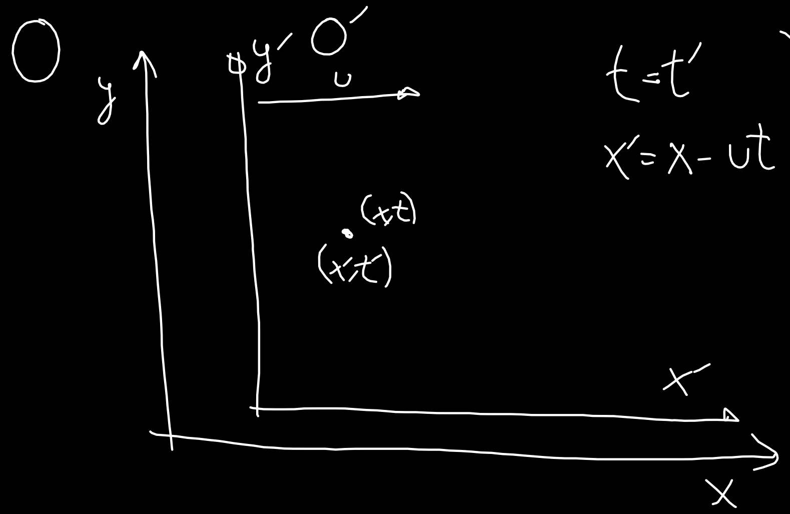
8/1/2018 SPECIAL RELATIVITY

$C = 3\times10^8\, m/s$

C'

NEWTON'S MECHANICS



$$\left.\begin{array}{l} t = t' \\ x' = x - ut \end{array}\right\}$$

$$V = \frac{dx}{dt}$$

$$V' = \frac{dx'}{dt'} = \frac{d(x-ut)}{dt} = V - U$$

$V = U \rightarrow V' = 0$

$V' = V - U$

$C' = C - U$

C IS THE SAME IN ANY REFERENCE FRAME

LORENTZ

NEWTON

$$\begin{cases} x' = x - ut \\ t' = t \end{cases}$$

LORENTZ

$$\begin{cases} t' = \gamma\left(t - \frac{ux}{c^2}\right) \\ x' = \gamma\,(x - ut) \end{cases}$$

$$\gamma \equiv (1 - u^2/c^2)^{-1/2}$$

$$\beta = \frac{u}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1 + \frac{\beta^2}{2} + \cdots$$

$$t' = \left(1 + \beta^2/2\right)\left(t - \beta x/c\right) = t$$

$$x' = \left(1 + \beta^2/2\right)\left(x - ut\right)$$

$$= x - ut$$

NOTHING CAN MOVE FASTER THAN C

NEWTON

* $V' = V - U = \frac{3}{4}c + \frac{3}{4}c = \frac{6}{4}c = \frac{3}{2}c$

$V = \frac{3}{4}c$

$U = -\frac{3}{4}c$

EINSTEIN

* $V' = \frac{V - U}{1 - UV/c^2}$ $V' = 0.96c < c$

ENERGY AND MOMENTUM

NEWTON'S

$E_{KIN} = \frac{1}{2}mv^2$

$p_x = mv$

EINSTEIN

$E = \tilde{\gamma} mc^2$

$\tilde{\gamma} =$

$E_{KIN} = \frac{1}{2} m v^2$

$\boxed{c=1}$ NATURAL UNITS

SR $E = \tilde{\gamma} m c^2 \simeq \boxed{mc^2} + \frac{1}{2} m v^2 + O(\beta^2)$

N: $p_x = mv$

$\tilde{\gamma} = \frac{1}{\sqrt{1 - v^2/c^2}} \sim 1 + \frac{\beta^2}{2} + O(\beta^4)$

SR: $p_x = \tilde{\gamma} m v$

$v \ll c \quad \tilde{\gamma} \to 1 \quad E \to mc^2 \quad \beta = \frac{v}{c}$

$E = E_{REST} + E_{KIN}$

$E_{REST} = mc^2$, $E_{KIN} = \underline{(\tilde{\gamma} - 1) mc^2}$

$E = mc^2$

REST MASS ENERGY

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- Motivatie (kort)
- ervaring

ANTIMATTER

ELECTRON e^- $\Rightarrow$ POSITRON e^+

$$m_{e^-} = m_{e^+}$$

$$Q_{e^-} = -Q_{e^+}$$

PROTON p $\Rightarrow$ ANTI-PROTON $\bar{p}$

$$m_p = m_{\bar{p}}$$

$$Q_p = -Q_{\bar{p}}$$

$$e^- + e^+ \rightarrow \gamma + \gamma$$

$$Q_{e^-} + Q_{e^+} = 0$$

$$Q_\gamma + Q_\gamma = 0$$

$$E_{KIN} = (\gamma - 1) mc^2$$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$E_{REST} = mc^2$$

$$\frac{E_{KIN}}{E_{REST}} = \gamma - 1 \simeq 0.01$$

e^- $\quad v = 0.1 \times c = 10^9 \, km/h$

$$\frac{E_{KIN}}{E_{REST}} \simeq 1 \Rightarrow \beta = 0.87$$

$m_{ANTIMATTER} = 0.25g$

$E = mc^2 = 2 \times 0.25g \times c^2 = 5 \times 10^{13} J$

$E_{ATOMIC} \simeq 6.3 \times 10^{12} J$

CERN produces 10^{-12} g/yr

$$O \longrightarrow O'$$

$$p'_x = \gamma\left(p_x - \frac{vE}{c^2}\right)$$

$$E' = \gamma\,(E - v p_x)$$

FOUR VECTORS

$$d=3 \quad \vec{A} = (A_x, A_y, A_z)$$

$$A^\mu = (A_t, A_x, A_y, A_z)$$

$$\mu = 0, 1, 2, 3$$
$$= t, x, y, z$$

NORM $|\vec{A}| = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{A_1^2 + A_2^2 + A_3^2}$

$$\boxed{|A| = \sqrt{A_t^2 - A_x^2 - A_y^2 - A_z^2}}$$

$$x^\mu = (ct, x, y, z) \qquad |x| = (c^2t^2 - x^2 - y^2 - z^2)^{1/2}$$

$$p^\mu = (E/c, p_x, p_y, p_z) \qquad |p| = \left(E^2/c^2 - |\vec{p}|^2\right)^{1/2}$$

$\boxed{\text{MASS-SHELL CONDITION}}$

THE NORM $|A|$ IS THE SAME IN ANY REF FRAME

$$E = \tilde{\gamma} m c^2$$

$$p_x = \tilde{\gamma} m v$$

$$p^\mu = (\tilde{\gamma} m c, \tilde{\gamma} m v) = \tilde{\gamma} m (c, v)$$

$$\underline{|p|} = \tilde{\gamma} m \sqrt{c^2 - v^2} = \underline{\tilde{\gamma}}\, m c \underline{\sqrt{1 - \frac{v^2}{c^2}}} = \underline{mc}$$

$$A \longrightarrow B + C \quad [\text{DECAY}]$$

$\boxed{E, \vec{p} \Rightarrow \text{CONSERVED}}$

GOAL $\Rightarrow$ COMPUTE β_B, β_C $\qquad m_A, m_B, m_C$

(*) REST FRAME OF A

$$p_A^\mu = (m_A c, 0, 0, 0)$$
$$p_B^\mu = (E_B/c, 0, 0, p_z)$$
$$p_C^\mu = (E_C/c, 0, 0, -p_z)$$

$$p_z = \frac{\beta_B}{\sqrt{1-\beta_B^2}} m_B c$$

$$-p_z = \frac{\beta_C}{\sqrt{1-\beta_C^2}} m_C c$$

$$\rightarrow m_A c = E_B/c + E_C/c$$

$$E_B/c = \sqrt{m_B^2 c^2 + p_z^2} \quad (\text{MASS-SHELL})$$

$m_A \simeq m_B + m_C$

$$P_{Z/C} = \left(2(m_A - m_B - m_C) \Big/ \left(1/m_B + 1/m_C \right) \right)^{1/2}$$