Introduction to Elementary Particle: Theory

Bachelor course in Applied Physics, Technical University of Delft.

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Numerical solutions to problem sets, current version: January 30, 2018

1 Numerical solutions to Problem Set I

1) We have a sample of 1.49 μg of pure ${}^{13}_{7}$N, characterized by a half-life of $\tau = 600$ s.
Answer the following questions:
a) How many atoms are there in this sample of pure nitrogen?
b) How large is the activity at $t = 0$?
c) How large is the activity after one hour?
d) How much ${}^{13}_{7}$N do we still have after one hour?
e) Assume that ${}^{13}_{7}$N undergoes a beta decay of the form ${}^{13}_{7}\mathrm{N} \to X + e^+ + \nu$. What is X here? Is this reaction actually possible?

Numerical solutions:
$A(t = 0) = 1.15 \times 10^{14}$ atoms$/s$, $A(t = 1\,h) = 2.8 \times 10^{11}$ atoms$/s$, $N(t = 1\,h) = 1.7 \times 10^{14}$ atoms.

2) Consider the radiative decay of polonium-210: ${}^{210}_{84}\mathrm{Po} \to \mathrm{X} + \alpha$.
a) What is X in this reaction, and why?

b) Determine the energy Q released in this decay (in MeV), using that $m_{\mathrm{Po}} = 209.93676$ u, $m_{\mathrm{X}} = 205.92945$ u, and $m_\alpha = 4.0012$ u.

c) Compute the kinetic energy that the α particle has in this decay process. Make use of the conservation of energy and momentum, and assume that the Po nuclei was at rest before decaying. Assume also Newtonian (non-relativistic) mechanics.

d) Compute the velocity of the radiated α particle. Should we worry about relativistic effects? Why?

Numerical solution:
$Q = 5.7$ MeV, $v_\alpha = 0.055c$

3) What is the energy that we need to supply to a proton in order to accelerate it from $\beta = 0$ to $\beta = 0.9$? And from $\beta = 0.9$ to $\beta = 0.99$? And from $\beta = 0.99$ to $\beta = 0.999$? Is is possible to accelerate then the proton up to $\beta = 1$? Why?

Numerical solution:
$\Delta E(0, 0.9) = 1214\,\mathrm{MeV}$, $\Delta E(0.9, 0.99) = 4498\,\mathrm{MeV}$, $\Delta E(0.99, 0.999) = 14330\,\mathrm{MeV}$.

4) An electron in a cathodic rays tube, starting from rest, experiences a potential difference of $\Delta V = 5 \times 10^4$ V between an initial position and a screen that can measure its position upon impact. With which speed does the electron reach the screen? Perform the calculation using both classical mechanics and relativistic mechanics. How do the results change if now the potential difference is increased up to $\Delta V = 2 \times 10^5$ V? And to $\Delta V = 10^6$ V? For which values of the potential difference is the classical approximation sensible and for which others one needs the relativistic expressions?

Numerical solutions:
In classical mechanics: $v = 0.44\,\mathrm{c}$, $v = 0.88\,\mathrm{c}$, $1.97\,\mathrm{c}$.
With relativistic kinematics, $v = 0.41\,\mathrm{c}$, $v = 0.70\,\mathrm{c}$, $v = 0.94\,\mathrm{c}$.

5) An unstable particle at rest, denoted by X, decays into two lighter particles A and B. We know that particle A has a mass of 6.67×10^{-27} kg and a speed of $\beta = 0.6$. We also know that particle B has a mass of 1.67×10^{-27} kg. Determine the speed of particle B. Compute also the mass of the initial particle X.

Numerical solution:
$m_X = 6557\,\mathrm{MeV}/c^2$.

6) Consider an anti-proton, traveling with momentum 1 TeV/c in the z direction.
a) Assume that this anti-proton collides head-on with a proton carrying the same momentum but traveling in the opposite direction. Assume that the result of this collision is a new particle. Using energy-momentum conservation, compute the four-momentum of this new particle. What is its mass?
b) Same situation as above, but now with the proton standing at rest.
c) Discuss what are the advantages and advantages of the two configurations described above.

Numerical solution:
$m_X = 2\,\mathrm{TeV}/c^2$, $m_X = 40\,\mathrm{GeV}/c^2$

2 Numerical solutions to Problem Set II

This set of exercises should be completed and send by email by **Wednesday 24th of January 2018**.

1) Consider that we have detected photons and neutrinos, both with an energy of 1 MeV, coming from a supernova explosion which is at a distance of 10^5 light years from us. We measure that the arrival time on Earth of these photons and neutrinos differ by less than 10 seconds. Given that photons are massless, derive from this observation an upper bound on the neutrino mass.

Numerical solutions:
$m_\nu c^2 \leq 3 \times 10^{-3}$ meV$/c^2$

2) Construct the four-momentum vector $(E/c, \vec{p})$ for the following particles. Verify explicitly in all cases that the mass-shell condition holds.
a) A proton at rest.
b) A neutron with a total energy of 1500 MeV, moving the z direction.
c) An electron with kinetic energy of 500 keV, moving in the z direction.
d) A proton moving in the z direction with speed $\beta = 0.9$.
e) A photon with wavelength $\lambda = 10^{-15}$ m, moving in the z direction.

3) Consider the Φ meson, see Table **??**. In the framework of the quark model, can you give a possible quark content of this meson? Is this choice unique? This Φ meson has a width (a fundamental uncertainty in its rest mass) of around 4.3 MeV/c^2. What can we say about its expected lifetime?

Numerical solutions:
$\Delta t \geq 7.6 \times 10^{-23}$ s.

4) Consider the following decay modes of the tau leptons, where X represents one or more unknown particles. Indicate which of these decay modes are actually possible, and if they are not possible why is so. For the allowed decays, indicate what particle(s) X stands for.

a) $\tau^- \to e^- + X$
b) $\tau^- \to \mu^- + X$
c) $\tau^+ \to \pi^+ + X$
d) $\tau^+ \to p + X$
e) $\tau^+ \to \nu_\mu + X$

5) The decay $\Delta^{++} \to p + \pi^+$ can be interpreted in terms of the constituent quark model. Draw the corresponding diagrams, taking into account the quark composition of the initial and final-state hadrons, for the following processes:

a) $\pi^0 + p \to n + \pi^+$
b) $\Phi \to K^+ + K^-$
c) $\pi^- + p \to \Lambda^0 + K^0$
d) $\Lambda^0 \to p + \pi^-$

In all cases describe any assumption that you might have used. Note that the first three processes are mediated by the strong interaction, and therefore the only allowed lines are those connecting the same quark in the initial and in the final state, and those connecting a quark and an antiquark of the same flavor in either the initial or the final state. Discuss why the last reaction is different and cannot be mediated by the strong interaction.