Quantum Field Theory Extension

MSc Physics and Astronomy, Theoretical Physics track

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1 Course guide

Quantum Field Theory (QFT) is the mathematical framework that describes the behaviour of subatomic particles as well as quasi-particles in condensed matter systems. It is built upon the combination of classical field theory, quantum mechanics, and special relativity. In particular, QFT is the language of the Standard Model of particle physics, one of the most successful physical theories ever constructed by humankind.

In this course, we discuss a number of topics in Quantum Field Theory, extending the topics covered in the preceding course. In particular, we study how infinities arises in QFTs and how they can be tamed using the renormalization procedure, and present the quantization of the photon field and its consequences for the description of the interactions between matter and gauge fields.

Embedding. The present course is part of the joint UvA and VU Master course (MSc) in Physics and Astronomy, in particular within the *Theoretical Physics* track. The course takes place during the first four teaching weeks of January, and consists on 4 hours of lectures plus 2 hours of tutorial work per week.

This *Quantum Field Theory extension* course is, as its name indicates, the natural continuation of the *Quantum Field Theory* course, and indeed here we will assume that all students have already followed it. The topics covered in the present extension are of general interest for all theoretical physics students, and those interested in further pursuing a deeper study of QFT after this course are encouraged to enroll in the *Particles and Fields* and/or the *Advanced Quantum Field theory* courses. Both courses continue this exploration of the rich nature of QFTs from various point of view, the first more focused towards phenomenological applications and the second concentrating on the more formal aspects of the theory.

The topics covered on this extension have been chosen to minimize the overlap between previous courses that the students have followed (in particular the QFT course) and future courses that the students might choose to follow (specifically aQFT and P&F). Therefore students interested in achieving a deep understanding of QFT in general should follow this course in addition to the previous QFT course and the two subsequent courses, since while some degree of repetition is unavoidable, we have made an effort to keep it to a minimum.

Course assessment. The assessment of the course will be based on homework problems, and therefore there will be no final exam. The completion of all the homework assignments is a requisite to pass the course. The exact dates for the homework problems submission will be announced during the first week of lectures.

The two homework problem sets are included in Apps. A and B of these notes, respectively.

Course outline. This course, and the corresponding lecture notes, are organized as follows. We start in Sect. 2 by introducing calculations in quantum theory beyond the Born approximation, showing how ultraviolet divergences arise in loop diagrams for the specific case of $\lambda\phi^4$ theory. We also demonstrate how in renormalizable quantum field theories these divergences can be absorbed into a redefinition of the Lagrangian bare (unphysical) parameters. Next, in Sect. 3 we present a language to relate the behaviour of quantum field theories at energy different scales, called the renormalization group equation. In the second part of the course, we move in Sect. 4 to discuss of the quantization of Quantum Electrodynamics (QED), the quantum field theory of the electromagnetic interactions, deriving in particular in canonical quantization of the photon field and deriving the Feynman rules of the theory. In this part we will also present for completeness a review of the classical symmetries of the Abelian gauge theory. We complete the course by performing calculations of simple scattering processes in scalar QED.

In addition, to make these lecture notes self-consistent, we have added two extra appendices, App. C with a brief review of some results in QFT that the students should have already encountered before, and App. D for a list of useful mathematical expressions and identities that are required at some point during the course.

Course materials. The main resource for this course are the lecture notes, that are available via my personal webpage:

http://juanrojo.com/teaching

These lecture notes will be updated as the course goes on. These notes are however not meant to be the only study resource, but rather they represent a guide to help the student to navigate within the course material and when needed consult additional references. In addition, the lecture notes have not been proof-read or cross-checked by other people: the students are encouraged to let me know of mistakes and typos that they might find. These lecture notes are naturally complemented by the corresponding lecture notes of Daniel Baumann's *Quantum Field Theory* course, also available online.

The topics covered in this course are inherited to three main textbooks, namely:

- *Quantum Field Theory*, Mark Srednicki, Cambridge University Press.

  This textbook is freely accessible online as a .pdf file:

  https://www.physics.utoronto.ca/~luke/PHY2403F/References.html

- *Quantum Field Theory and the Standard Model*, Matthew D. Schwartz, Cambridge University Press.

  More information about this textbook can be found here:

  http://users.physics.harvard.edu/~schwartz/teaching

- *An introduction to Quantum Field Theory*, Michael E. Peskin and Daniel V. Schroeder, Westview Press. A classic QFT textbook. The solutions for the exercises in some of the earlier chapters of the book can be found here:

  http://homerreid.dyndns.org/physics/peskin/index.shtml

For the interested students, other related online lectures notes that they might consider to also study are the following ones:

- David Tong's lecture notes on Quantum Field Theory:

  http://www.damtp.cam.ac.uk/user/tong/qft.html

- Sidney Coleman's Harvard Quantum Field Theory course:

  https://arxiv.org/pdf/1110.5013.pdf

- Michael Luke's QFT lecture notes

https://www.physics.utoronto.ca/~luke/PHY2403F/References.html

Finally, let me mention that a potentially useful overview of several QFT textbooks by Flip Tanedo can also be found here:

https://fliptomato.wordpress.com/2006/12/30/
from-griffiths-to-peskin-a-lit-review-for-beginners/

Learning objectives. At the end of the course, the student will be able to:

- Understand the physical origin of the infinities that arise in calculations of scattering processes in Quantum Field Theory beyond the Born approximation.
- Calculate finite one-loop processes in QFT by removing these infinities using the renormalization method in the case of the scalar $\lambda\phi^4$ theory.
- Relate physical phenomena at different scales by using the renormalization group flows.
- Quantize electromagnetism and perform calculations involving spin-1 photon fields.
- Understand what are the implications of electromagnetism's classical symmetries at the QFT level.
- Become familiar with simple interaction processes in Quantum Electrodynamics.

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