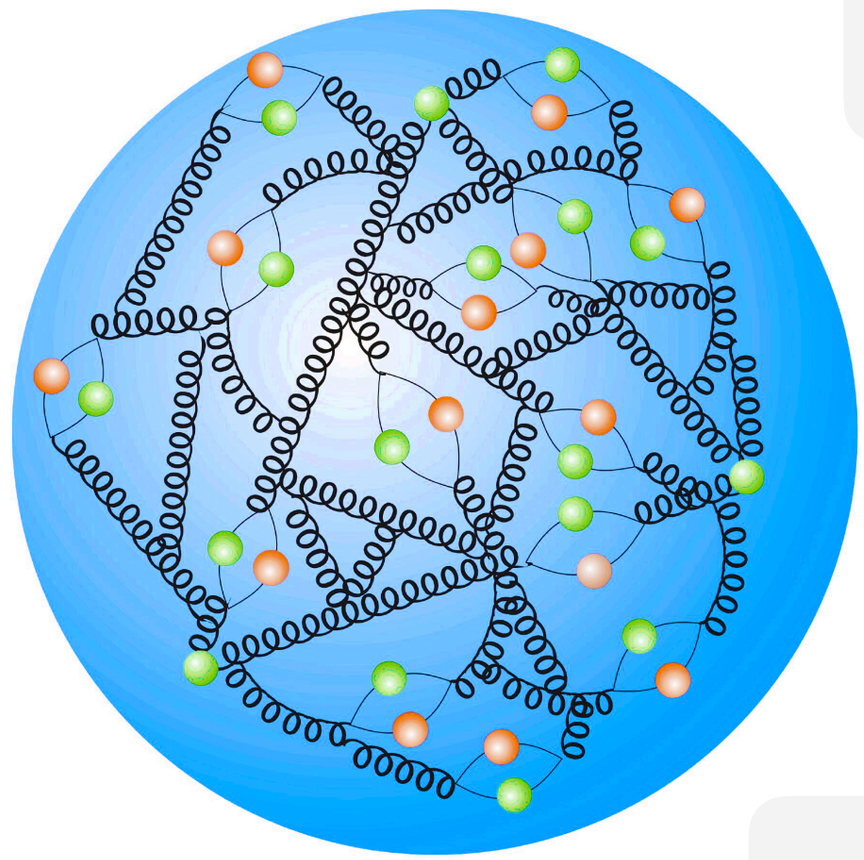
The inner life of protons

The many faces of the proton

Proton = QCD bound state of **quarks** and **gluons**

Origin of mass?

Origin of spin?



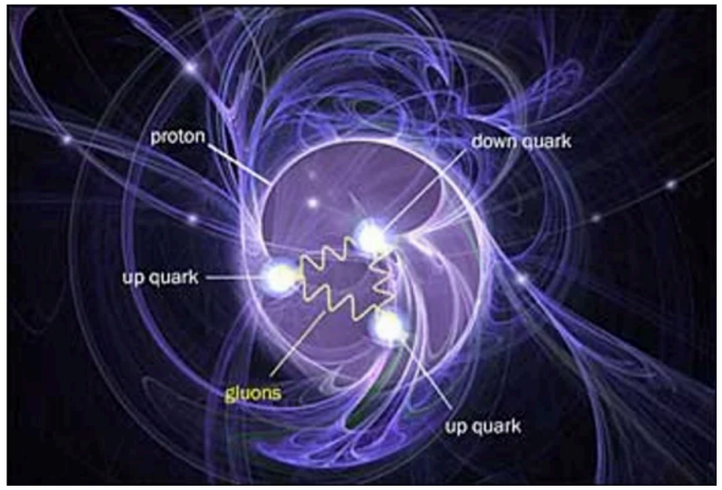
Intrinsic charm quarks?

Nuclear modifications?

The strong force in the spotlight

THE SCIENCES

Proton Spin Mystery Gains a New Clue



Non-zero gluon polarisation

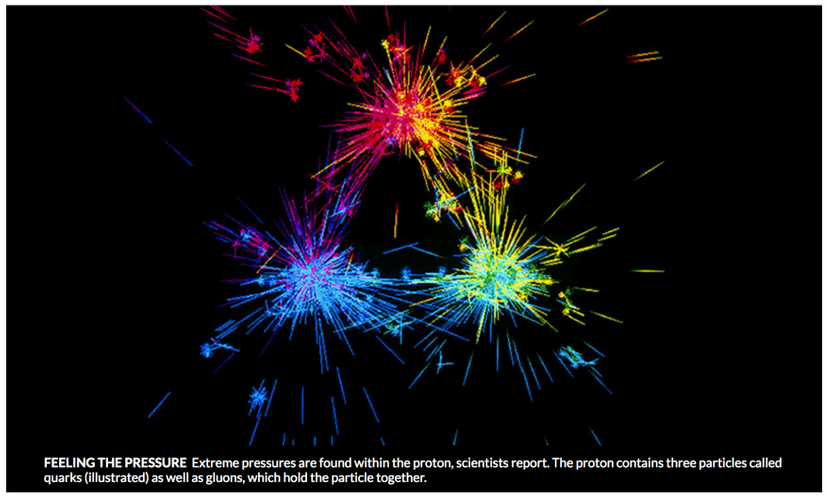
Scientific American
based on *Rojo et al, NPB* (2014)

NEWS PARTICLE PHYSICS

The inside of a proton endures more pressure than anything else we've seen

For the first time, scientists used experimental data to estimate the pressure inside a proton

BY EMILY CONOVER 1:18PM, MAY 16, 2018

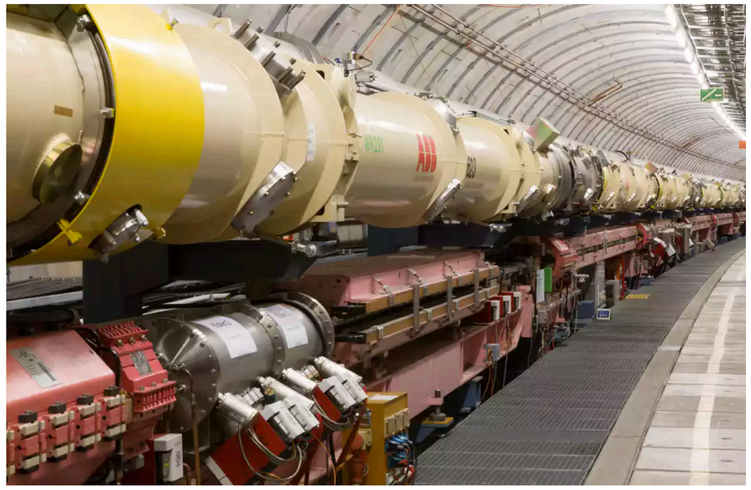


FEELING THE PRESSURE Extreme pressures are found within the proton, scientists report. The proton contains three particles called quarks (illustrated) as well as gluons, which hold the particle together.

Science News
based on *Burkert et al., Nature* (2018)

After 40 years of studying the strong nuclear force, a revelation

This was the year that analysis of data finally backed up a prediction, made in the mid 1970s, of a surprising emergent behaviour in the strong nuclear force

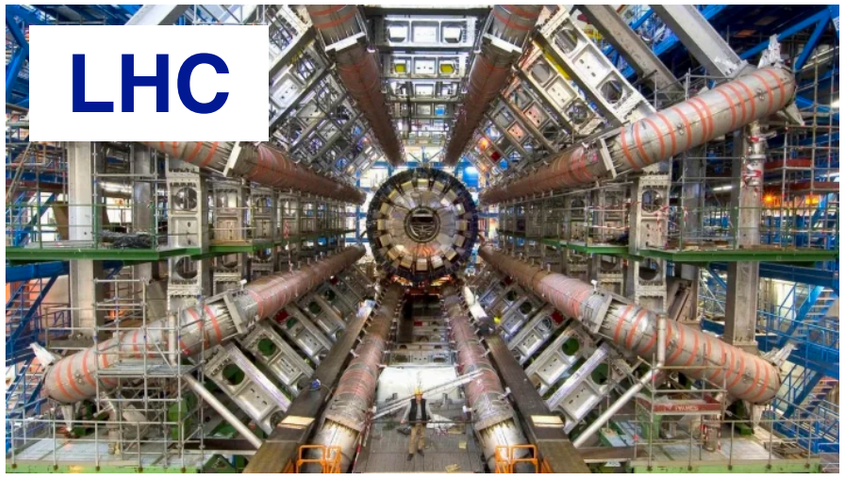


gluon-dominated matter

The Guardian
based on *Rojo et al,* EPJC (2018)

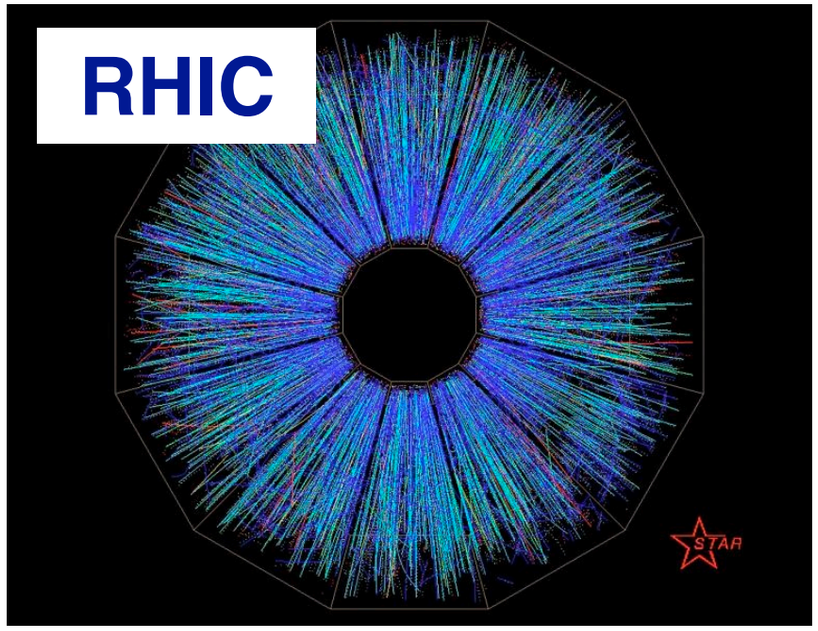
4 decades with **Quantum Chromodynamics** (**QCD**):
still uncovering **novel phenomena!**

From colliders to the cosmos



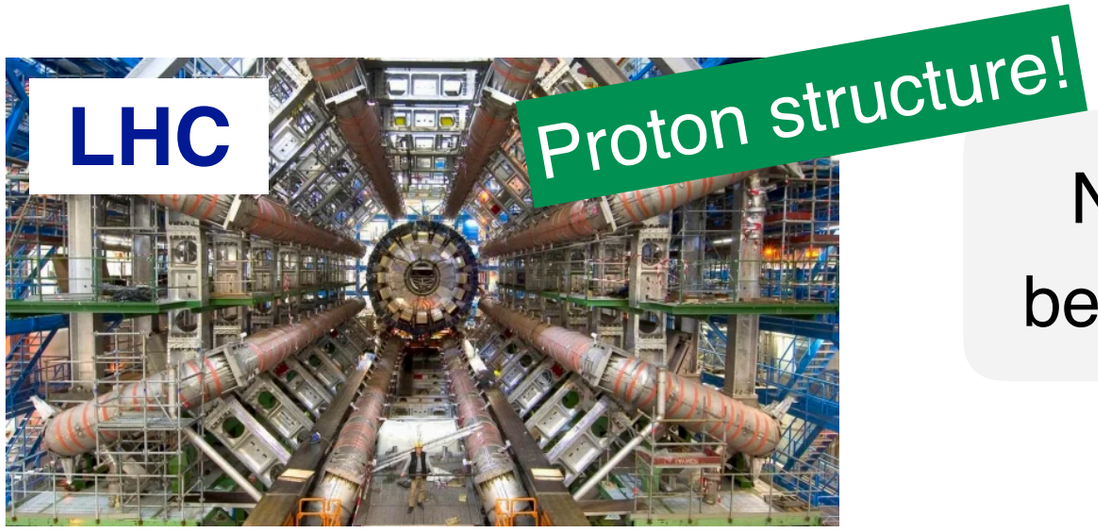
New **elementary particles** beyond the **Standard Model?**

Origins and properties of **cosmic neutrinos**?



Nature of **Quark-Gluon Plasma** in **heavy-ion collisions?**

From colliders to the cosmos



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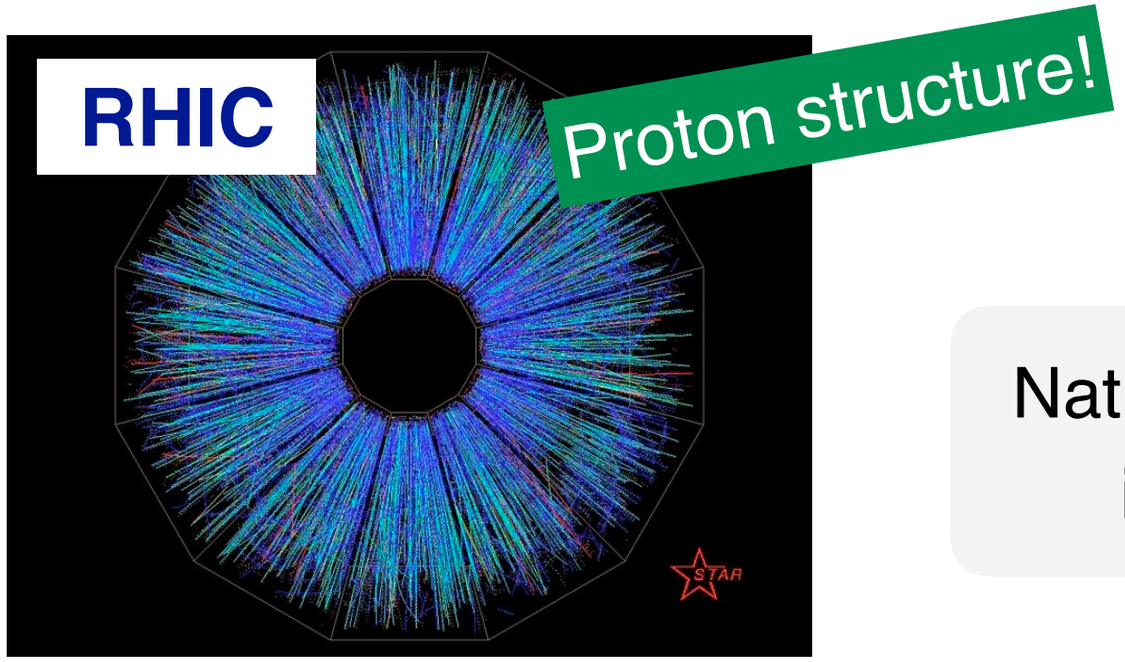
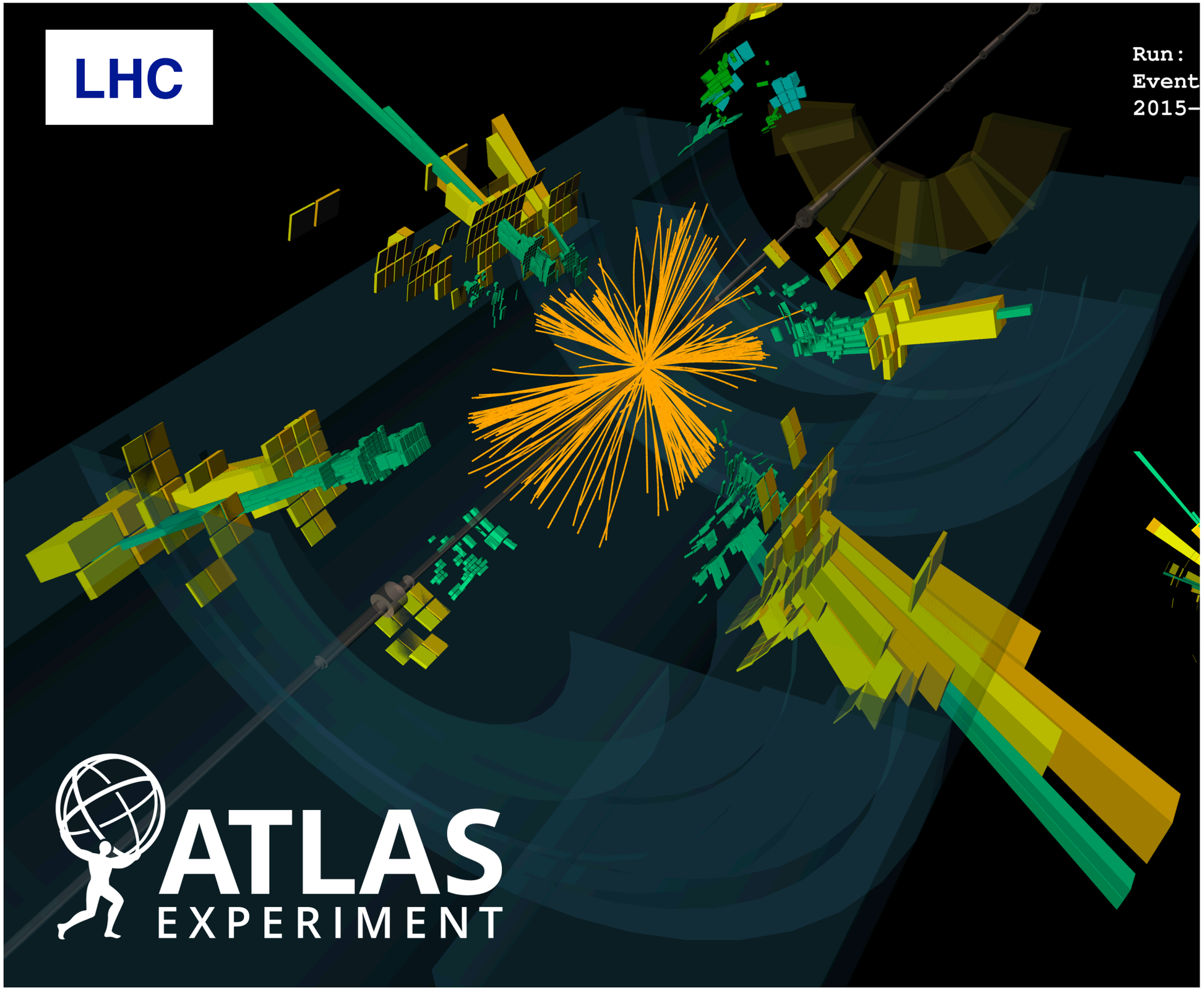


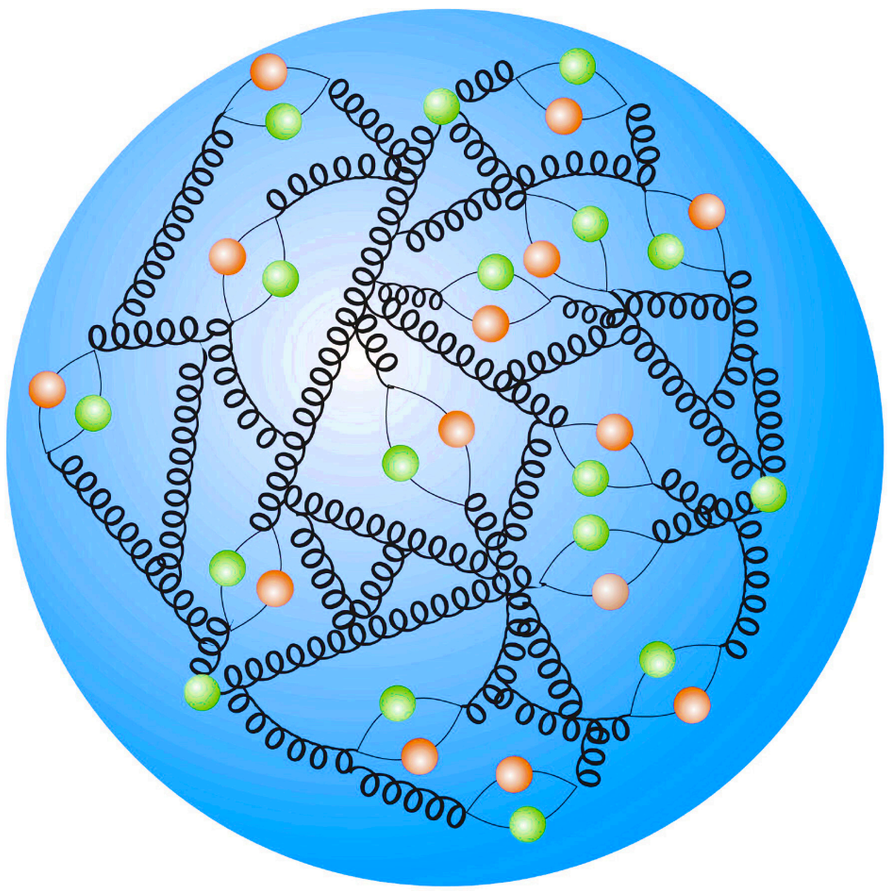
Nature of **Quark-Gluon Plasma** in **heavy-ion collisions?**

QCD in collisions



Parton Distributions

Proton energy divided among constituents: **quarks** and **gluons**



Parton Distribution Functions (PDFs)

Determine from **data***:

Global QCD analysis

Mass? Spin? Heavy quarks?

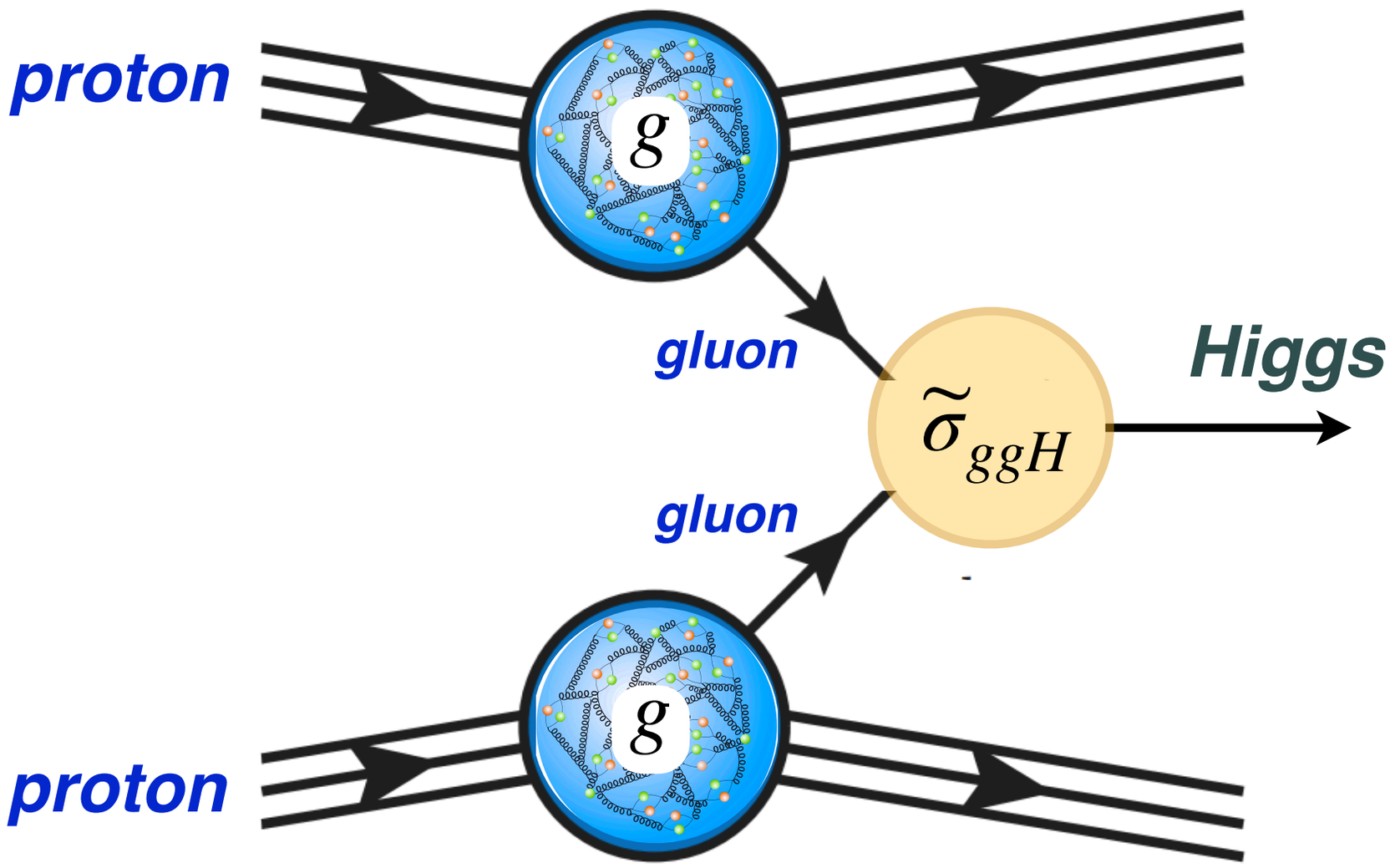
Event rates LHC, RHIC, IceCube?

* *also lattice QCD data*

Parton Distributions

$$N_{\mathrm{LHC}}(H) \sim g \otimes g \otimes \widetilde{\sigma}_{ggH}$$

Parton Distributions



All-order structure: **QCD factorisation theorems**

Parton Distributions

$$g(x, Q)$$

Energy of hard-scattering reaction**:** inverse of resolution length

x: fraction of proton momentum carried by gluon

Probability of **finding a gluon inside a proton**, carrying a fraction ***x*** of the proton momentum, when probed with energy ***Q***

Dependence on *x* fixed by **non-perturbative QCD dynamics**: extract from experimental data

- **Energy conservation**: momentum sum rule

$$\int_0^1 dx\, x \left(\sum_{i=1}^{n_f} \left[q_i((x, Q^2) + \bar{q}_i(x, Q^2) \right] + g(x, Q^2) \right) = 1$$

- **Quark number conservation**: valence sum rules

$$\int_0^1 dx\, \left(u(x, Q^2) + \bar{u}(x, Q^2) \right) = 2$$

Parton Distributions

$$g(x, Q)$$

Energy of hard-scattering reaction**:** inverse of resolution length

x: fraction of proton momentum carried by gluon

Probability of **finding a gluon inside a proton**, carrying a fraction ***x*** of the proton momentum, when probed with energy ***Q***

Dependence on ***Q*** fixed by **perturbative QCD dynamics**: computed up to $\mathcal{O}\left(\alpha_s^4\right)$

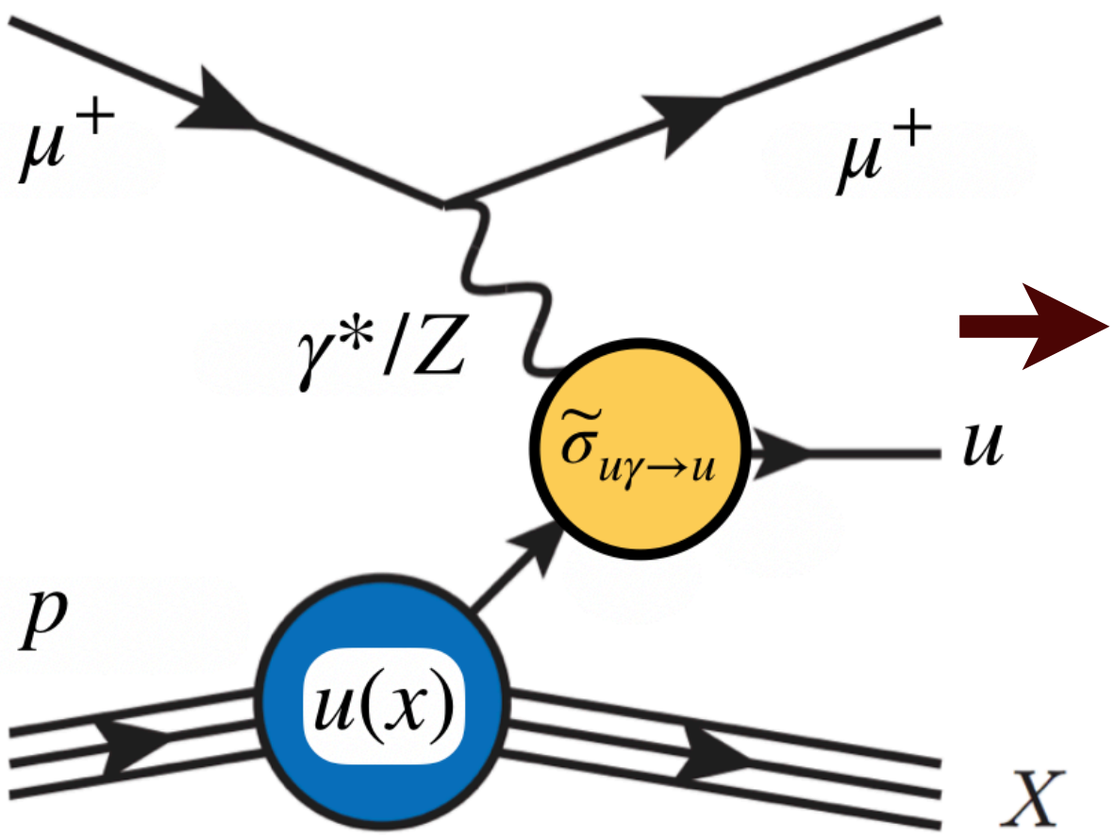
$$\frac{\partial}{\partial \ln Q^2} q_i(x, Q^2) = \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}, \alpha_s(Q^2)\right) q_j(z, Q^2)$$

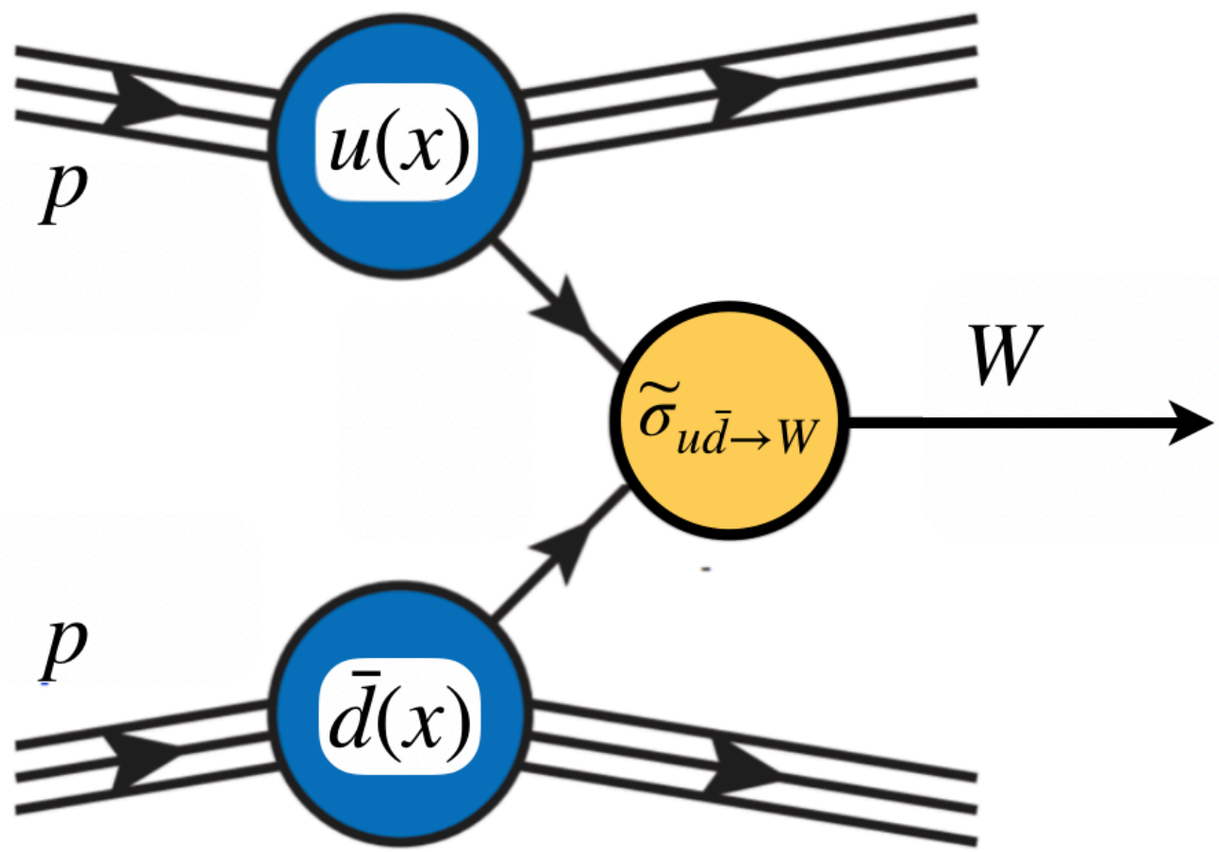
DGLAP *parton evolution equations*

The Global QCD analysis paradigm

QCD factorisation theorems**: PDF universality**

$$\sigma_{l\,p\to\mu\,X} = \widetilde{\sigma}_{u\gamma\to u} \otimes u(x) \;\rightarrow\; \sigma_{p\,p\to W} = \widetilde{\sigma}_{u\bar{d}\to W} \otimes u(x) \otimes \bar{d}(x)$$

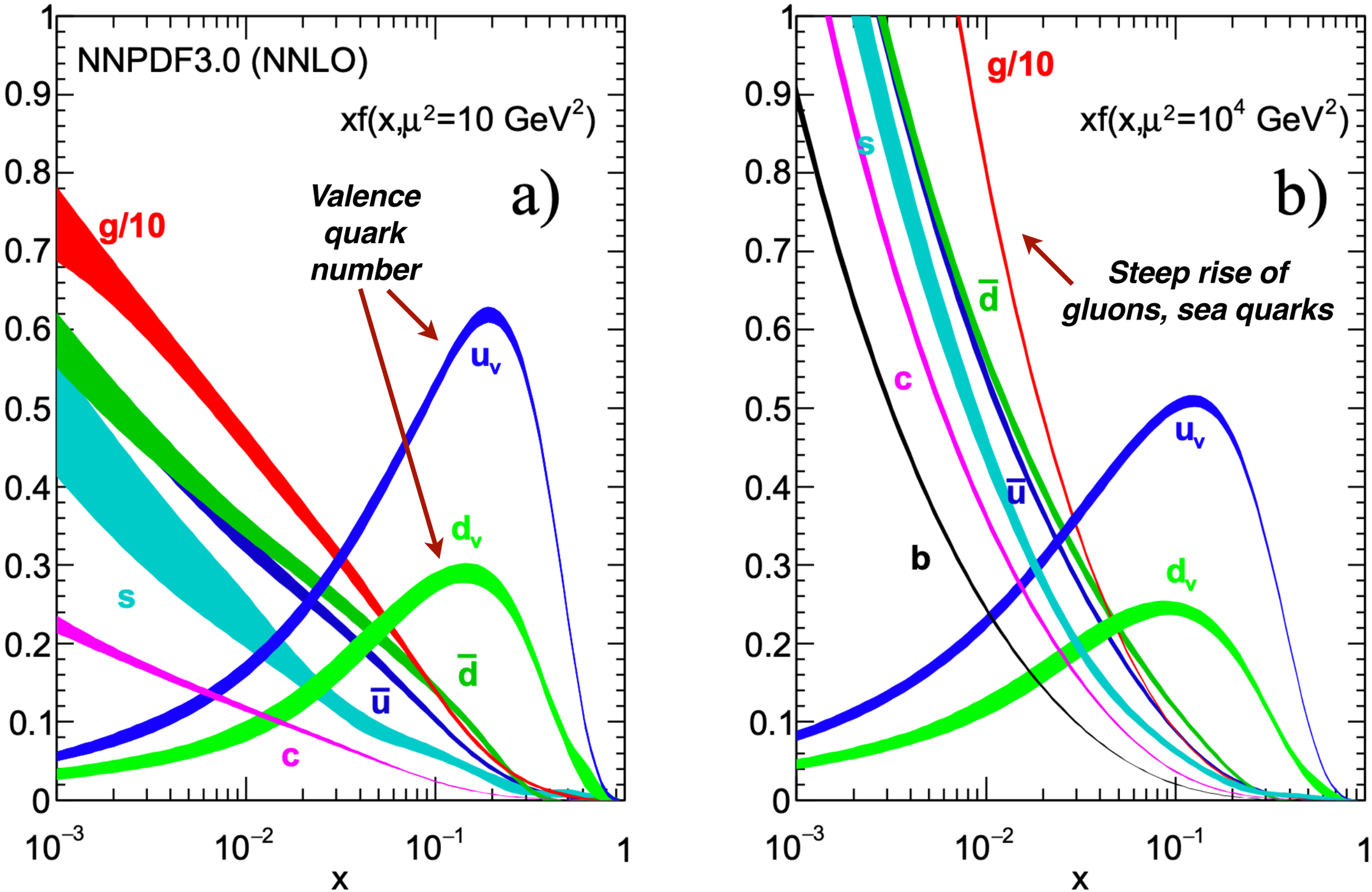




*Determine PDFs in **lepton-proton collisions** (deep-inelastic scattering) …*

*… and use them to compute predictions for **proton-proton collisions***

A proton structure snapshop



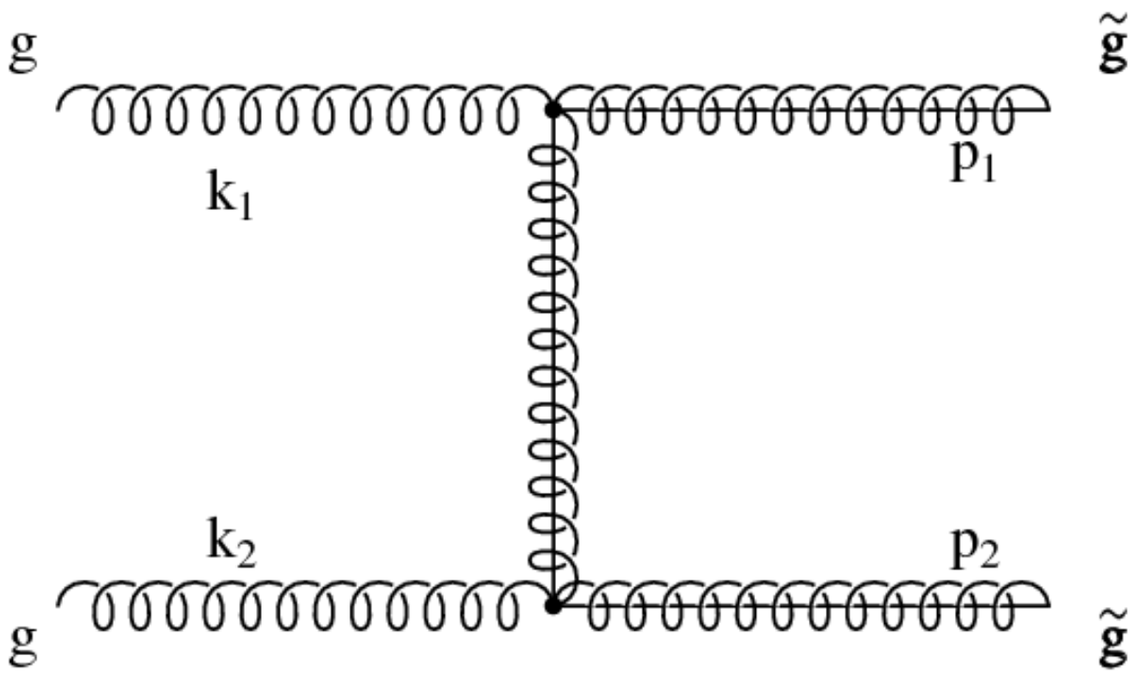
***x**: parton momentum fraction*

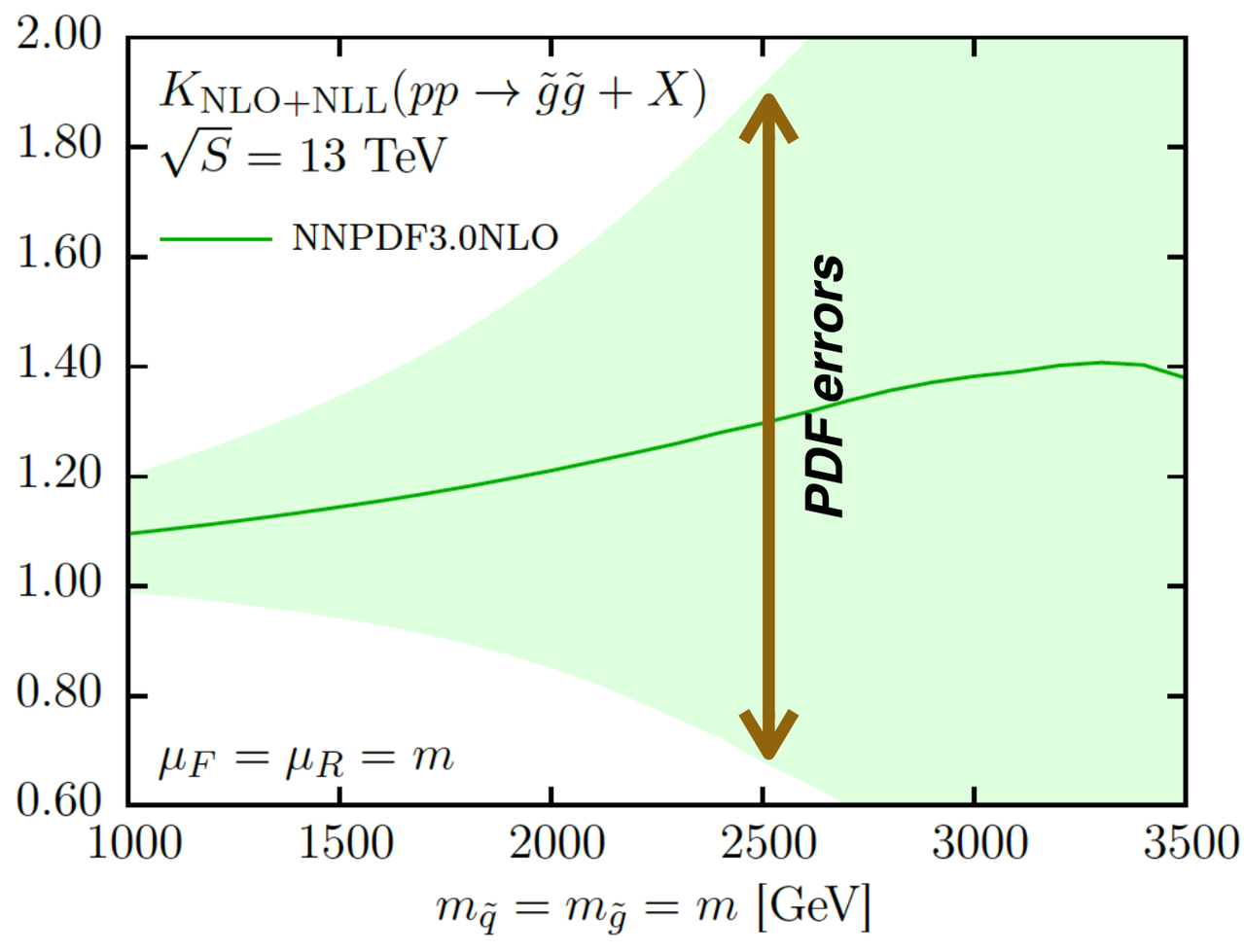
Why do we need better PDFs?

PDF uncertainties in the production of **New Physics heavy resonances** up to **100%**

Due to limited coverage of the **large Bjorken-*x* region**

gluino-pair production in supersymmetry



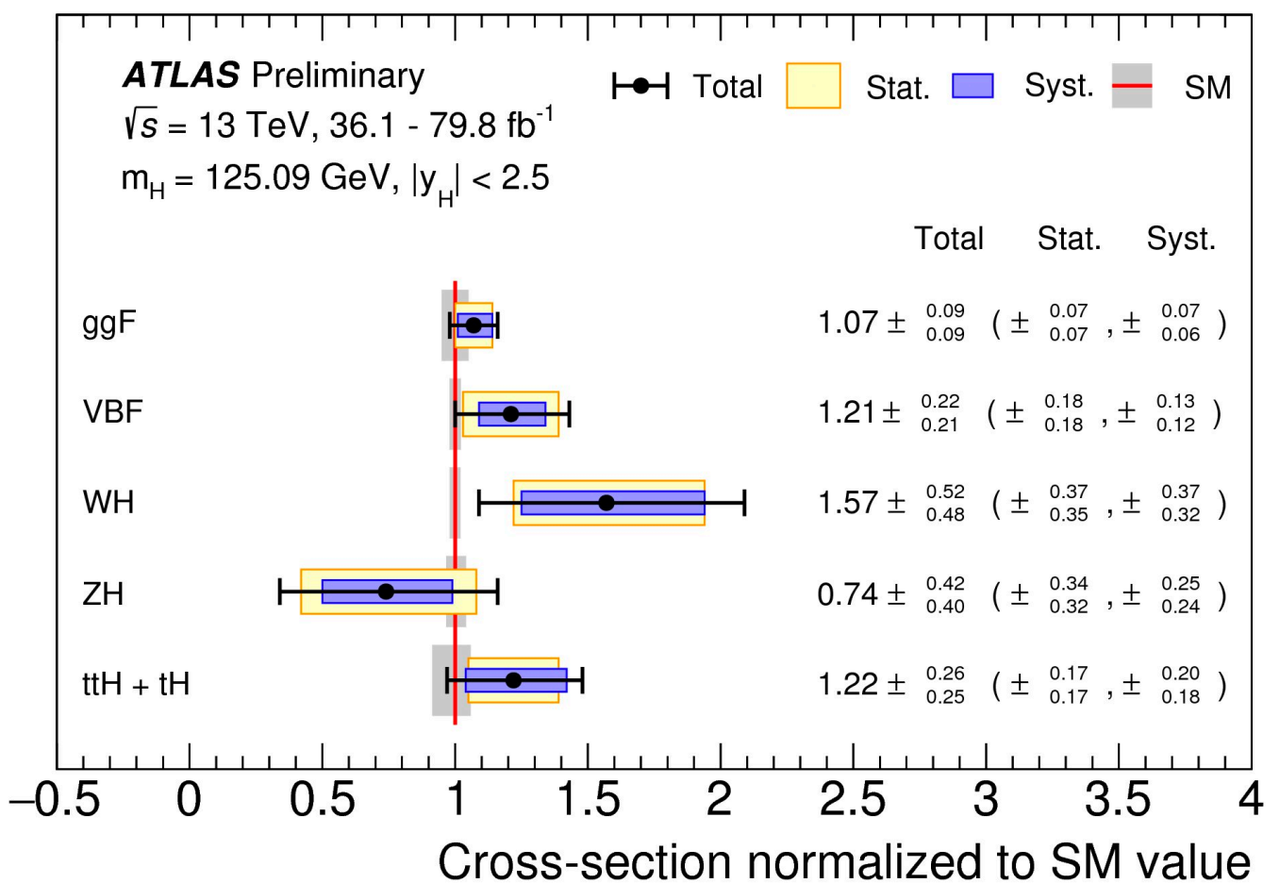


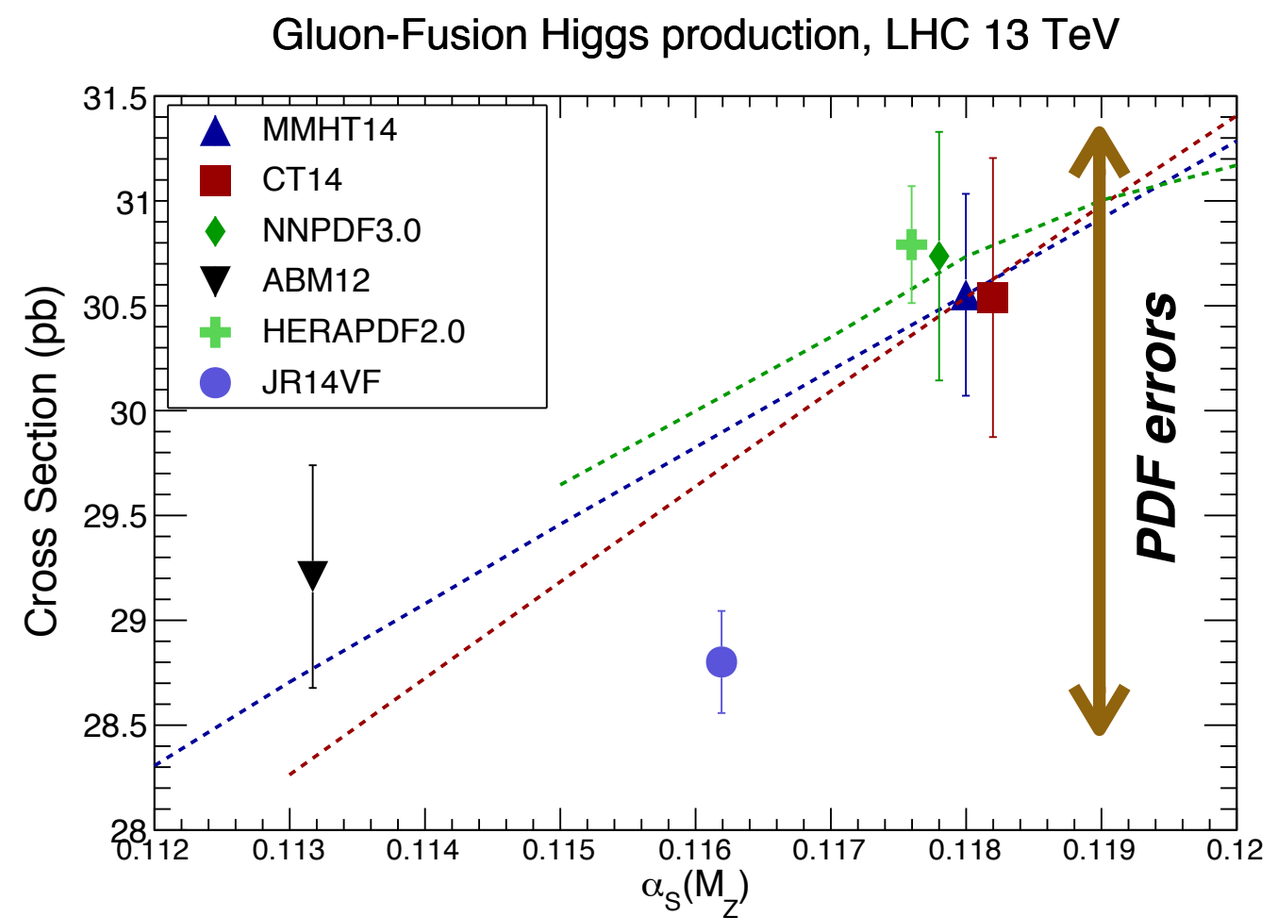
Why do we need better PDFs?

PDF uncertainties one of dominant theory errors in Higgs production cross-sections

Even small **deviations of Higgs couplings** from SM predictions: smoking gun for BSM

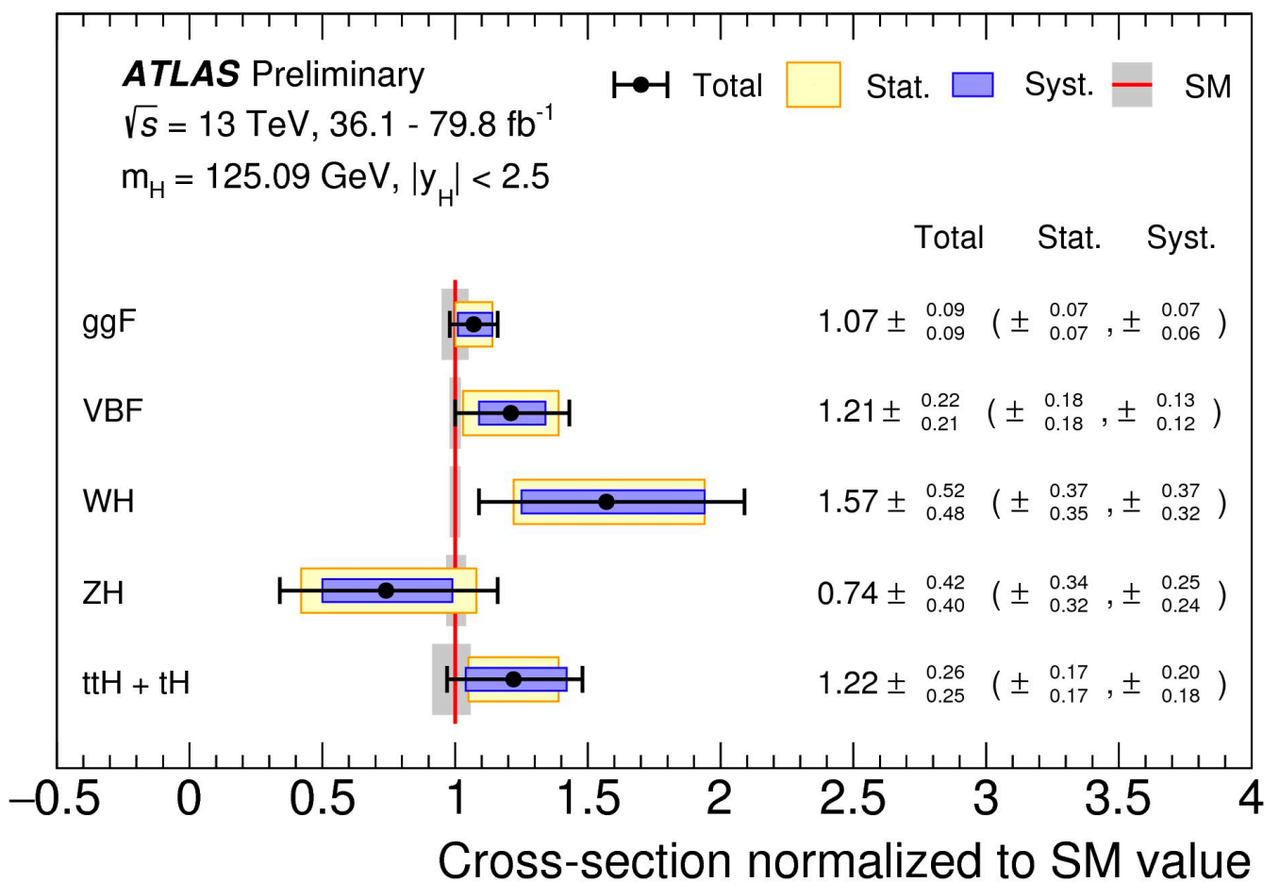
Inclusive Higgs production rates



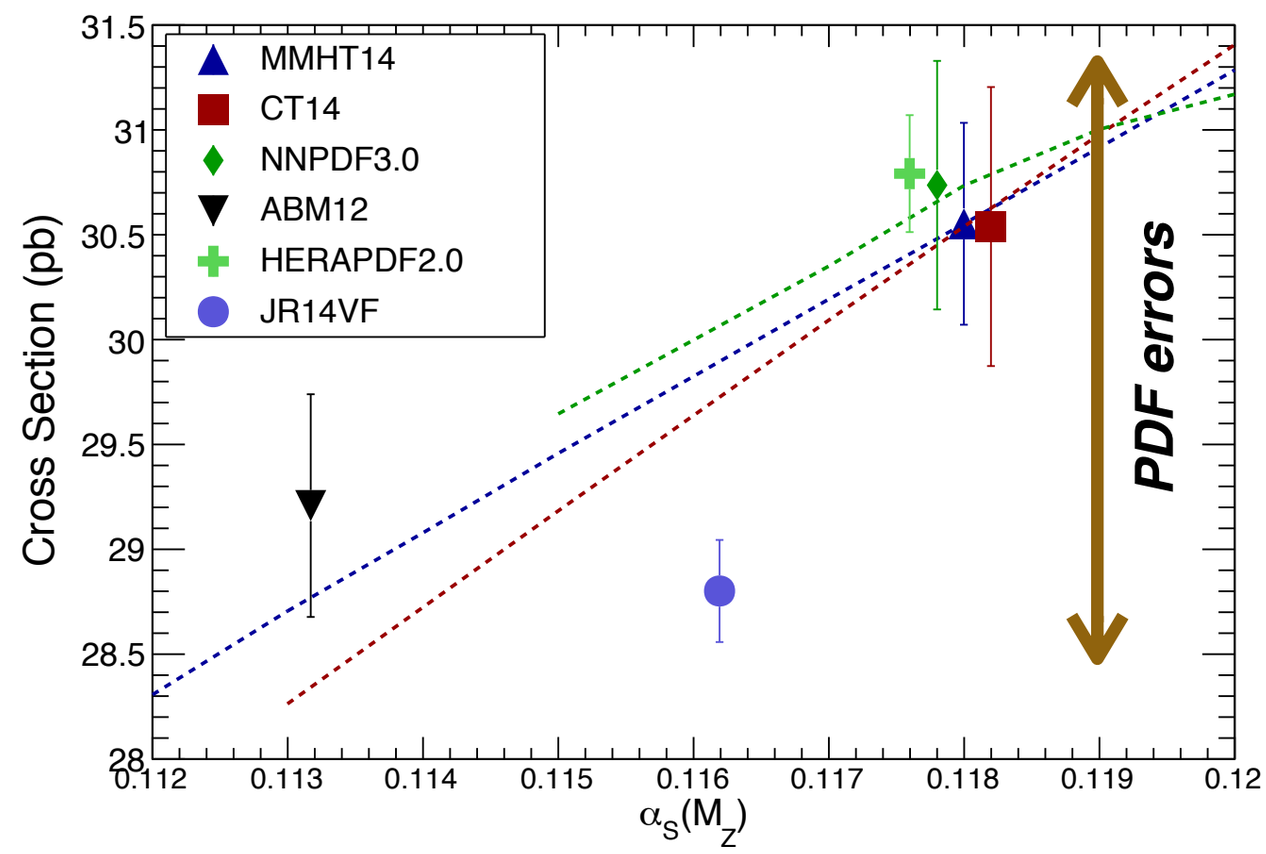


Why do we need better PDFs?

BSM model	Deviations in Higgs coupling to		
	W, Z weak bosons	bottom quarks	photons
New heavy Higgs boson	6%	6%	6%
Two-Higgs Doublet model	1%	10%	1%
Composite Higgs	-3%	-9%	-9%
New heavy top-like quark	-2%	-2%	+2%



Gluon-Fusion Higgs production, LHC 13 TeV



Why do we need better PDFs?

- Heavy bSM physics beyond the direct reach of the LHC can be **parametrised in a model-independent** in terms of complete basis of **higher-dimensional operators:** this is the Standard Model Effective Field Theory

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i^{N_{d6}} \frac{c_i}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j^{N_{d8}} \frac{b_j}{\Lambda^4}\mathcal{O}_j^{(8)} + \dots ,$$

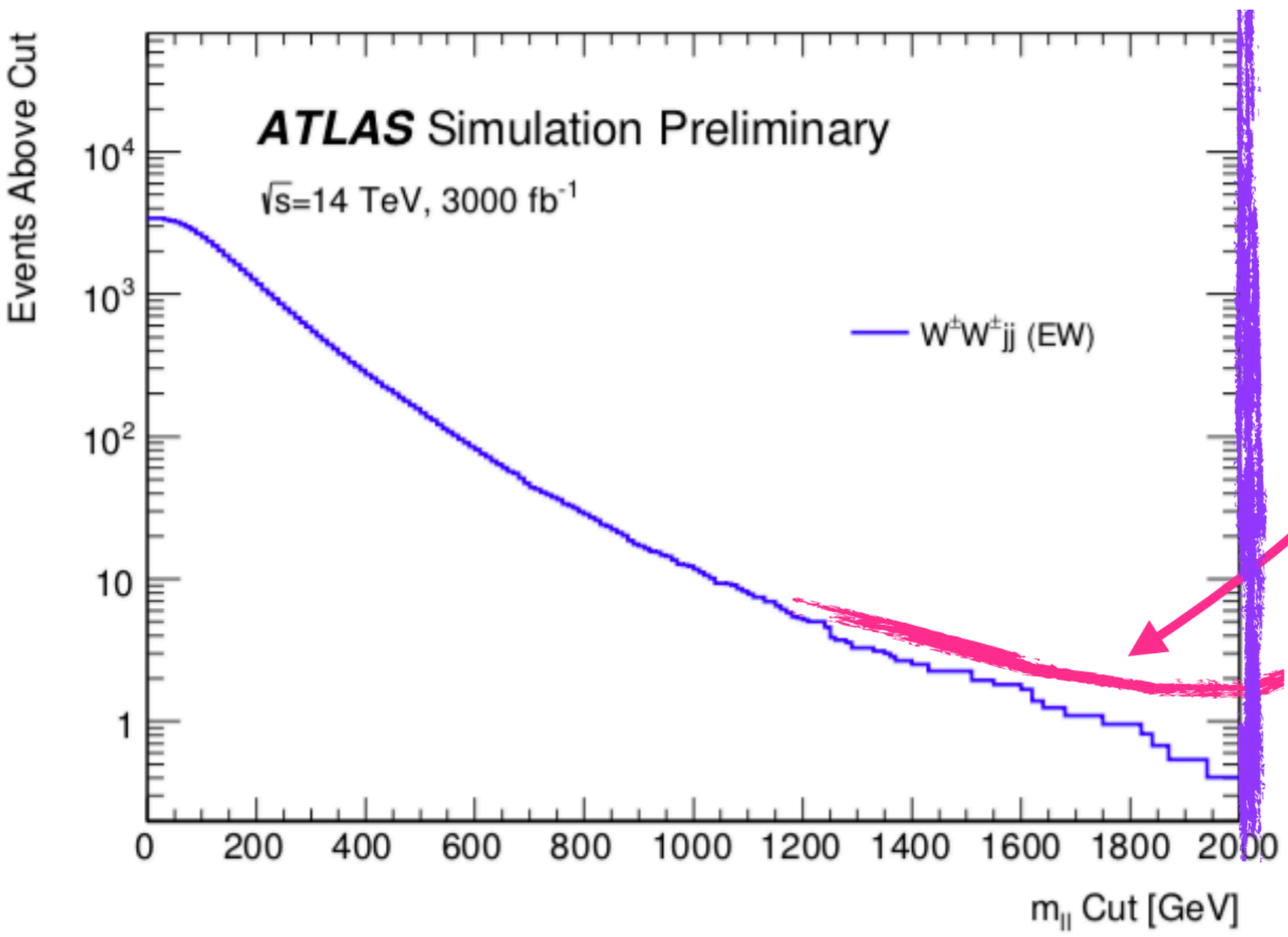
- Some operators induce **growth with the partonic centre-of-mass energy**: increased sensitivity in LHC cross-sections in the TeV region

$$\sigma(E) = \sigma_{\rm SM}(E)\left(1 + \sum_i^{N_{d6}} \omega_i \frac{c_i\, m_{\rm SM}^2}{\Lambda^2} + \sum_i^{N_{d6}} \widetilde{\omega}_i \frac{c_i\, E^2}{\Lambda^2} + \mathcal{O}\left(\Lambda^{-4}\right)\right)$$

*enhanced sensitivity from **TeV-scale processes:***
unique feature of LHC

Why do we need better PDFs?

Deviation from SM prediction in high energy tails?



CA Lee, HL/HE-LHC Jamboree, 1 March 2019

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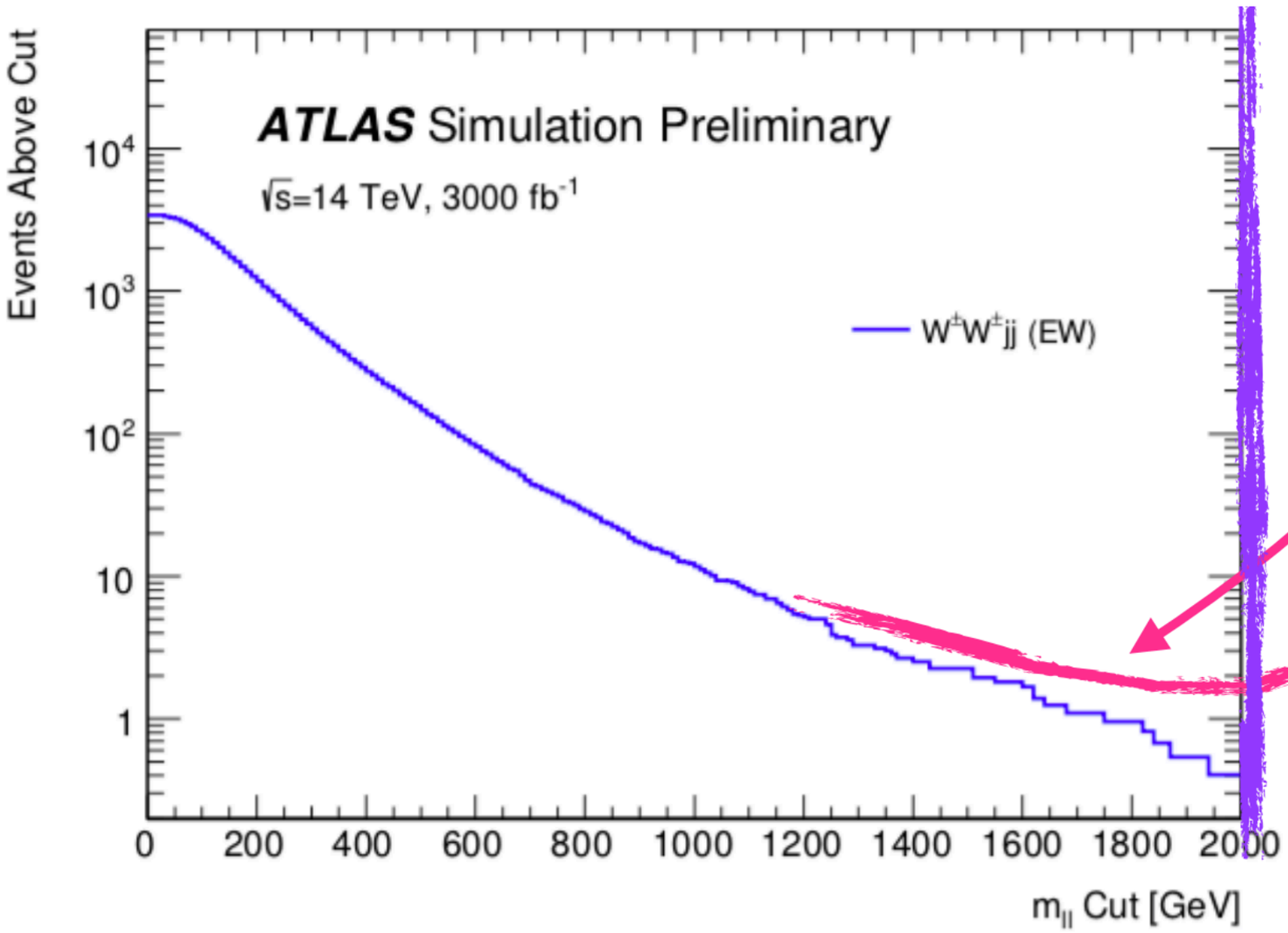
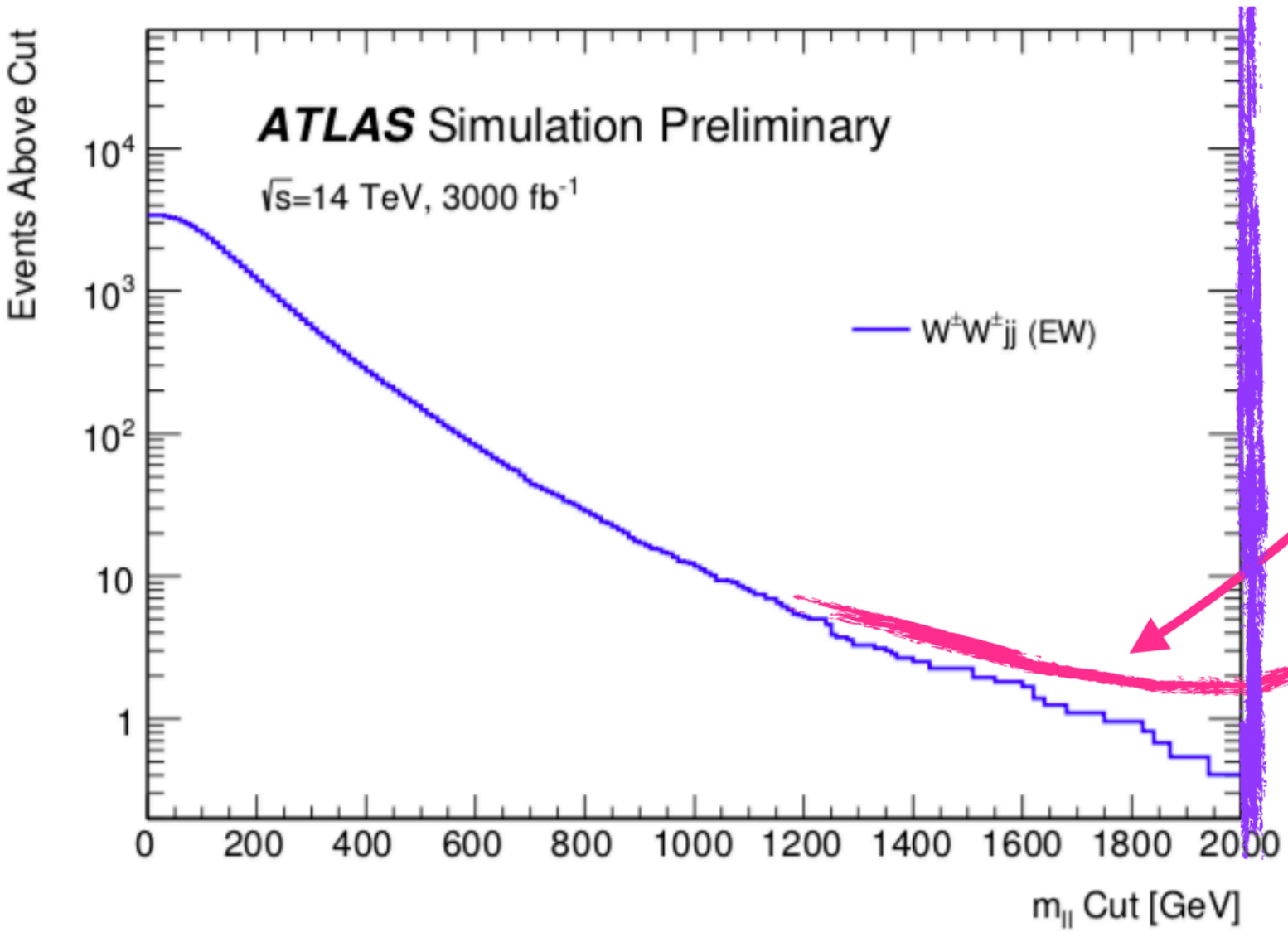


CA Lee, HL/HE-LHC Jamboree, 1 March 2019

SMEFT interpretation: from a massive particle at high energies …

Why do we need better PDFs?

Deviation from SM prediction in high energy tails?



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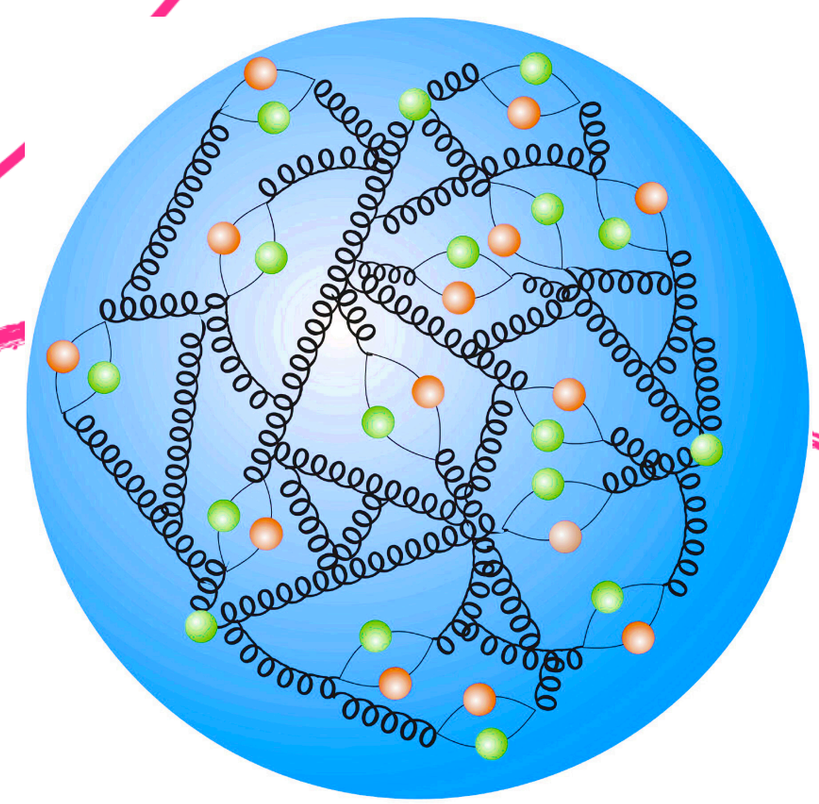
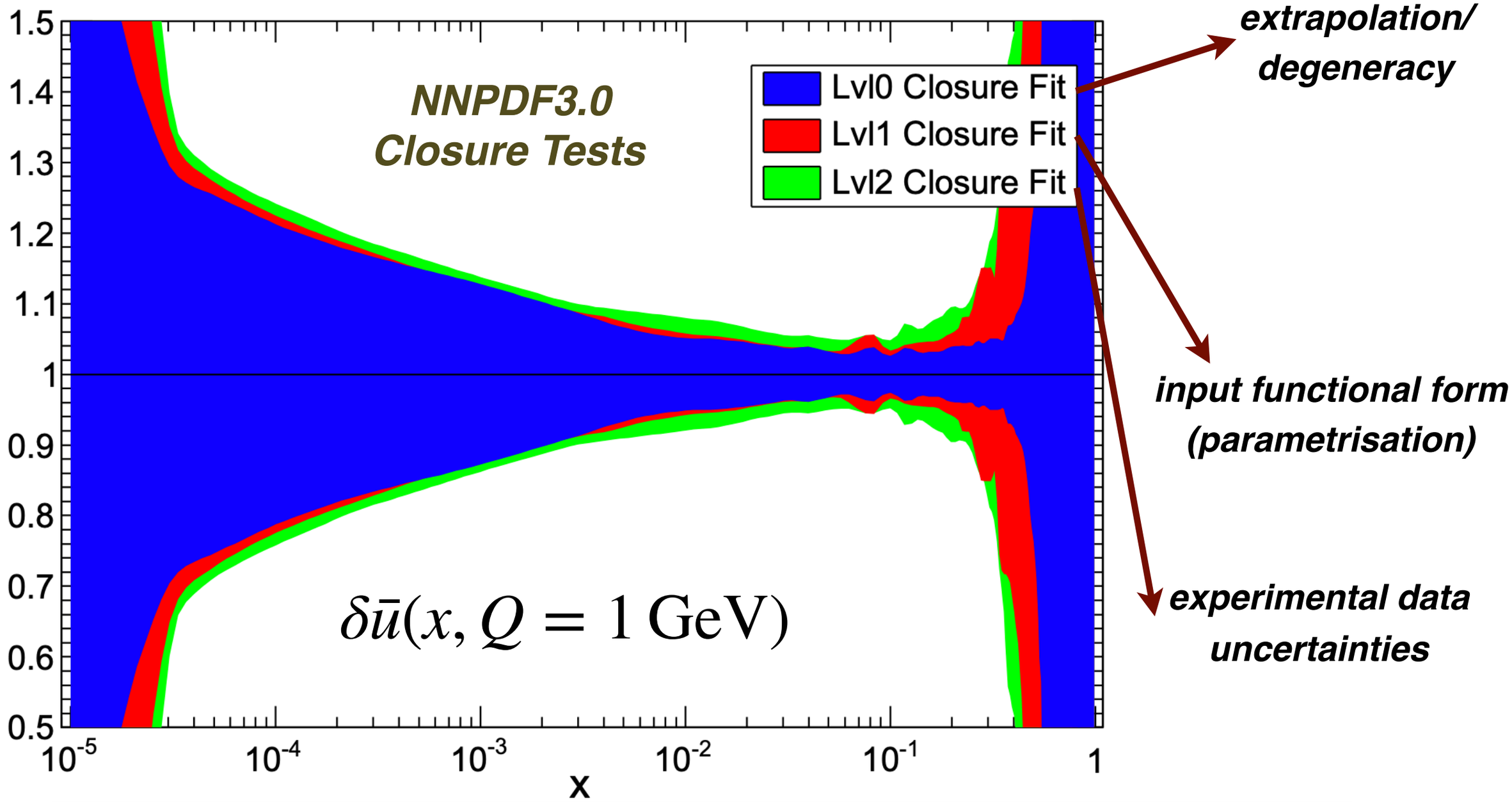


…or reflecting our limited understating of proton structure?

PDF uncertainties

PDF uncertainties

PDF uncertainties receive contributions from **different sources**:



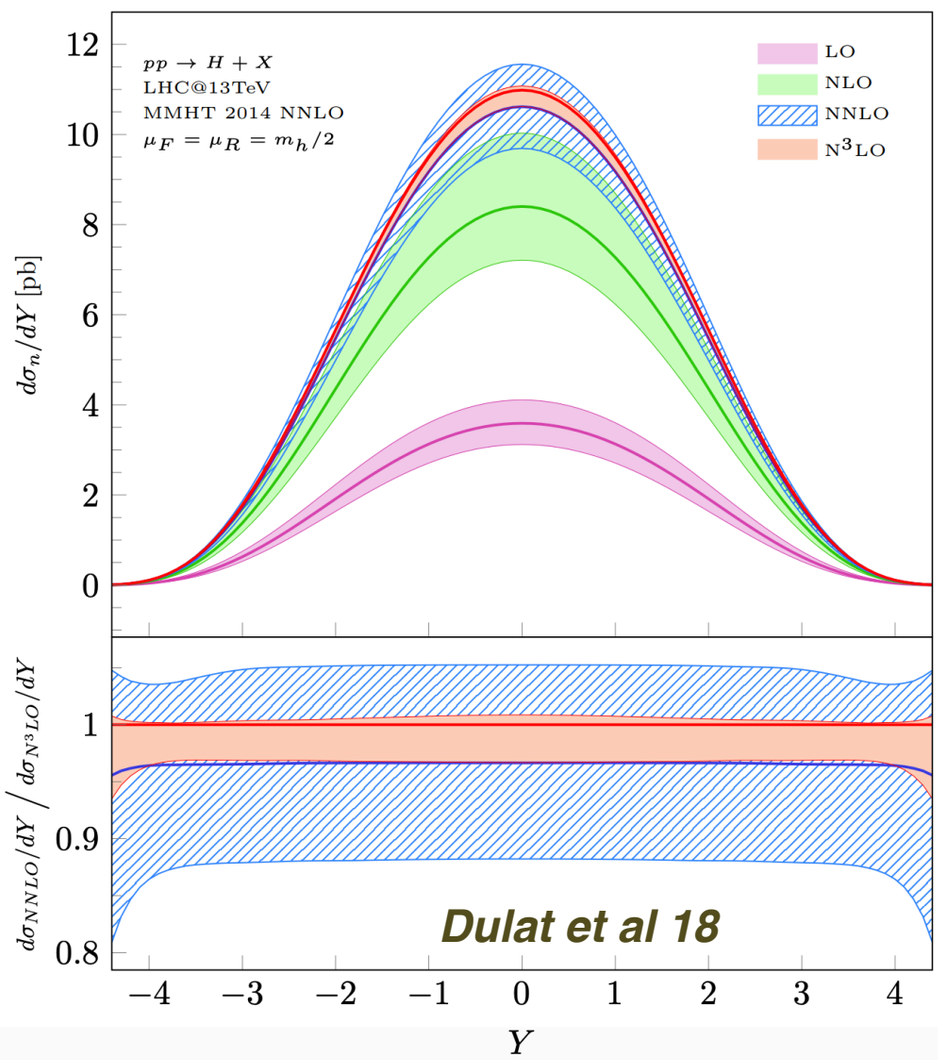
What about **theory uncertainties**? Parametric theory errors from $\delta\alpha_S$, δm_c are routinely accounted for, but other relevant contributions are missing ….

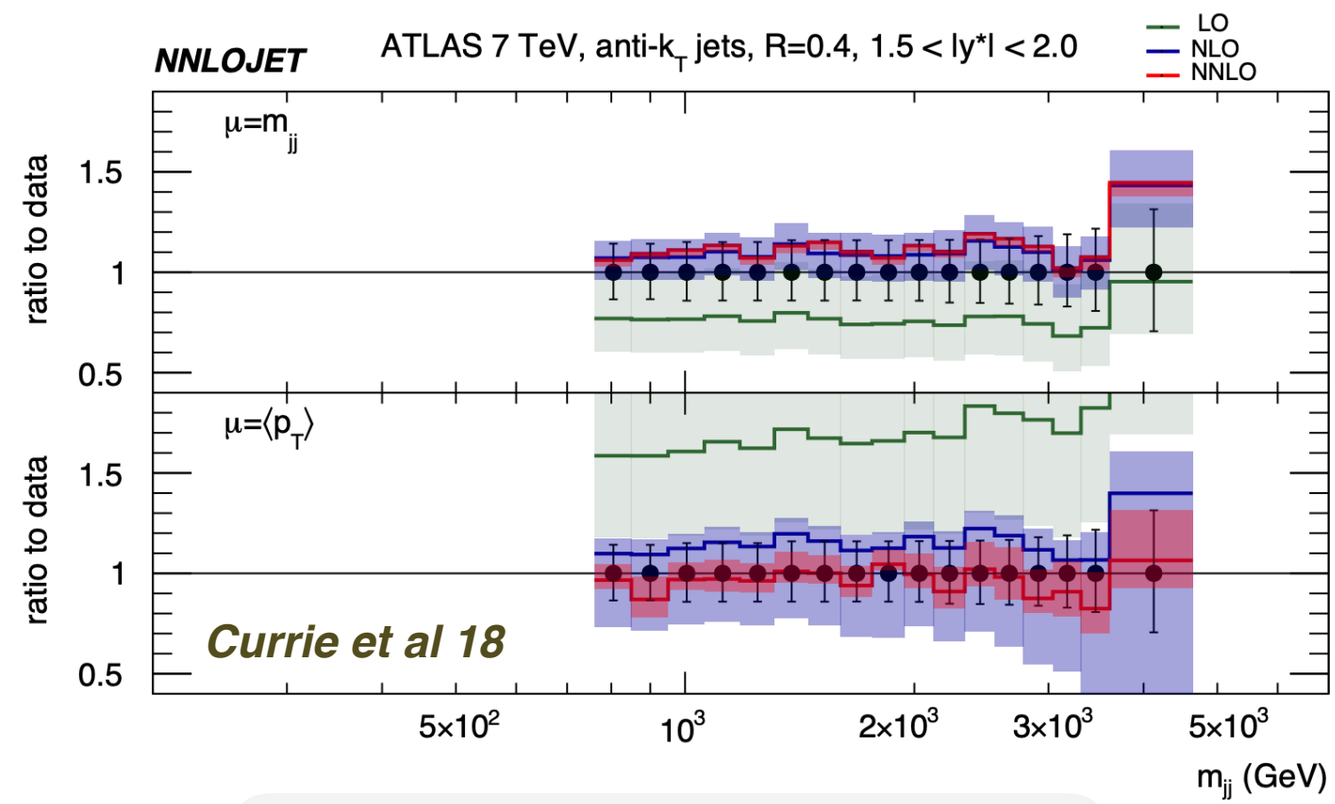
Theory uncertainties from MHOs

Global PDF fits are based on **fixed-order QCD calculations**

$$\sigma = \alpha_s^{p}\sigma_0 + \alpha_s^{p+1}\sigma_1 + \alpha_s^{p+2}\sigma_2 + \mathcal{O}(\alpha_s^{p+3})$$

The truncation of the perturbative series has associated a theoretical uncertainty known as the **Missing Higher Order (MHO)** uncertainty





*What is the **impact of MHOUs** in a global PDF fit?*

Theory uncertainties from MHOs

At any finite order, perturbative QCD calculations depend on the unphysical **renormalisation** and **factorisation scales**

$$\sigma(\mu_R, \mu_F) = \sum_{k=0}^{n} \sum_{i,j}^{n_f} \alpha_s^{p+k}(\mu_R)\, \widetilde{\sigma}^{(k)}(\mu_R, \mu_F) \otimes q_i(\mu_F) \otimes q_j(\mu_F) + \mathcal{O}\left(\alpha_s^{p+n+1}\right)$$

In PDF fits, both scales are set to a given fixed value, the typical **momentum transfer of the process *Q*,** and MHOUs are neglected

$$\sigma(\mu_R = Q, \mu_F = Q) = \sum_{k=0}^{n} \sum_{i,j}^{n_f} \alpha_s^{p+k}(Q)\, \widetilde{\sigma}^{(k)}(Q) \otimes q_i(Q) \otimes q_j(Q)$$

At order **N^{k}LO**, the scale dependence of physical cross-sections is expressed in terms the **N^{k-1}LO** splitting functions and partonic cross-sections by imposing:

$$\sigma(\mu_R, \mu_F) = \sigma(Q, Q) + \mathcal{O}\left(\alpha_s^{p+k+1}\right)$$

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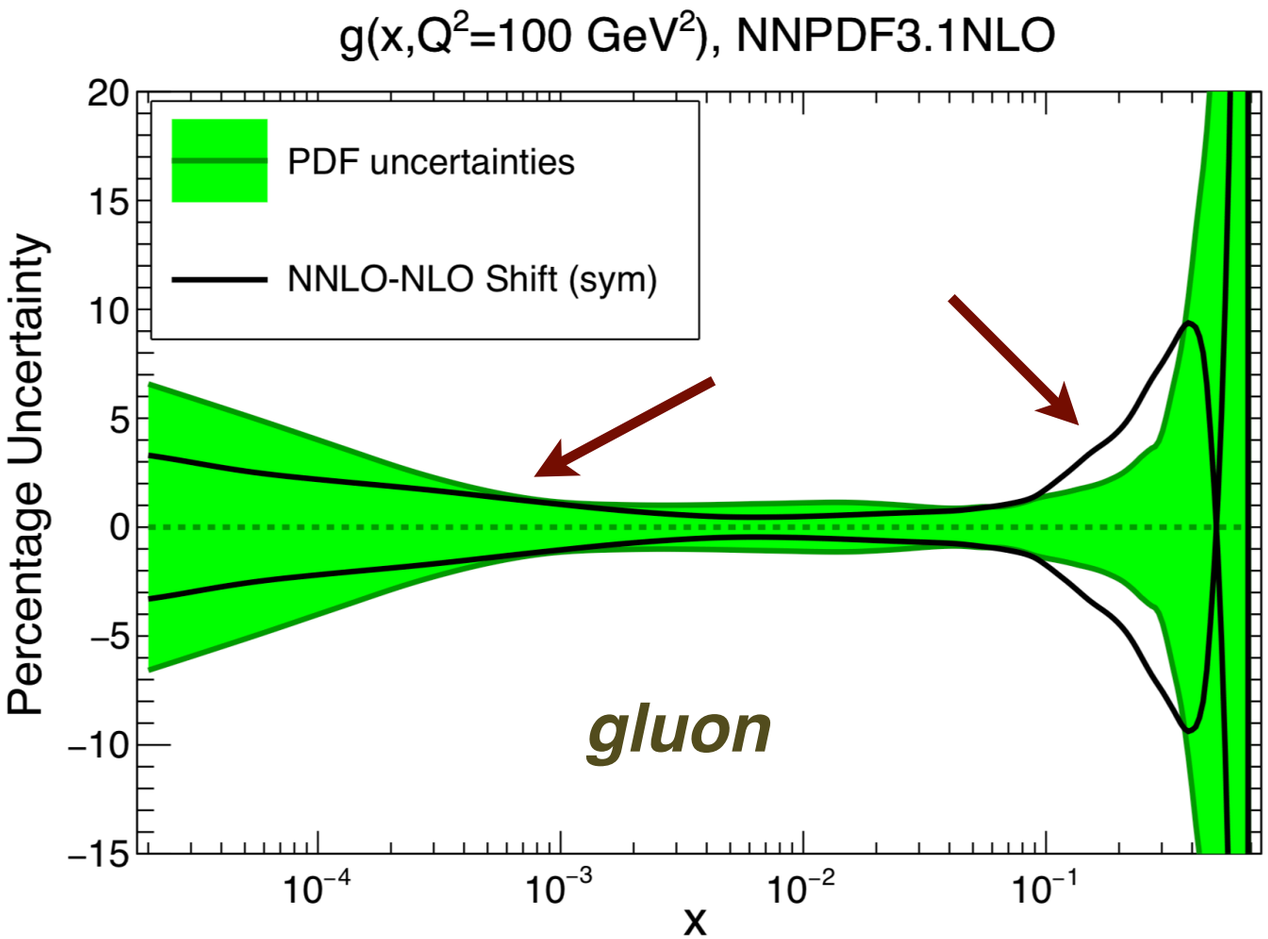
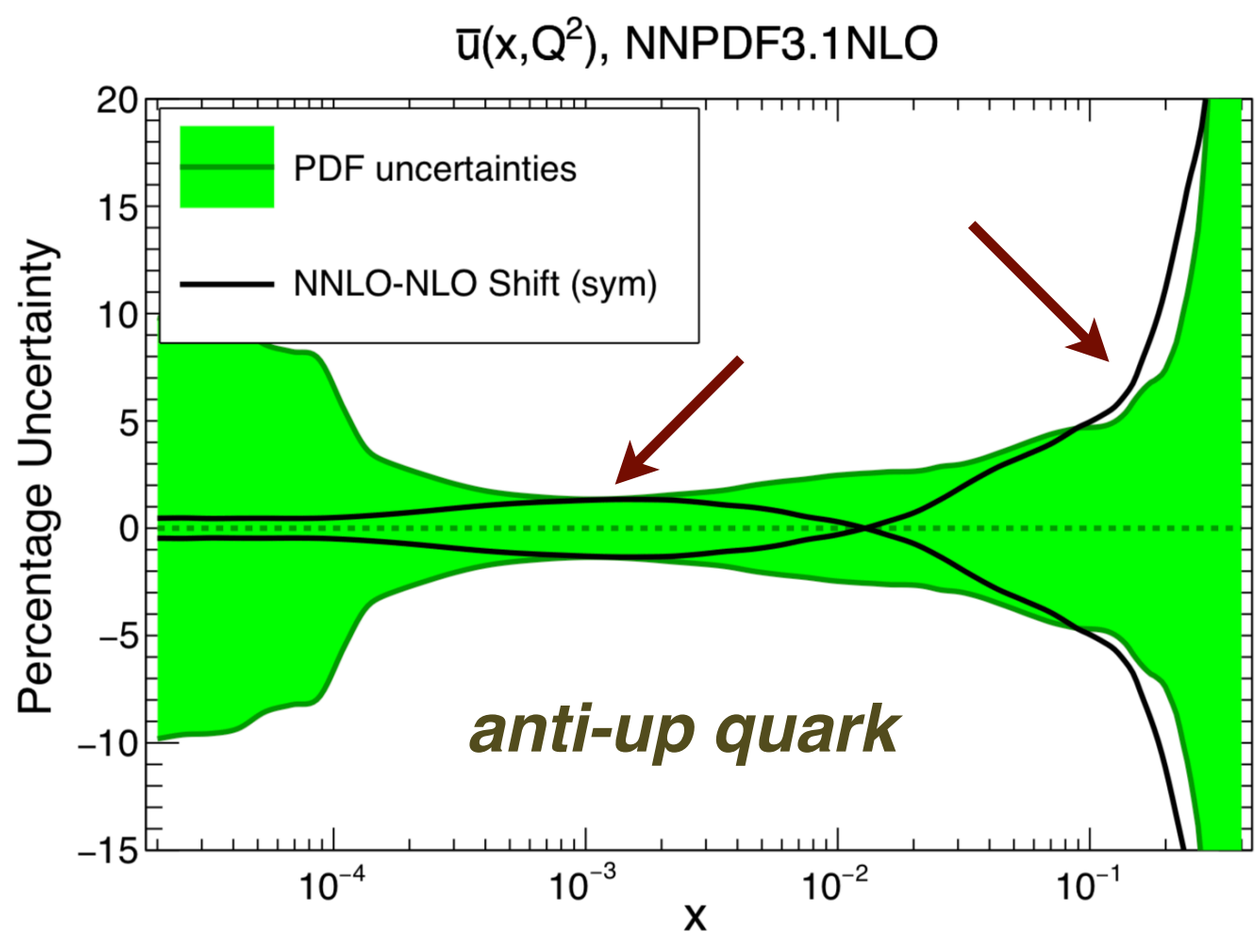
Scale-dependent terms at **N^{k}LO** predicted from **N^{k-1}LO** results: varying $\boldsymbol{\mu_R}$ and $\boldsymbol{\mu_F}$ within a certain range provides an estimate of MHOUs

$$\Delta_{\mathrm{MHO}}^{(\max)}\sigma \equiv \max\left((\sigma(\mu_R^{(1)}, \mu_F^{(1)}) - \sigma(Q,Q)), \sigma(\mu_R^{(2)}, \mu_F^{(2)}) - \sigma(Q,Q), \ldots \right)$$

Theory uncertainties from MHOs

How severe is **ignoring MHOUs** in modern global PDFs fits?





Shift between **NLO and NNLO PDFs** comparable or larger than **PDF errors**

Given the high precision of modern PDF determinations, **accounting for MHOUs** is most urgent!

Parton Distributions with Theory Uncertainties: general strategy

Based on ***NNPDF Collaboration:*** *R. Abdul Khalek, R. D. Ball, S. Carrazza, S. Forte, T. Giani, Z. Kassabov, R. L. Pearson, E. R. Nocera, J. Rojo, L. Rottoli, M. Ubiali, C. Voisey, M. Wilson*

arXiv:1905.04311, *submitted to PRL*

arXiv:1906.10698, *submitted to EPJC*

A theoretical covariance matrix

Construct a **theory covariance matrix** from **scale-varied cross-sections** and combine it with the experimental covariance matrix

$$\chi^2 = \frac{1}{N_{\rm dat}} \sum_{i,j=1}^{N_{\rm dat}} \left(D_i - T_i\right) (C + S)^{-1}_{ij} \left(D_j - T_j\right)$$

experimental ***theoretical***

assumption: theory errors are **Gaussianly distributed** around **true value**

Formally the theory covariance matrix is defined as

$$S_{ij} = \left\langle (\mathcal{T}_i - T_i)(\mathcal{T}_j - T_j) \right\rangle \equiv \left\langle \Delta_i \Delta_j \right\rangle$$

true result *actual calculation*

How to estimate these **theory systematic shifts**?

A theoretical covariance matrix

Here we use **scale variations** to estimate the MHOUs

$$\Delta_{i_a}(\kappa_f, \kappa_{r_a}) \equiv T_{i_a}(\kappa_f, \kappa_{r_a}) - T_{i_a}(0,0)$$

theory shifts

labels data points in process a

$\kappa_{f(r_a)} = \ln \mu^2_{f(r_a)}/Q^2$

central prediction

note: renormalisation scale variations are only correlated within the *same* process

$$S_{ij} = n_m \sum_{V_m} \Delta_{i_a}(\kappa_f, \kappa_{r_a}) \Delta_{i_b}(\kappa_f, \kappa_{r_b})$$

theory covariance matrix

normalisation factor

sum over scale variations

Different prescriptions for scale variations possible:

Need to validate which ones exhibit the best performance

A theoretical covariance matrix

Here we use **scale variations** to estimate the MHOUs

$$\Delta_{i_a}(\kappa_f, \kappa_{r_a}) \equiv T_{i_a}(\kappa_f, \kappa_{r_a}) - T_{i_a}(0,0)$$

theory shifts

labels data points in process a

$\kappa_{f(r_a)} = \ln \mu^2_{f(r_a)}/Q^2$

central prediction

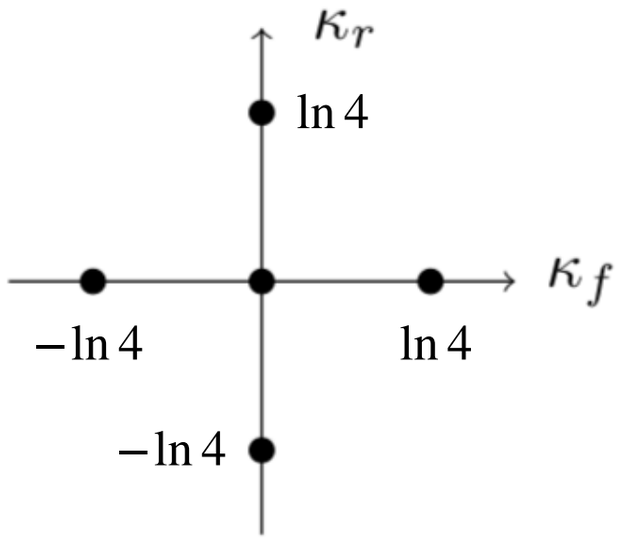
- Scale-varied theories evaluated with **APFEL** for DIS structure functions at NLO and NNLO, and with **APPLgrid/HOPPET/APFELgrid** for hadronic processes at NLO
- *Ren and Fact scales*: associated to **MHOUs** in **hard cross-section** and in **PDF evolution**

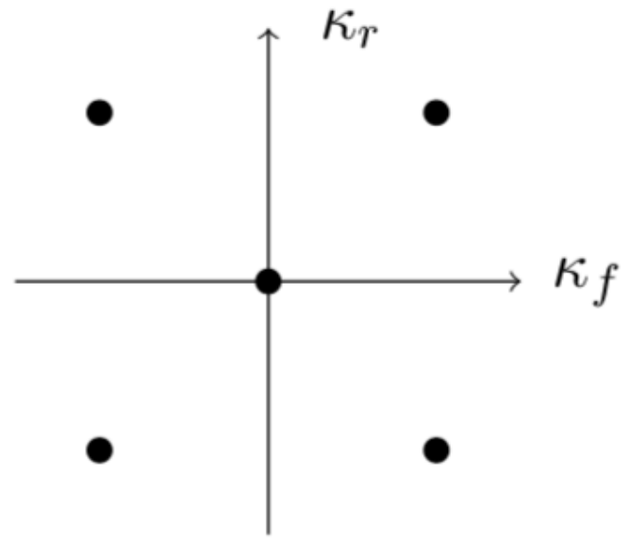
Scale	MHOU	'Traditional' name [17, 18, 21–23]	'Modern' name [24],[PDG]
μ_r	in hard xsec	—	renormalization scale
μ_f	in PDF evolution	renormalization scale	factorization scale
$\widetilde{\mu}$	in physical xsec	factorization scale	scale of the process

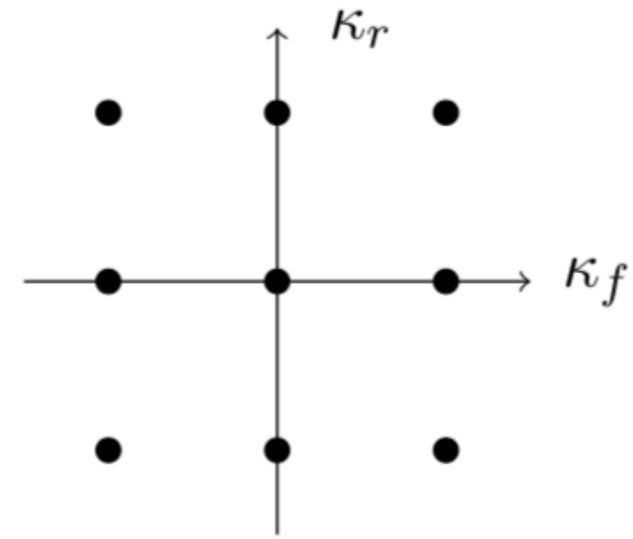
Point prescriptions

single process

symmetric prescriptions



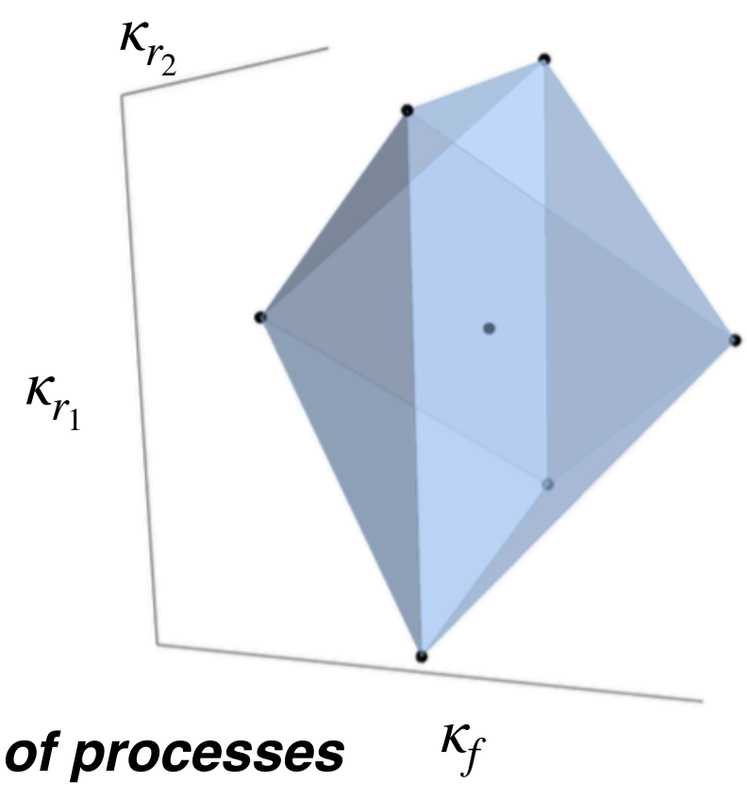


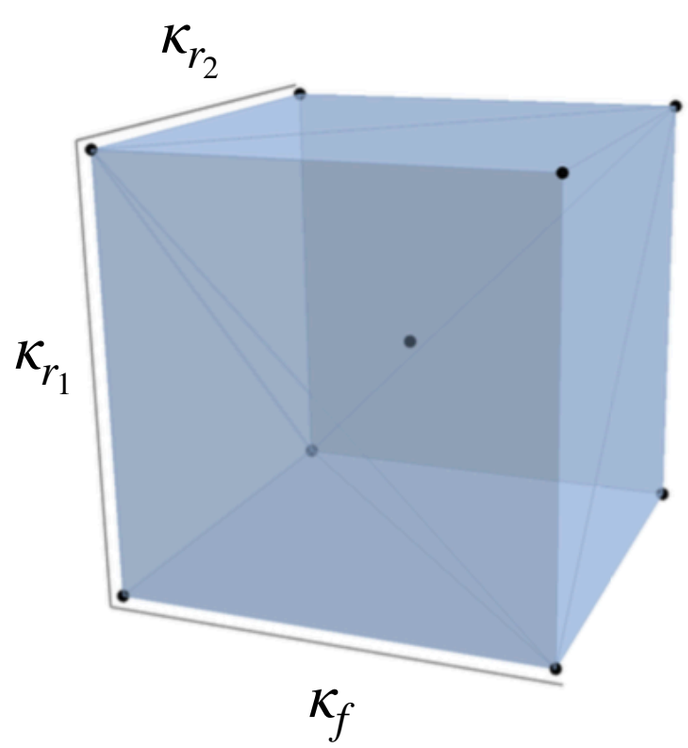


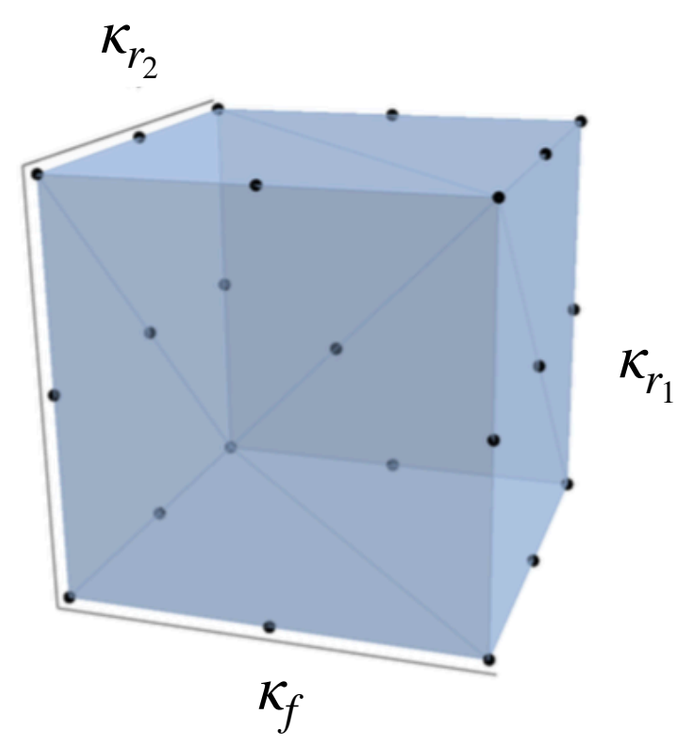
5-points

5bar-points

9-points







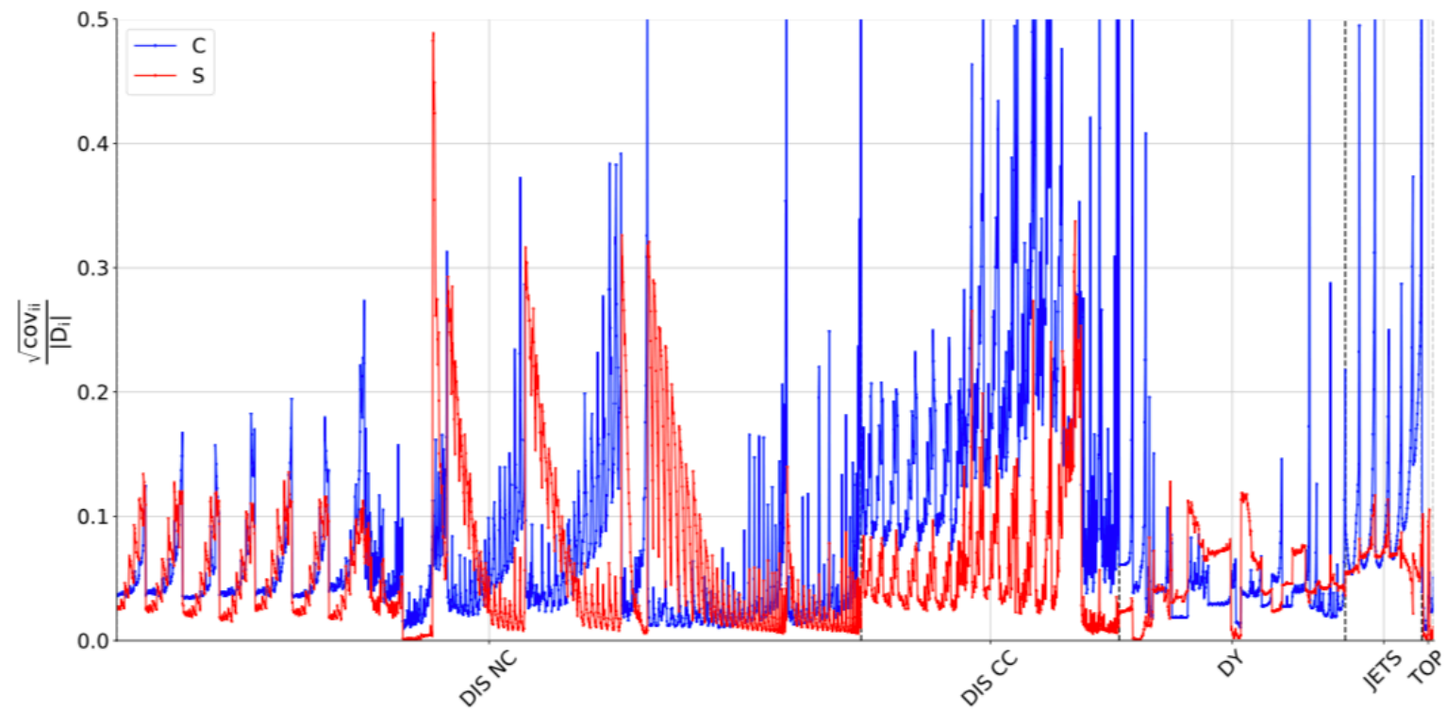
Pair of processes

Results

grouping by process

Process Type	Dataset	Reference	$N_{\rm dat}$	$N_{\rm dat}$ (total)
DIS NC	NMC	[25, 26]	134	1593
	SLAC	[27]	12	
	BCDMS	[28, 29]	530	
	HERA σ^p_{NC}	[33]	886	
	HERA σ^c_{NC}	[34]	31	
DIS CC	NuTeV dimuon	[30, 31]	41	552
	CHORUS	[32]	430	
	HERA σ^p_{CC}	[33]	81	
DY	ATLAS W, Z, 7 TeV 2010	[39]	30	484
	ATLAS W, Z, 7 TeV 2011	[40]	34	
	ATLAS low-mass DY 2011	[41]	4	
	ATLAS high-mass DY 2011	[42]	5	
	ATLAS Z p_T 8 TeV (p_T^{ll}, M_{ll})	[43]	44	
	ATLAS Z p_T 8 TeV (p_T^{ll}, y_Z)	[43]	48	
	CMS Drell-Yan 2D 2011	[48]	88	
	CMS W asy 840 pb	[49]	11	
	CMS W asy 4.7 pb	[50]	11	
	CMS W rap 8 TeV	[51]	22	
	CMS Z p_T 8 TeV (p_T^{ll}, M_{ll})	[52]	28	
	LHCb Z 940 pb	[57]	9	
	LHCb $Z \to ee$ 2 fb	[58]	17	
	LHCb $W, Z \to \mu$ 7 TeV	[59]	29	
	LHCb $W, Z \to \mu$ 8 TeV	[60]	30	
	CDF Z rap	[35]	29	
	D0 Z rap	[36]	28	
	D0 $W \to e\nu$ asy	[37]	8	
	D0 $W \to \mu\nu$ asy	[38]	9	
JET	ATLAS jets 2011 7 TeV	[44]	31	164
	CMS jets 7 TeV 2011	[53]	133	
TOP	ATLAS $\sigma_{tt}^{\rm top}$	[45, 46]	3	26
	ATLAS $t\bar{t}$ rap	[47]	10	
	CMS $\sigma_{tt}^{\rm top}$	[54, 55]	3	
	CMS $t\bar{t}$ rap	[56]	10	
Total			2819	2819

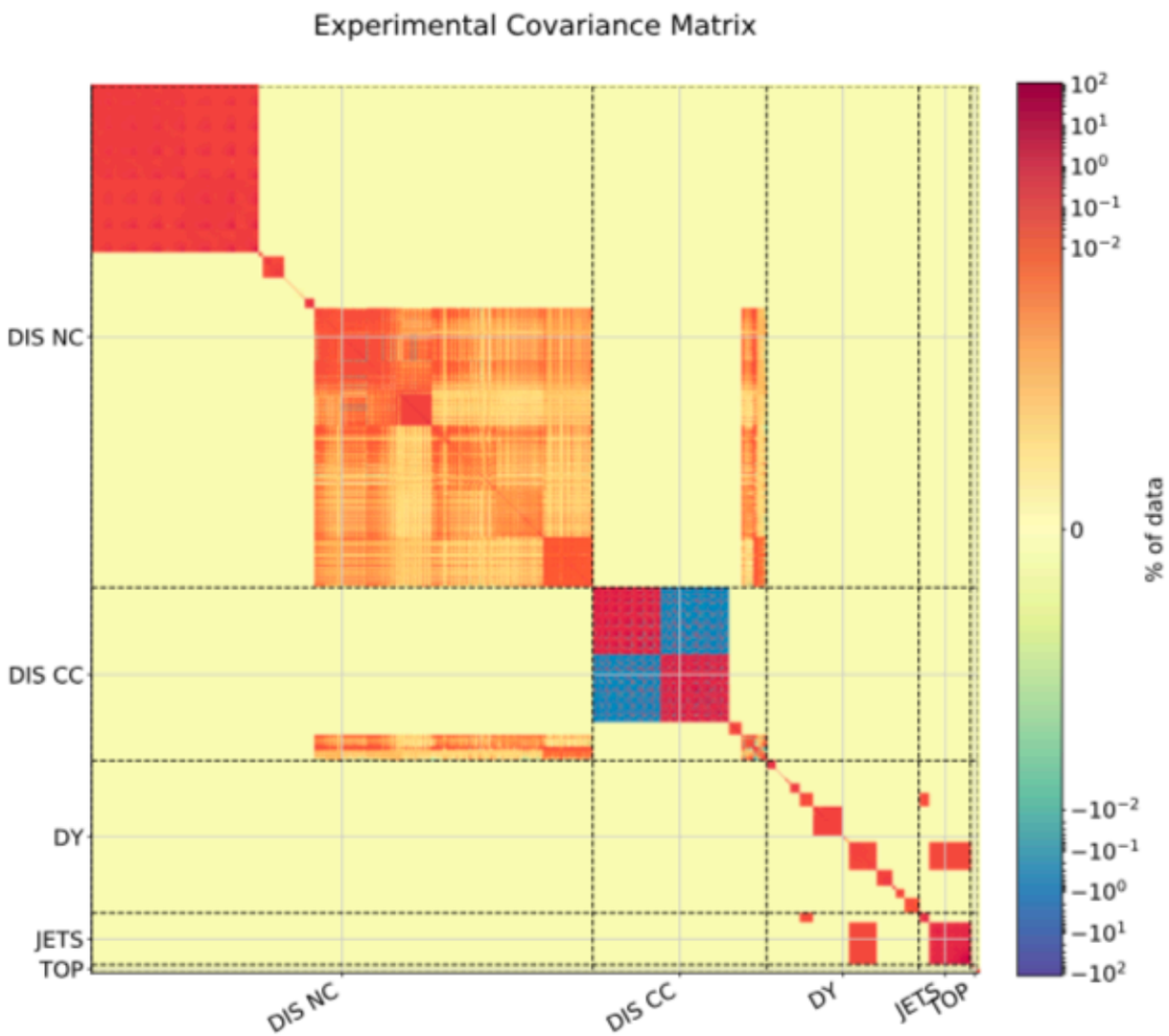
NLO, 9-point prescription

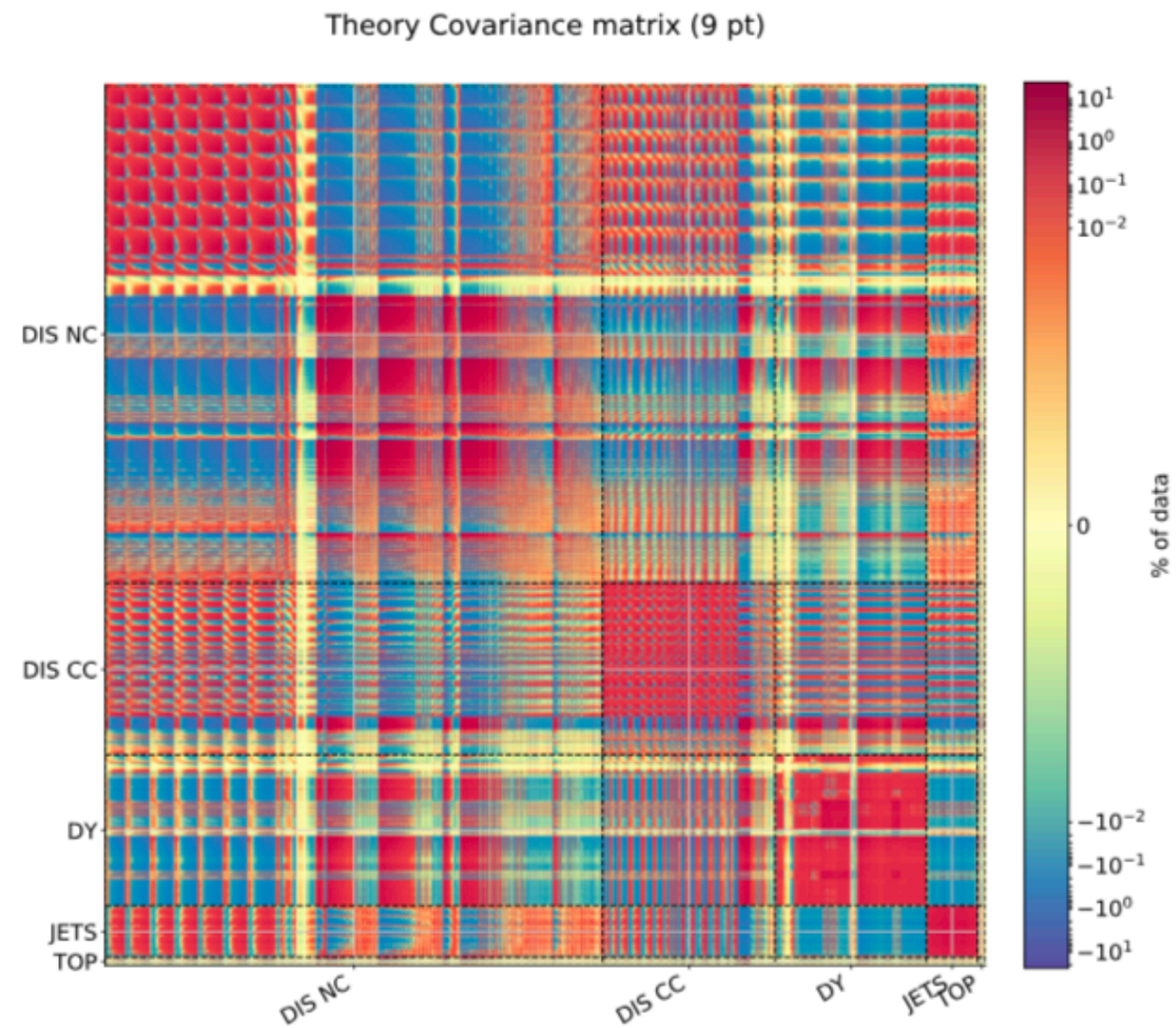


- MHOUs **comparable or larger** in many cases as compared to experimental errors
- MHOU modify the **relative weight** that each dataset carries in the global fit
- The effect of MHOUs is more striking at the level of correlations, since they completely change the pattern

Results

covariance matrices

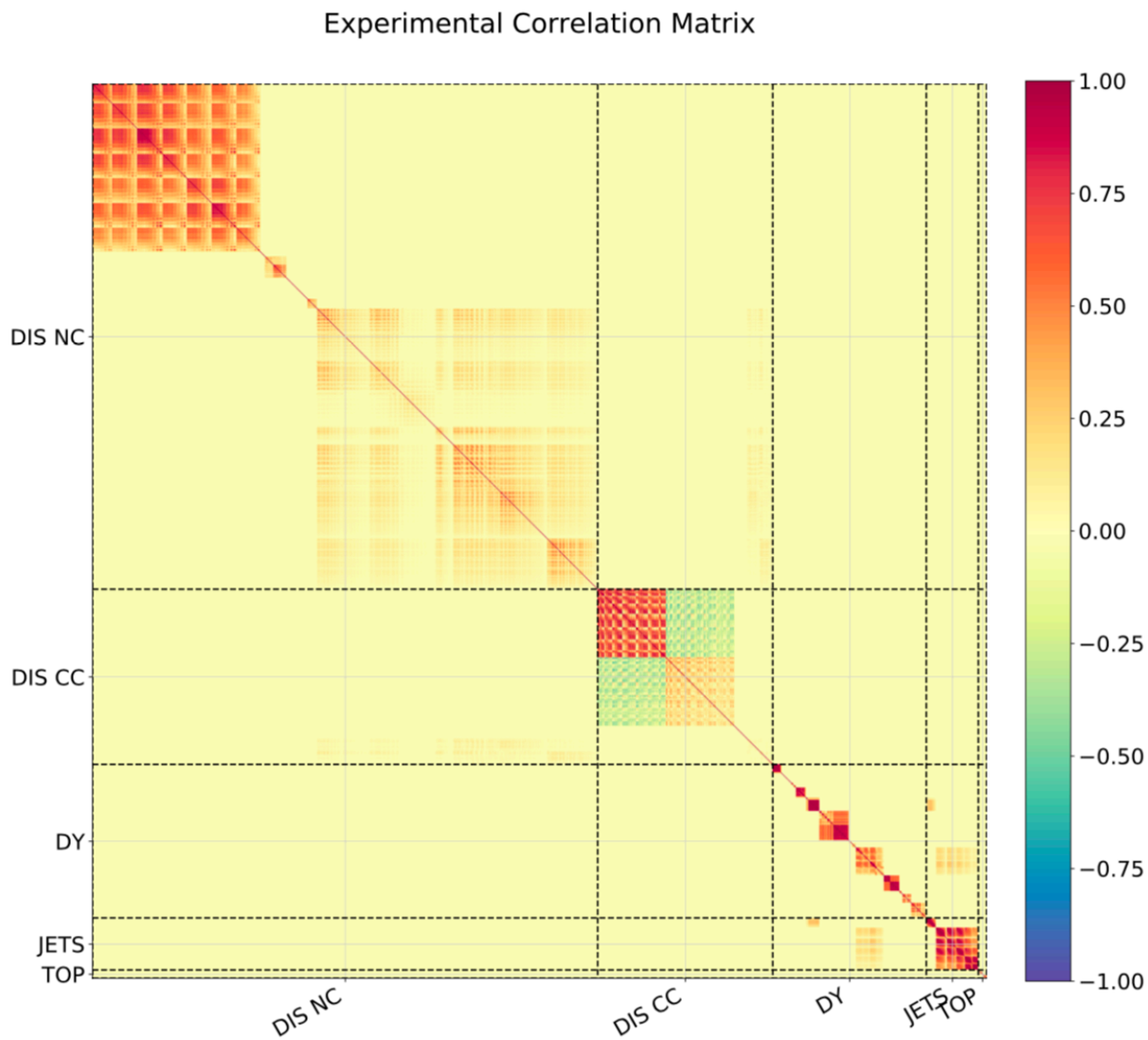


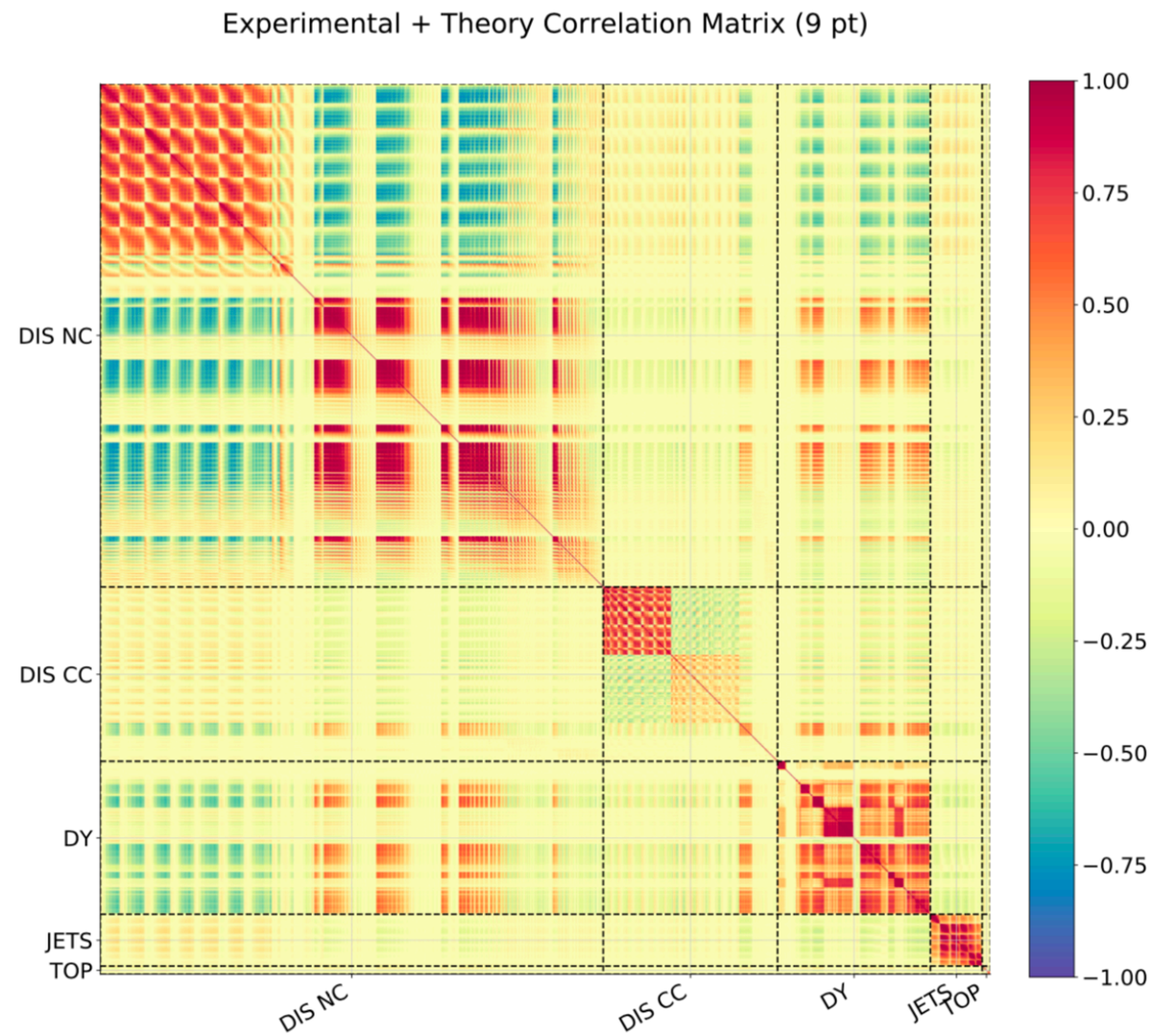


Rich pattern of **theory-induced correlations:**
Absent if only experimental errors considered

Results

correlation matrices

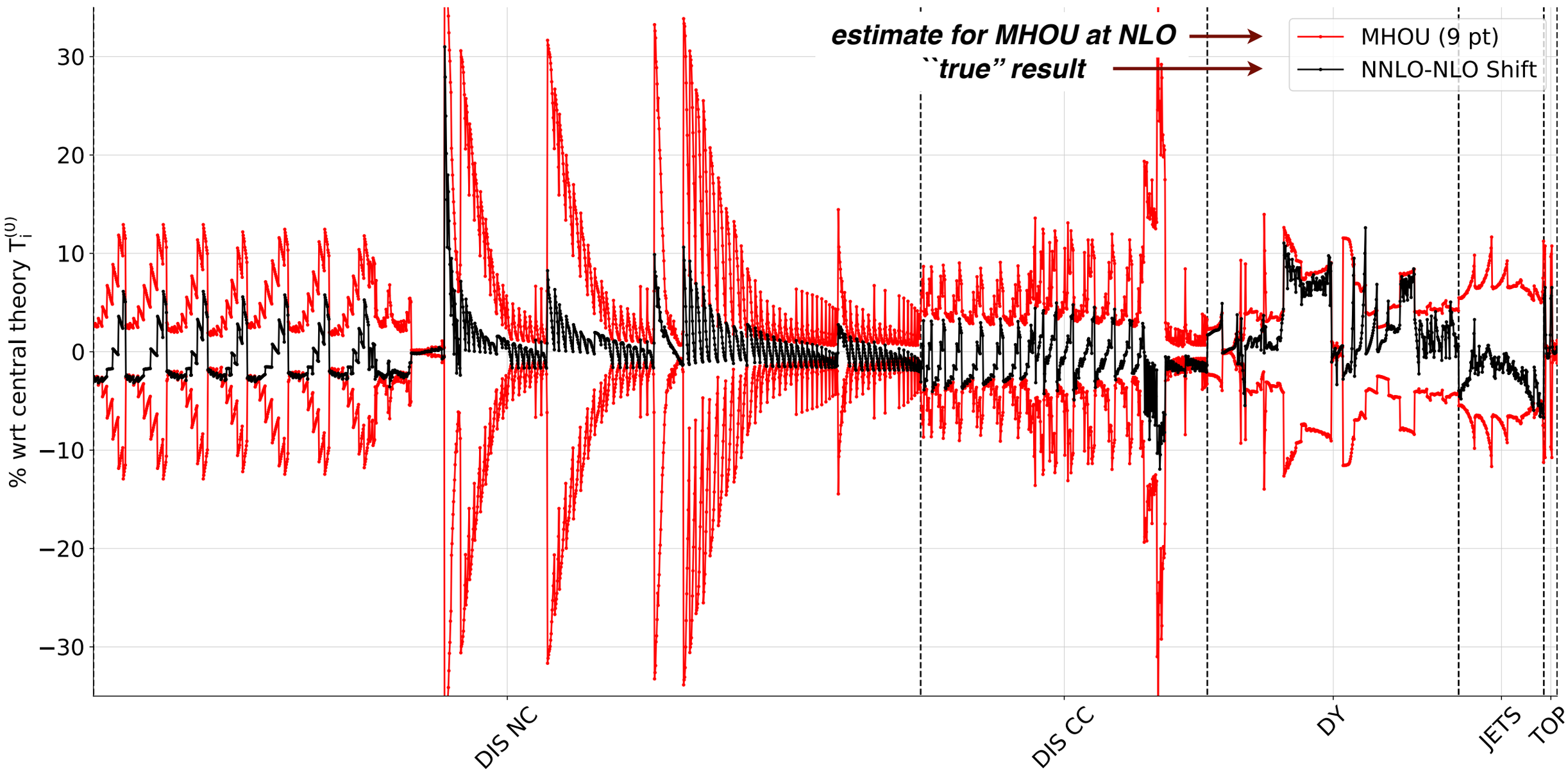




Rich pattern of **theory-induced correlations:**
Absent if only experimental errors considered

Validation

Systematic validation of NLO theory covariance matrix on the `exact' result, the **NLO=>NNLO shift**, for **O(3000) data points** of the global PDF fit



Scale variations: good estimate of MHOU for processes of relevance in PDF fits

Validation

- We can validate the full theory covariance matrix, including **correlation**s, in terms of the NNLO-NLO shift vector as follows
- Normalise the NLO theory covariance matrix so that its elements are dimensionless

$$\widehat{S}_{ij} = S_{ij} \Big/ \left(T_i^{(\mathrm{NLO})} T_j^{(\mathrm{NLO})} \right)$$

- Now define a **normalised shift vector** with components (with same input PDF in all cases)

$$\delta_i = \left(T_i^{(\mathrm{NNLO})} - T_i^{(\mathrm{NLO})} \right) \Big/ T_i^{(\mathrm{NLO})}$$

- Diagonalise the theory covmat: only a small number of eigenvalues **non-zero**

$$\widehat{S}_{ij} \rightarrow e_i^{\alpha}, \quad \alpha = 1, \ldots, N_{\mathrm{sub}} \qquad \textit{(eigenvectors)}$$

$$\widehat{S}_{ij} \rightarrow \lambda_{\alpha}, \quad \alpha = 1, \ldots, N_{\mathrm{sub}} \qquad \textit{(eigenvalues)}$$

$$N_{\mathrm{sub}} \ll N_{\mathrm{dat}}$$

Validation

- Next we **project the shift vector onto the eigenvectors**, and resolve its component lying in the **subspace *S*** spanned by the non-zero eigenvectors of the theory covariance matrix

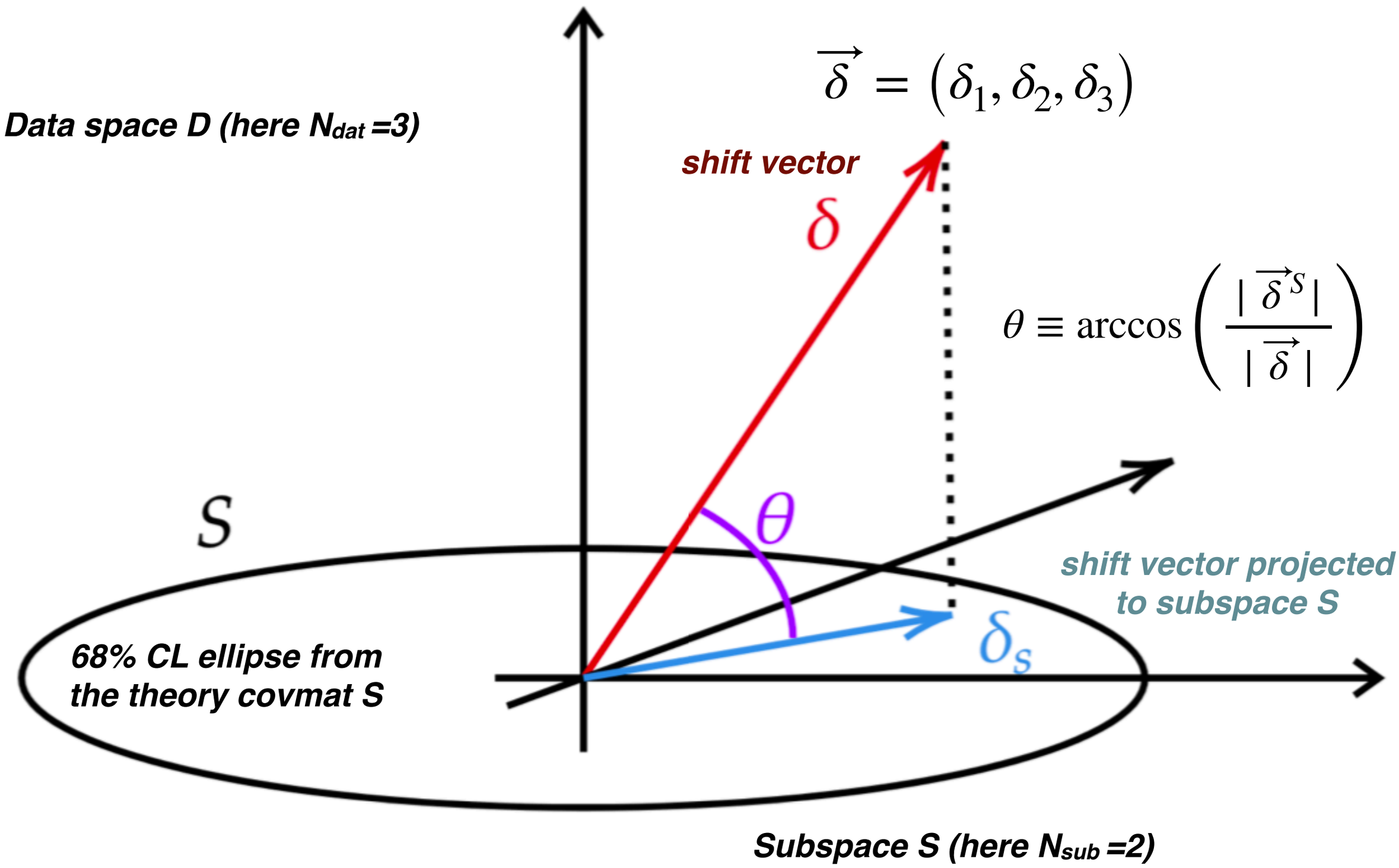
$$\delta^\alpha = \sum_{i=1}^{N_{\rm dat}} \delta_i \, e_i^\alpha \qquad\qquad \delta_i^S = \sum_{\alpha=1}^{N_{\rm sub}} \delta^\alpha \, e_i^\alpha$$

- A succesful validation requires that the components $\boldsymbol{\delta_i^S}$ **lie mostly on *S***, which implies that the following angle must be reasonably small

$$\theta \equiv \arccos\left(\frac{|\,\delta_i^S\,|}{|\,\delta_i\,|} \right)$$

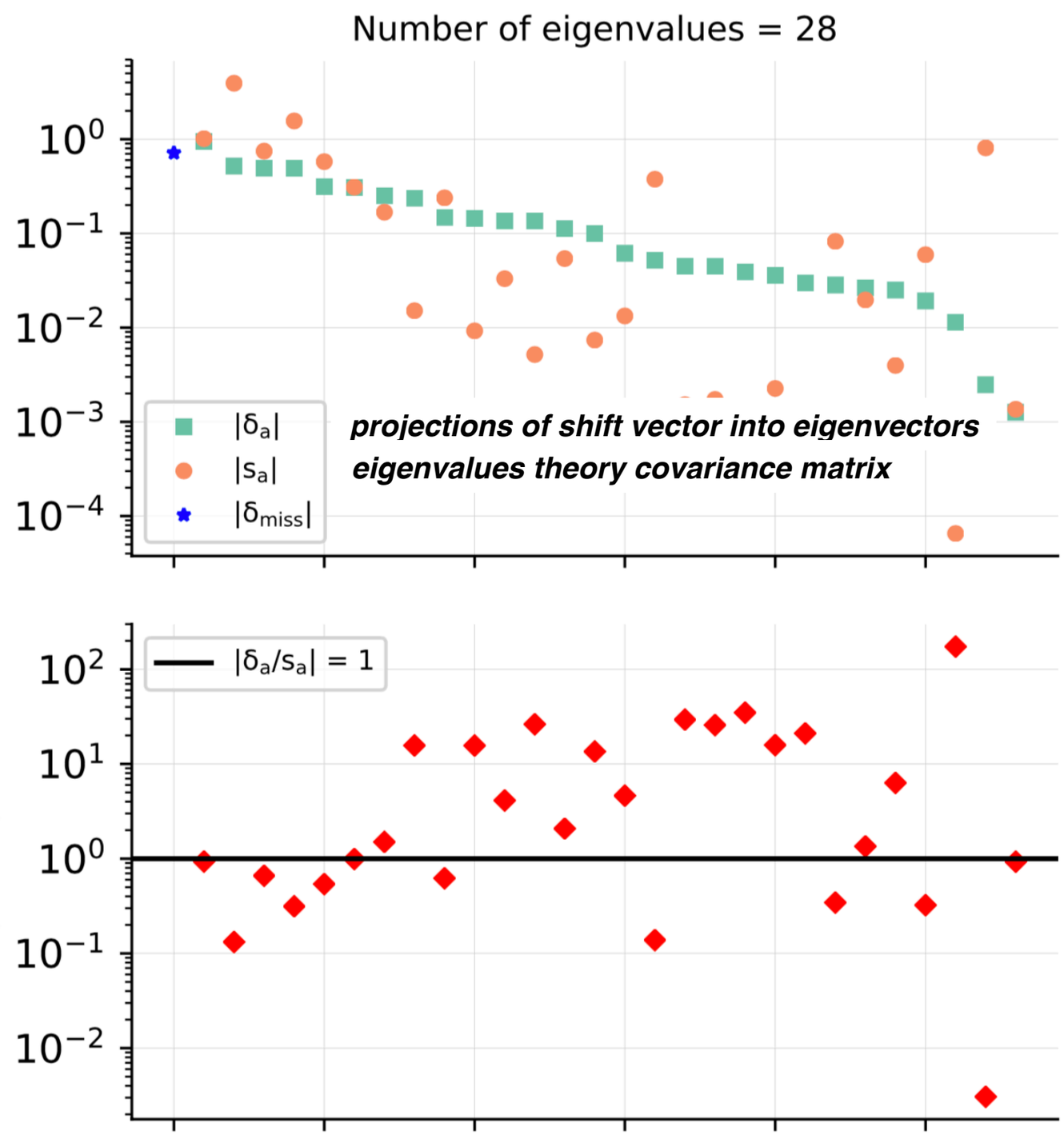
- Measure of how globally our estimate of the theory covariance matrix reproduces the actual pattern of **higher-order perturbative correction**
- Highly non-trivial validation, since $N_{dat} = 3000$ while $N_{sub} = 30$

Validation


Data space D (here N_{dat} =3)
$\vec{\delta} = (\delta_1, \delta_2, \delta_3)$
shift vector
δ
$\theta \equiv \arccos\left(\frac{|\vec{\delta}^S|}{|\vec{\delta}|}\right)$
S
θ
shift vector projected to subspace S
δ_S
68% CL ellipse from the theory covmat S
Subspace S (here N_{sub} =2)

Validation

Prescription	$N_{\rm sub}$	θ
5-pt	8	33°
$\overline{5}$-pt	12	31°
9-pt	28	26°
3-pt	6	52°
7-pt	14	29°



- **9-pt prescription best**, with 7-pt close
- The NLO theory covariance matrix built from scale-variations **reproduces well the shift from NLO to NNLO** including correlations

Parton Distributions with Theory Uncertainties: results

Fits with MHOUs

Label	Dataset	Order	Cov. Mat.	Comments
NNPDF31_nlo_as_0118_dis_kF_1_kR_1	DIS	NLO	C	baseline DIS-only NLO
NNPDF31_nlo_as_0118_dis_scalecov_9pt	DIS	NLO	$C + S^{(9\mathrm{pt})}$	
NNPDF31_nnlo_as_0118_dis_kF_1_kR_1	DIS	NNLO	C	baseline DIS-only NNLO
NNPDF31_nnlo_as_0118_dis_scalecov_9pt	DIS	NNLO	$C + S^{(9\mathrm{pt})}$	
NNPDF31_nlo_as_0118_kF_1_kR_1	Global	NLO	C	baseline Global NLO
NNPDF31_nlo_as_0118_scalecov_9pt	Global	NLO	$C + S^{(9\mathrm{pt})}$	
NNPDF31_nlo_as_0118_scalecov_7pt	Global	NLO	$C + S^{(7\mathrm{pt})}$	
NNPDF31_nlo_as_0118_scalecov_3pt	Global	NLO	$C + S^{(3\mathrm{pt})}$	
NNPDF31_nlo_as_0118_scalecov_9pt_fit	Global	NLO	$C + S^{(9\mathrm{pt})}$	S only in χ^2 definition
NNPDF31_nlo_as_0118_scalecov_9pt_sampl	Global	NLO	$C + S^{(9\mathrm{pt})}$	S only in sampling
NNPDF31_nnlo_as_0118_kF_1_kR_1	Global	NNLO	C	baseline Global NNLO

*DIS-only fits at **NLO** and **NNLO***

*Global fits at **NLO,** based on NNPDF3.1 with modified dataset*

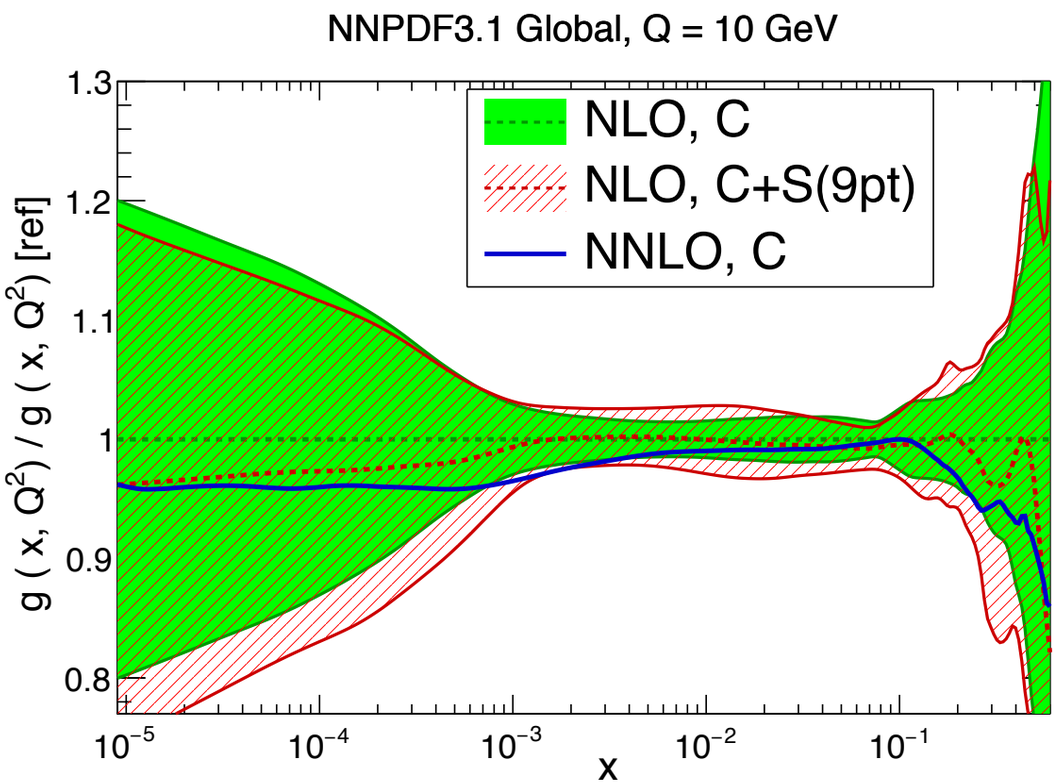
Fit quality

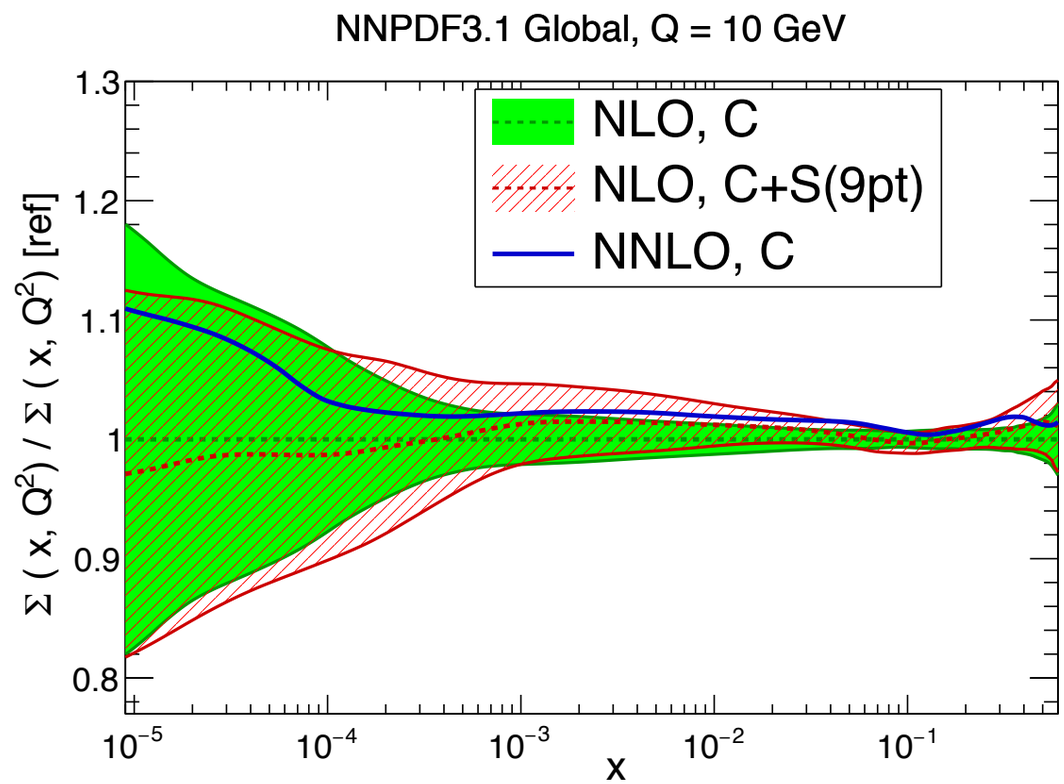
Dataset	$n_{\rm dat}$	$\chi^2/n_{\rm dat}$ in the NNPDF3.1 global fits			
		NLO			NNLO
		C	$C+S^{(9\rm pt)}$	$C+S^{(7\rm pt)}$	C
DIS NC	1593	1.088	1.079	1.086	1.084
DIS CC	552	1.012	0.928	0.933	1.079
DY	484	1.486	1.447	1.485	1.231
JETS	164	0.907	0.839	0.858	0.950
TOP	26	1.260	1.012	1.016	1.068
Total	2819	**1.139**	**1.109**	**1.129**	**1.105**

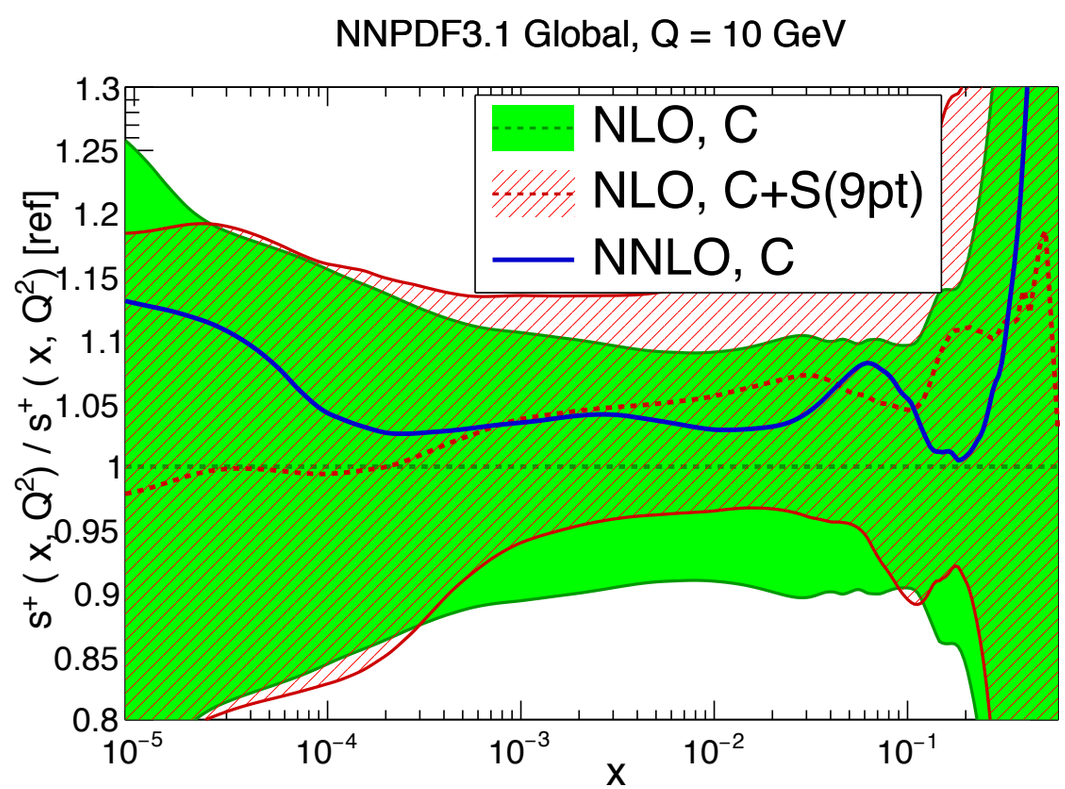
- **Improved fit quality** for all processes
- Moderate impact of varying the point prescription for *S*
- Fit quality for **NLO C+S(9pt)** close to **NNLO C** for the total dataset

Expect both **increase of total PDF uncertainties** as well as **shifts in central values** from rebalancing between experiments

Impact on PDFs

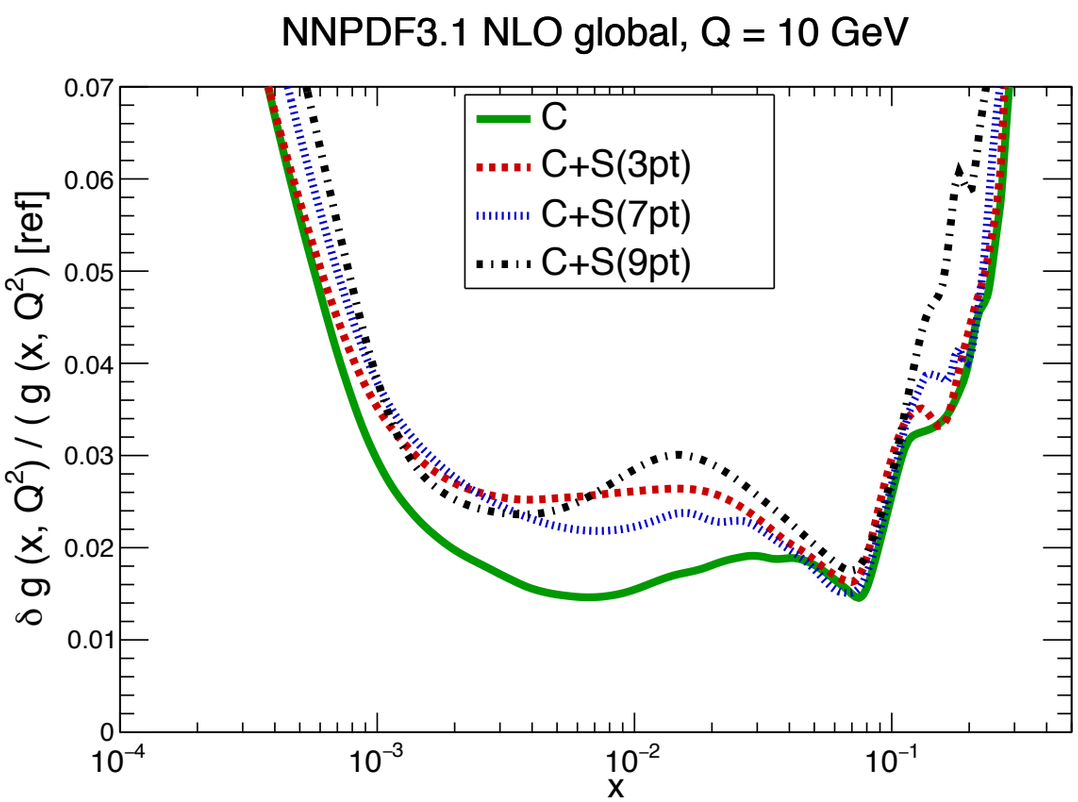


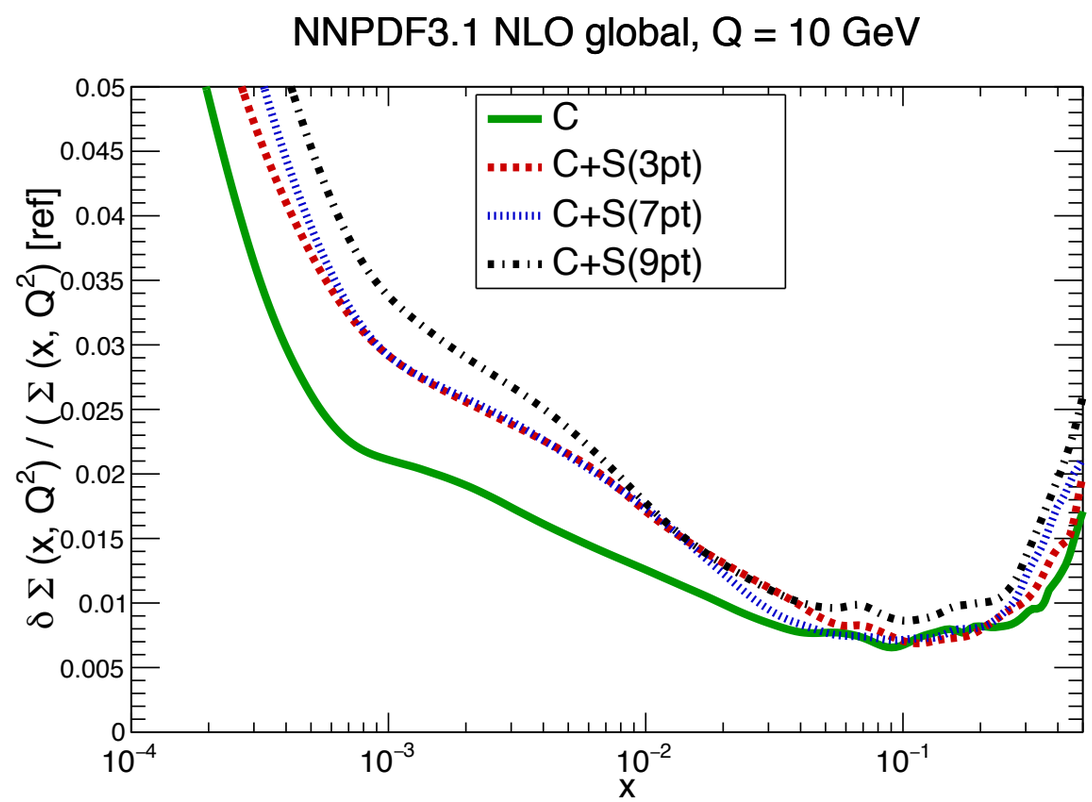


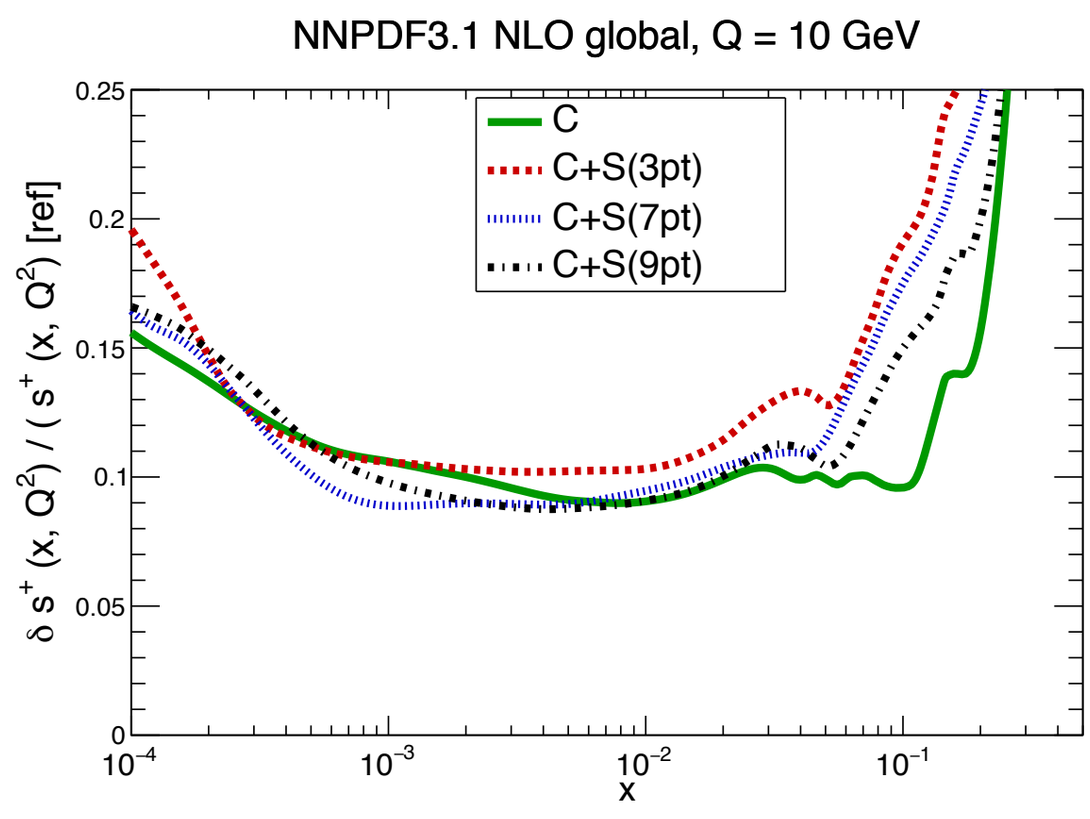


- **True result** (NLO to NNLO shift) contained within the NLO C+S band
- Central value of NLO C+S **shifts towards true result** in most cases
- Moderate (but not dramatic) **increase** in overall PDF uncertainties

Impact on PDFs







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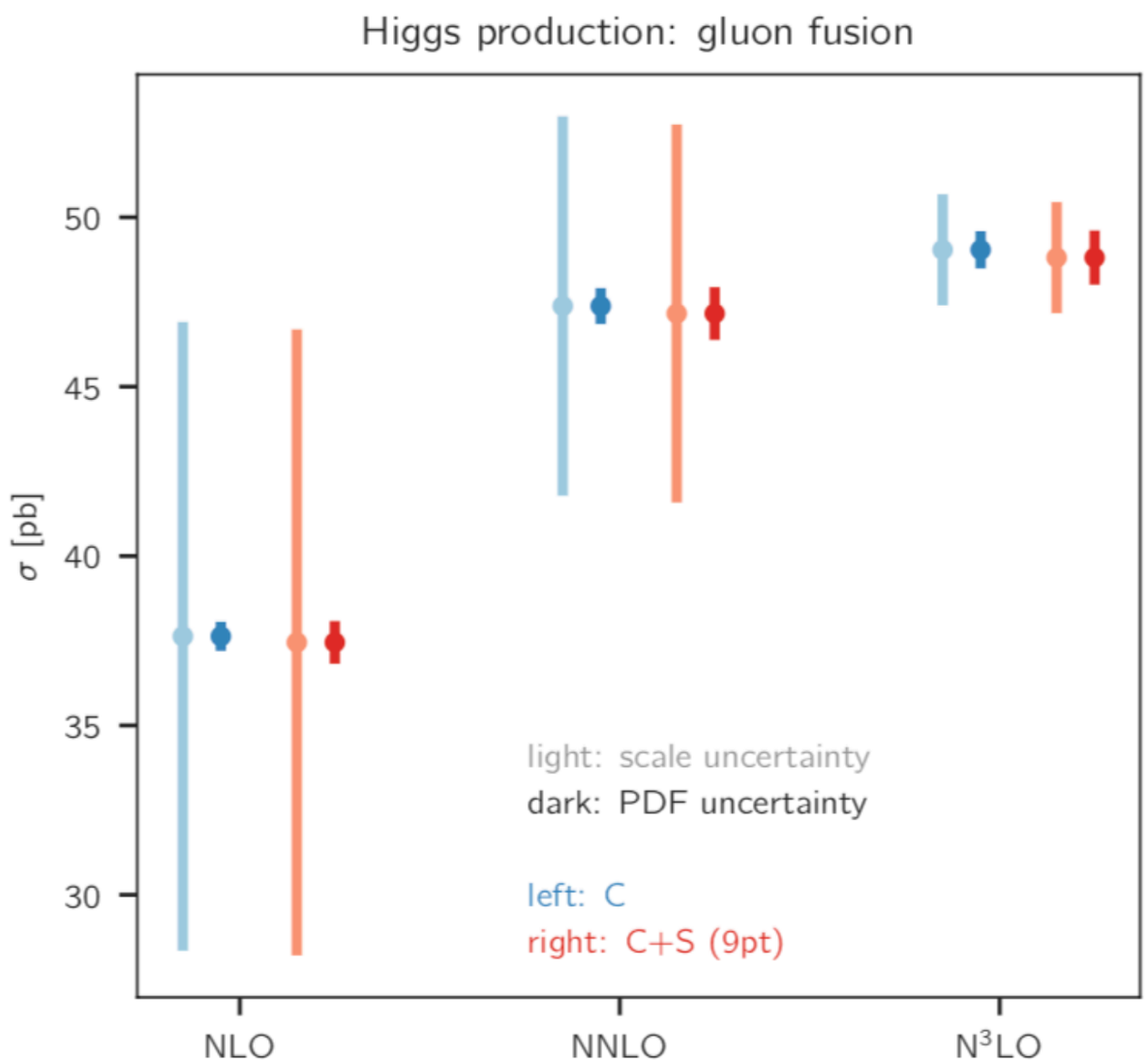
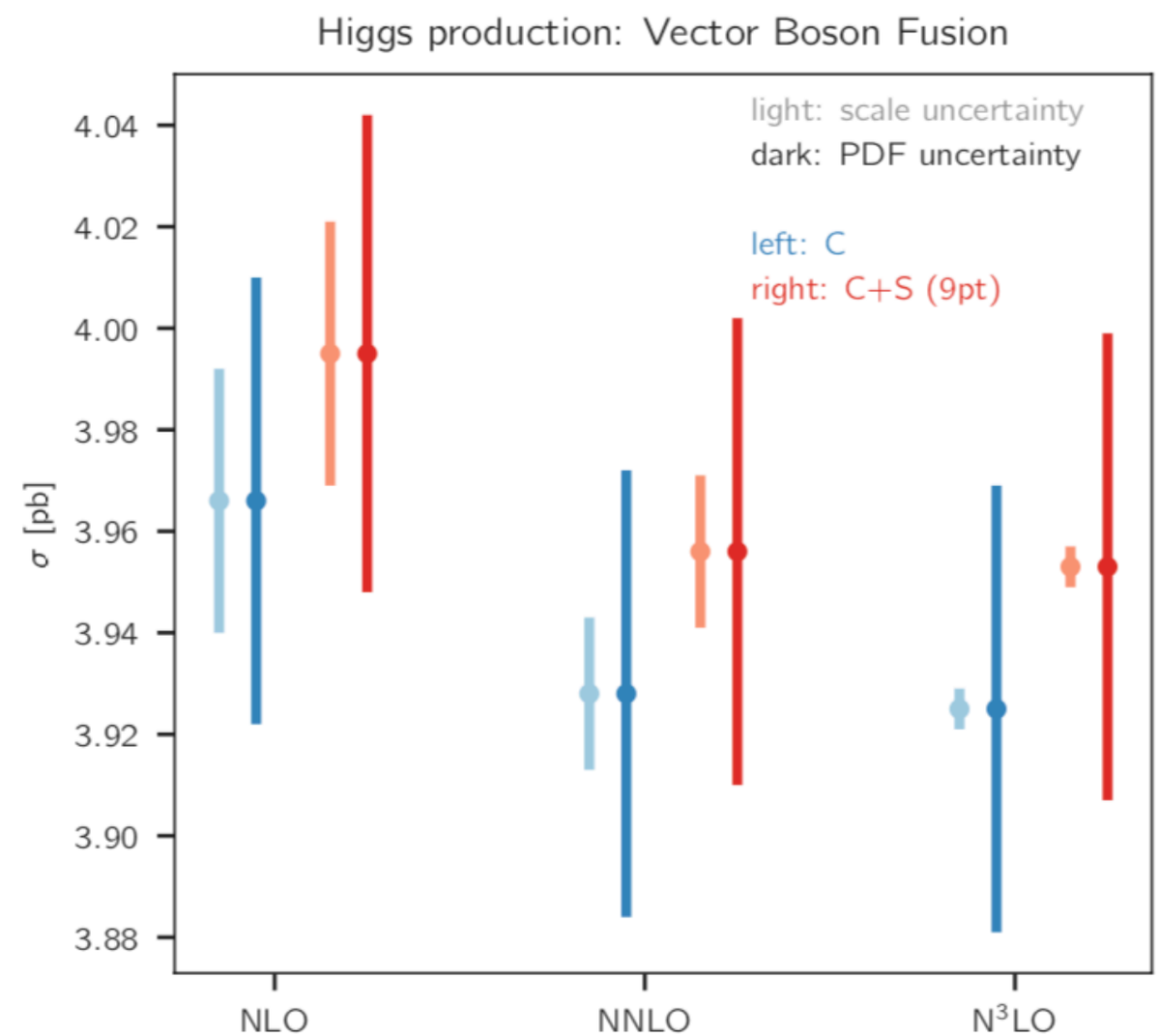
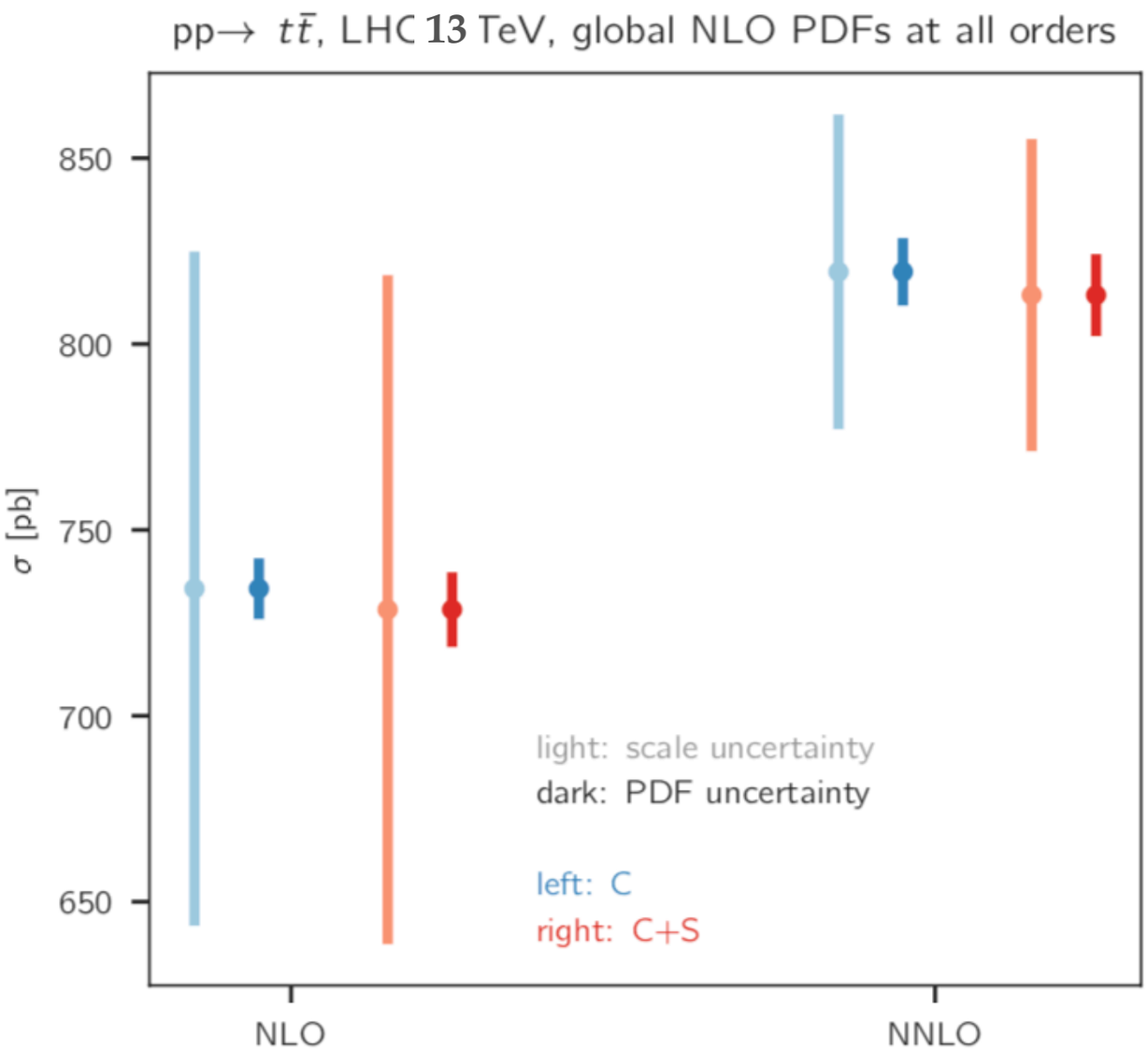
Impact for LHC phenomenology

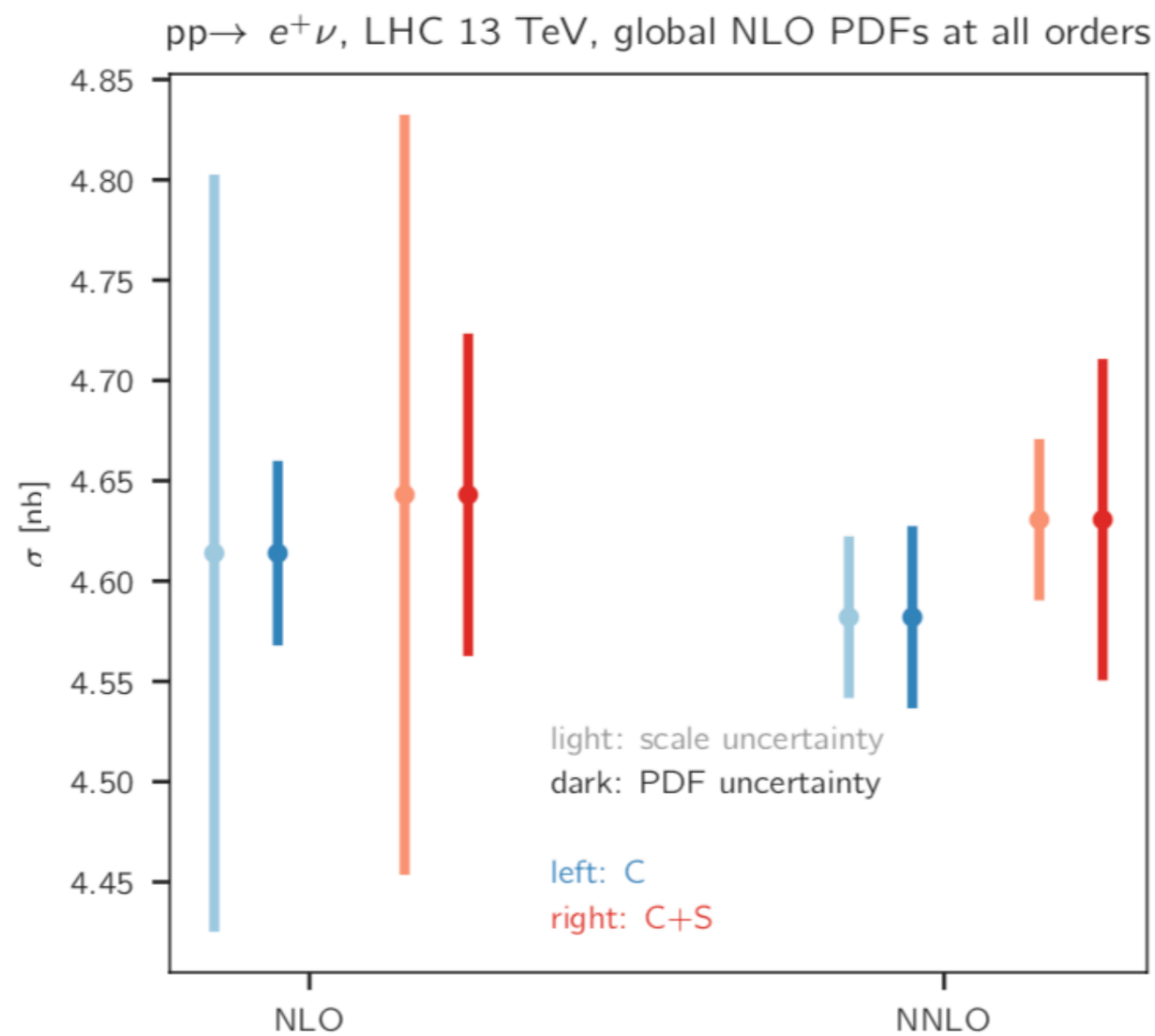




Depending on process, main consequence of **MHOUs in PDF fit for LHC pheno** is shift in central values, increase in overall PDF uncertainties, or both

Impact for LHC phenomenology





Depending on process, main consequence of **MHOUs in PDF fit for LHC pheno** is shift in central values, increase in overall PDF uncertainties, or both

Usage

How to construct now the **total theory error** on LHC cross-sections?

- **PDF uncertainties** (including MHOUs in the processes used to fit the PDFs): as usual!

$$\sigma_{\mathscr{F}}^{\rm PDF} = \left(\frac{1}{N_{\rm rep}-1} \sum_{k=1}^{N_{\rm rep}} \left(\mathscr{F}[\{q^{(k)}\}] - \langle \mathscr{F}[\{q\}] \rangle \right)^2 \right)^{1/2}$$

- **MHOU** on hard process: as usual, either with **scale envelope prescription** or with **theory covariance matrix** approach *eg*

$$\sigma_{\mathscr{F}}^{\rm th} = \left(S_{\mathscr{F}\mathscr{F}}^{(9{\rm pt})} \right)^{1/2}$$

- **Total theory uncertainty:** add in quadrature

$$\sigma_{\mathscr{F}}^{\rm tot} = \left(\left(\sigma_{\mathscr{F}}^{\rm PDF}\right)^2 + \left(\sigma_{\mathscr{F}}^{\rm th}\right)^2 \right)^{1/2}$$

(slight overestimate of total MHOU by neglecting μ_F correlations between PDFs and hard process)

Parton Distributions from scale-varied theories

PDF fits from scale-varied theories

Perform **multiple PDF fits** for a range of values of μ_R and μ_F
MHOUs on the PDFs estimated as the **envelope of fits** with different scales

3-points

$\sigma(\mu_R = Q, \mu_F = Q)$ central scales

$\sigma(\mu_R = 2Q, \mu_F = 2Q)$ $\sigma(\mu_R = Q/2, \mu_F = Q/2)$

7-points

$\sigma(\mu_R = 2Q, \mu_F = Q)$ $\sigma(\mu_R = Q, \mu_F = 2Q)$

$\sigma(\mu_R = Q/2, \mu_F = Q)$ $\sigma(\mu_R = Q, \mu_F = Q/2)$

9-points

$\sigma(\mu_R = 2Q, \mu_F = Q/2)$ $\sigma(\mu_R = Q/2, \mu_F = 2Q)$

Assume that μ_R and μ_F variations are fully correlated for all data points

PDF fits from scale-varied theories

Theory covariance matrix approach

$$\chi^2 = \frac{1}{N_{\rm dat}} \sum_{i,j=1}^{N_{\rm dat}} \left(D_i - T_i^{(0)}\right) (C + S)^{-1}_{ij} \left(D_j - T_j^{(0)}\right)$$

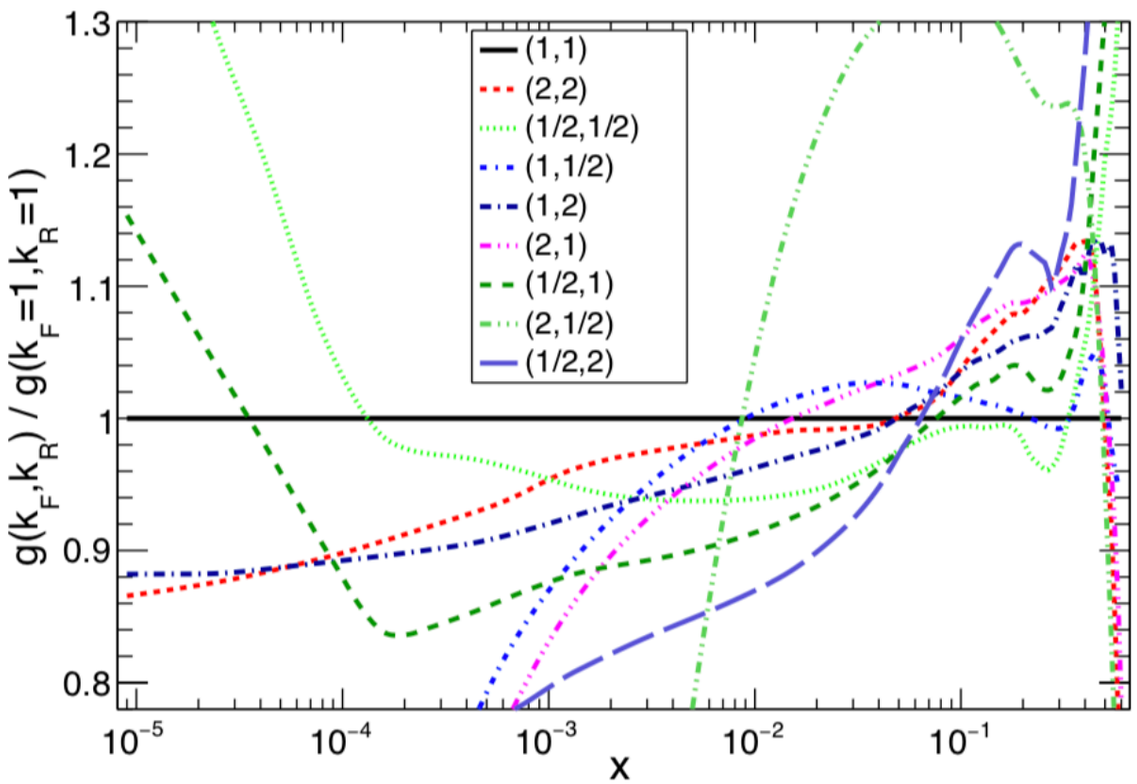
Scale-varied theories approach

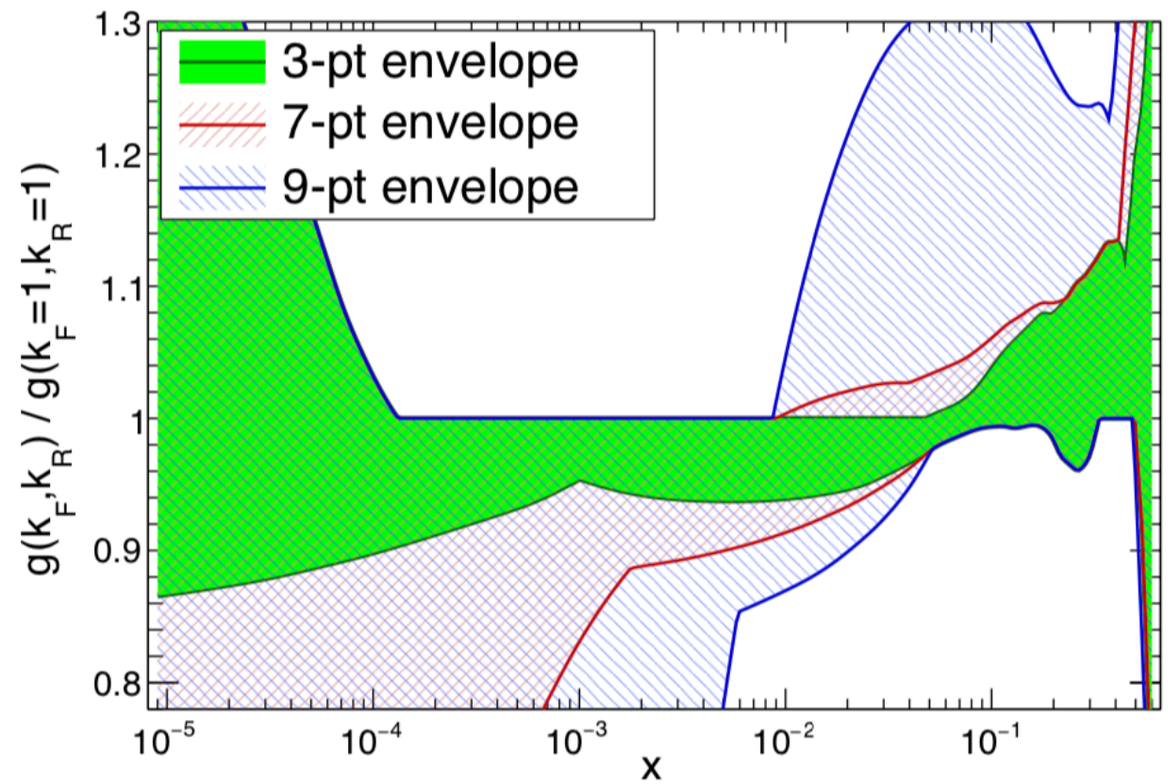
$$\chi^{2(s)} = \frac{1}{N_{\rm dat}} \sum_{i,j=1}^{N_{\rm dat}} \left(D_i - T_i^{(s)}\right) C^{-1}_{ij} \left(D_j - T_j^{(s)}\right)$$

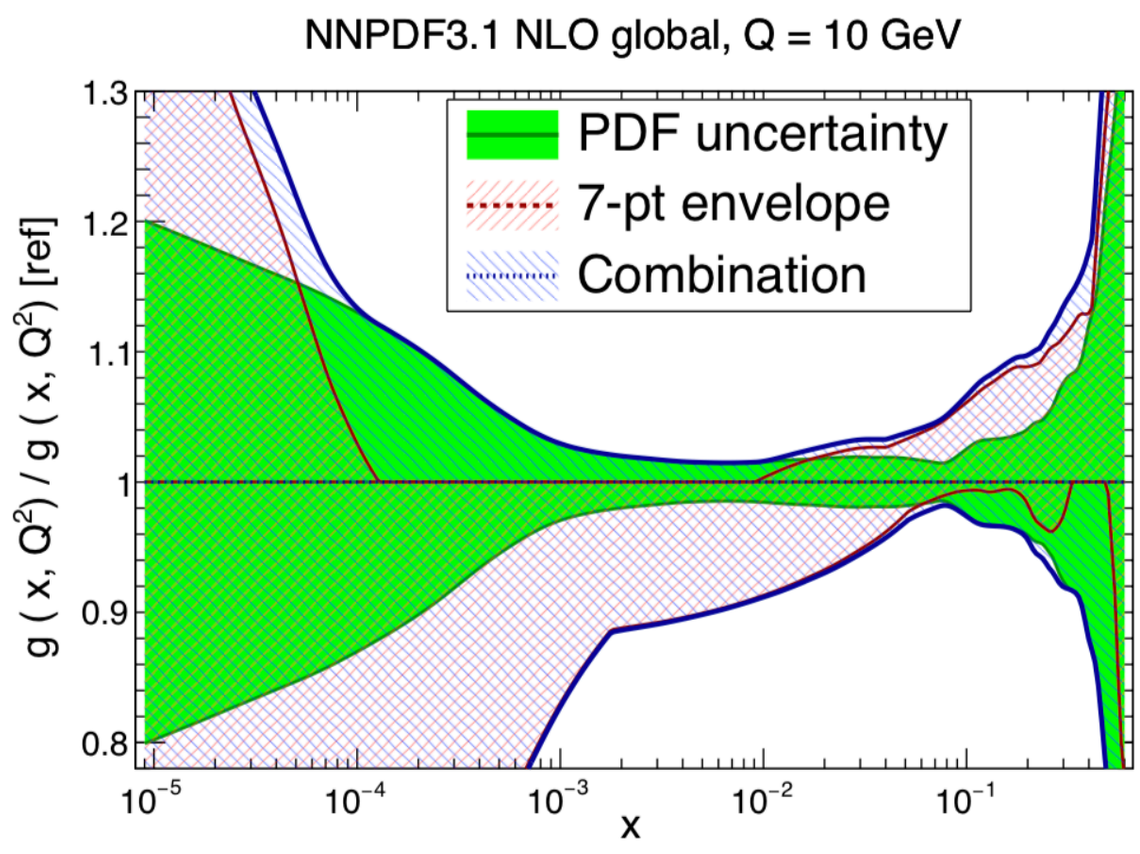
where *(s)* labels each specific combination of scale variations

Estimate MHOU on PDFs from envelope of fits from scale-varied theories

Results

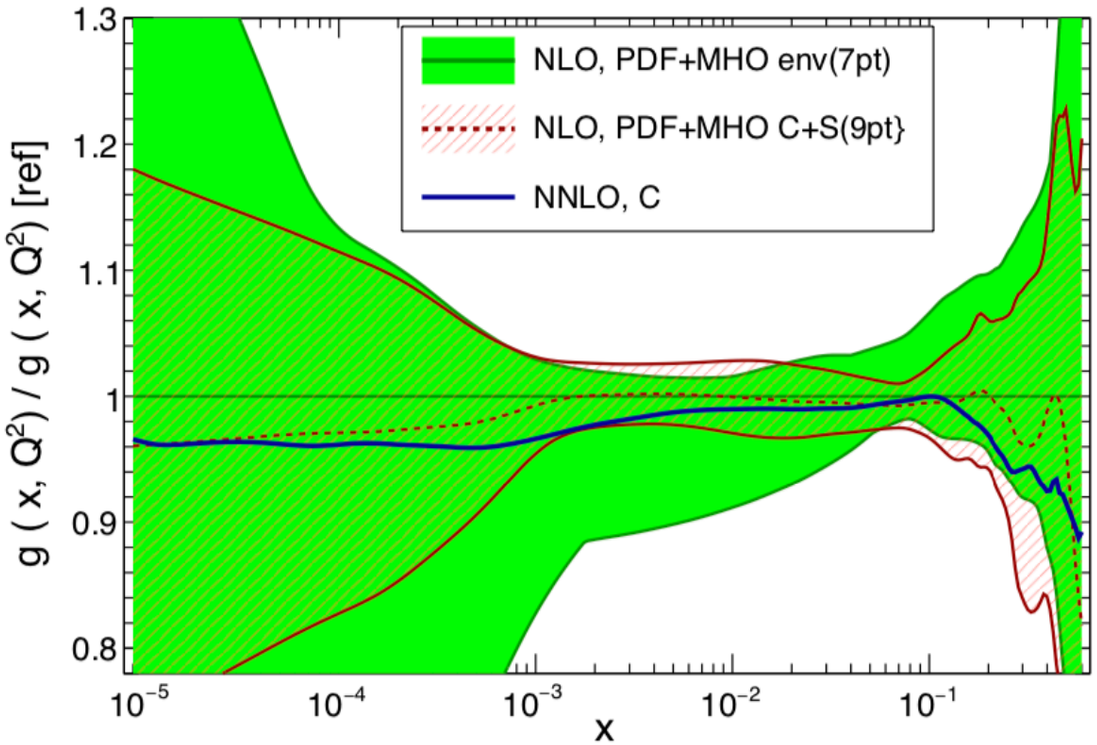


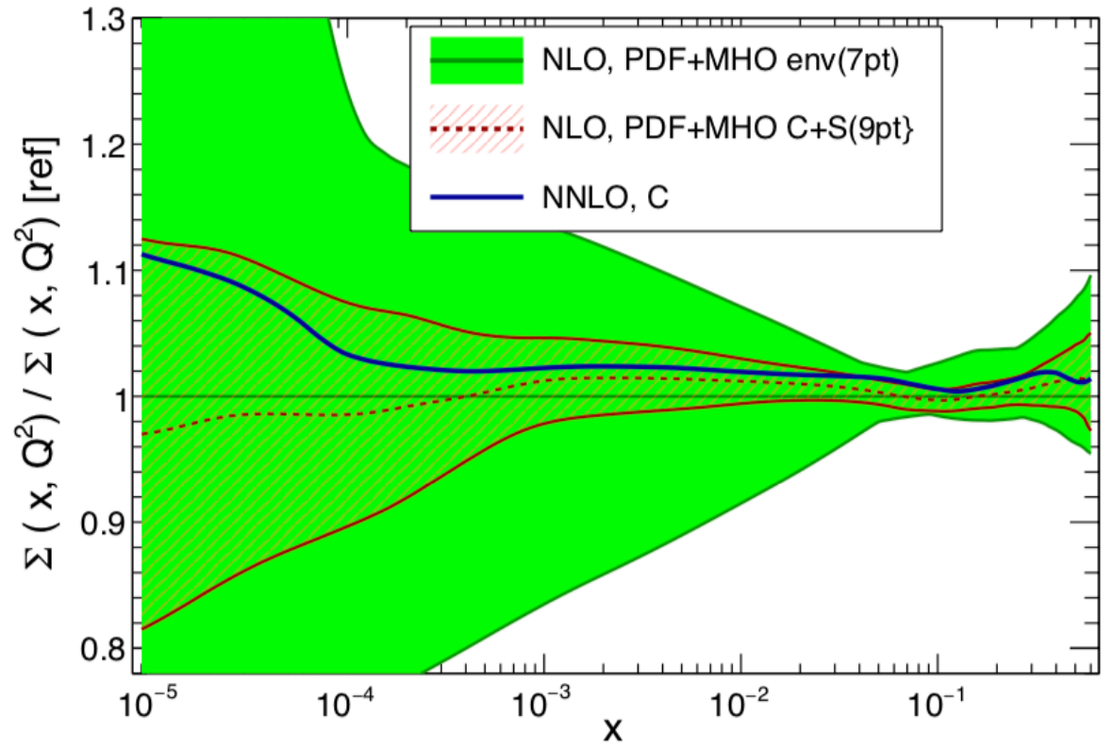


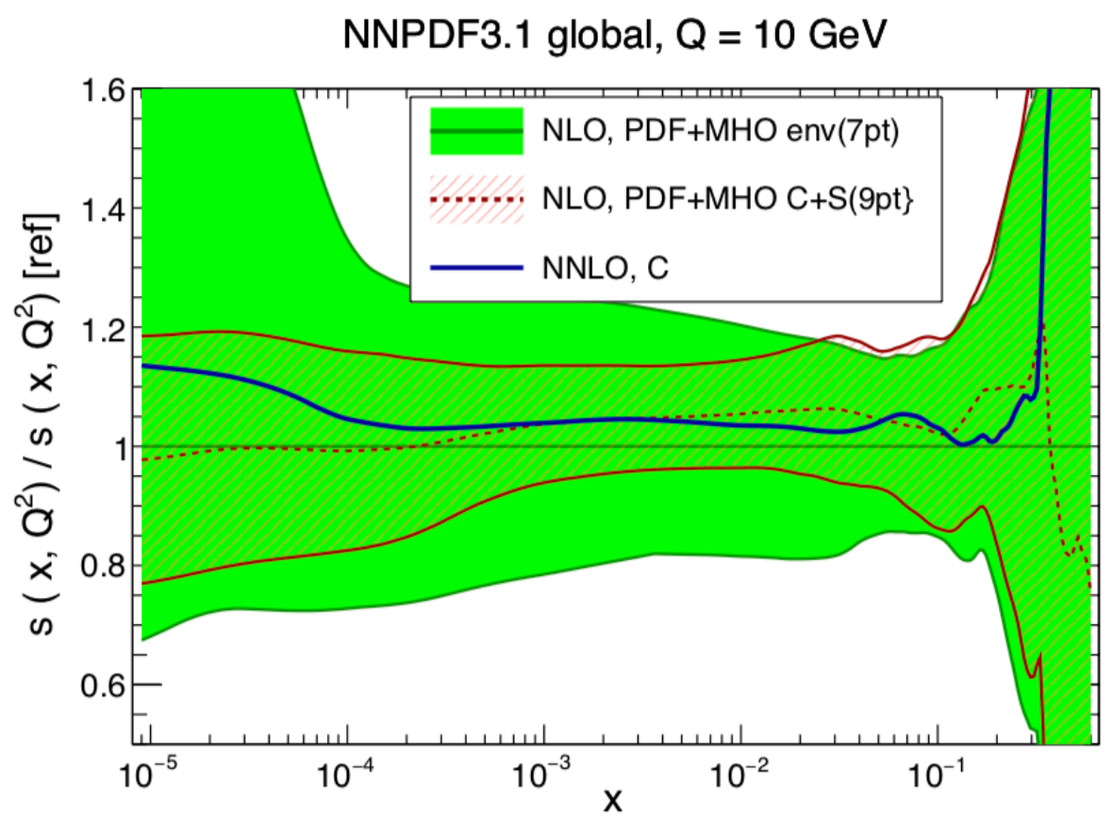


- Envelope prescription **unstable**: large differences with choices of scales
- Some scale variations clearly lead to **pathological results** in the PDF fit
- MHOUs on PDFs obtained with this approach can be combined in quadrature with standard PDF uncertainty

Results







- The 7-pt envelope for the scale-varied fits is agreement with the theory covmat prescription but clearly over conservative
- Moreover the envelope approach leaves central value unchanged: dies not account for the rebalancing effect that MHOUs have

Summary and outlook

- Systematically quantifying the **impact of MHOUs in global PDF fits** is an important ingredient for the precision phenomenology program at the LHC
- We have developed a novel approach to estimate MHOUs in PDF fits based on constructing **a theory covariance matrix** based on scale variations
- This approach is validated, for both diagonal elements and for correlations, by means of known **NLO to NNLO shift**
- First exploration of pheno implications for representative LHC processes
- Same approach can be used for **other theory uncertainties:** nuclear corrections
- Next step is the global NNLO fits with MHOUs: **NNPDF4.0**

MHOUs on PDFs represent the next frontier in global QCD fits!

Summary and outlook

- Systematically quantifying the **impact of MHOUs in global PDF fits** is an important ingredient for the precision phenomenology program at the LHC
- We have developed a novel approach to estimate MHOUs in PDF fits based on constructing **a theory covariance matrix** based on scale
- This approach is validated, for both diagon correlations, by means of known **NLO to NNLO**
- First exploration for representative LHC processes
- S used for **other theory uncertainties:** nuclear corrections
- Next step is the global NNLO fits with MHOUs: **NNPDF4.0**

Thanks for your attention!

MHOUs on PDFs represent the next frontier in global QCD fits!