Introduction to Elementary Particle: Theory

Bachelor course in Applied Physics, Technical University of Delft.

Dr Juan Rojo
VU Amsterdam and Nikhef Theory Group
http://www.juanrojo.com/
j.rojo@vu.nl

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Study guide

To begin these lecture notes, we provide some general information about the theory part of the course. More information can be found in the `Brightspace` page of the course, together with all the teaching materials related to both the Theory and the Instrumentation parts of the course.

General context

This course is divided into a Theory part, with instructor Dr. Juan Rojo, and an Experimental/Instrumentation part, with instructor Prof. Harry van der Graaf. This course is offered as part of the first year of the *Applied Physics* bachelor course of the Technical University of Delft. Both the theory and experimental lectures will be recorded and made available in *Collegerama* shortly after:

http://collegeramacolleges.tudelft.nl/online/welcome/

Course language

The theory part of the course will be given in English, while the experimental part will be given in Dutch. The problem sets (assignments) of the theory part will be provided in English, but the students can complete them either in Dutch or in English, as they prefer. Both English or Dutch are equally fine for the communication with the course instructors and with the teaching assistants.

Course assessment

The assessment of the Theory part of course is divided into the following components

- A final exam.
- Three problem sets

The three problem sets need to be completed and submitted via `Brightspace` by Wednesday 17, 24, and 31, respectively, by 17.00 at the latest.

Schedule of the theory lectures

There are in total 8 theory lectures, each during 2 hours (so a total of 16 hours of theory lectures). As indicated in the TU Delft Rooster, the theory lectures take place on Thursday and Fridays of weeks 2.6 to and 2.9, between 8.45 and 10.30 in lecture room **Aula CZ D**:

http://www.roosters.tudelft.nl/

Teaching materials

The main reference for the theory part of the course will the lecture notes, which will be available before each lesson on the `Brightspace` page of the course. In addition, there are a number of textbooks that interested students might want to read to delve deeper into some specific topics, though we emphasize that the lecture notes are self-consistent, and contain all the material needed to follow the topics covered in the course. Moreover the level of these books is mostly at the advanced undergraduate level (where particle physics courses are mostly taught). Some of these textbooks include:

- *Modern Particle Physics*, Mark Thomson, Cambridge University Press (2013).
- *Introduction to Elementary Particles*, David Griffiths, Wiley (2008).
- *An Introduction to the Standard Model of Particle Physics*, W. N. Cottingham and D. A. Greenwood, Cambridge University Press (2007).

In addition, popular science books that provide gentle introductions to the world of elementary particle physics include:

- *Smashing physics: inside the world's biggest experiment*, John Butterworth, Headline Book Publishing (2014).
- *The particle at the end of the universe*, Sean Carroll, Dutton (2013)
- *Higgs discovery*, Lisa Randall, Ecco (2013).
- *Why string theory?*, Joseph Conlon, CRC Press (2015).
- *Particle Physics: a very short introduction*, Frank Close, Oxford University Press (2013).

Course outline

The Theory part of the *Introduction to Elementary Particles* course is divided into four weeks, with two lectures of two hours each per week. The topics that will be covered are roughly distributed in five main conceptual blocks:

(a) We start with a general overview of the topic of elementary particle physics, including a discussion of the relevant distance and energy scales relevant for the description of particle physics. We will here introduce a number of important concepts such as atomic structure and radioactivity.

(b) We will then introduce the basic formalism of Einstein's Special Relativity, which is the appropriate mathematical language to describe elementary particles, as well as the concept of antimatter and antiparticles. We will also present the basic formalism of quantum physics, including crucial concepts such as the Heisenberg's uncertainty principle. This will include a discussion of a new property of elementary particles, their *spin* (intrinsic angular momentum), showing how particles with different spin behave very differently.

(c) Next we will then discuss the different possible types of elementary particles, such as fermions and bosons, and leptons and quarks. We explain how hadrons, particles that interact via the strong nuclear force, such as protons are composed by quarks, and the crucial role of symmetry principles to describe hadron structure.

(d) Here we study the fundamental interactions between elementary particles. We will show how these interactions can be understood as mediated by the exchange of the force carriers: the photons (for electromagnetism), the gluons (for the strong interaction) and the W, Z bosons for the weak interaction.

We will introduce a powerful method to represent particle interactions, known as *Feynman diagrams*. Then we will present what are the experimental consequences of the strong and weak interactions. We will introduce the Standard Model of particle physics, including the role that the recently discovered Higgs boson plays on it.

(e) In the final part of the course, we discuss a number of open questions within the Standard Model, such as the nature of dark matter and dark energy, or the asymmetry between matter and antimatter. Naturally, this will lead us to discuss the interplay between elementary particles and cosmology, that is, between the smallest and the largest distances in the Universe.

This course is self-contained, and does not assume any additional physics or mathematics beyond the standard high school level. For completeness, the lecture notes include additional background material on topics such as quantum physics and special relativity, with some advanced topics beyond the course learning objectives. Students interested in taking a closer looks at some of such more advanced topics of the course can contact me for further references.

1 The particle zoo: from atoms to electrons and quarks

We start this course with a general qualitative overview of the topic of elementary particle physics, including a discussion of the relevant distance and energy scales appropriate for the description of particle physics. We will show which are the appropriate scales to describe sub-atomic objects such an protons and electrons. We also present a first exploration of the elementary particles zoo. In the next lectures we will provide self-contained introductions to both special relativity and quantum theory, after which we will be able to explore in a rather more quantitative way the world of elementary particles.

The **Learning Goals** of this lecture are:

(a) To become familiar with the distance and energy scales appropriate to the description of elementary particles.

(b) To be able to explain the underlying physics of atomic phenomena such as radioactivity.

(c) To understand the properties of antimatter and of its interaction with normal matter.

(d) To identify the basic types of elementary particles as well as the interactions among them.

1.1 Traveling to the heart of matter

What do we understand by *elementary particle physics*? This term denotes generically the physics of anything which is smaller than the size of an atom. It is also known as *subatomic physics*, and you can also find the names nuclear physics or *high-energy physics*, depending on the specific application.

Crucially, the laws of physics in the world of elementary particles are very far from our everyday intuition, and therefore we will need to introduce first a brand new mathematical framework to describe phenomena involving elementary particles. But before that, let us take a look at the relevant distance scales involves, and review some of the evidence we have of the particle structure of matter. Therefore, in order to start our journey in the world of elementary particles, it is useful to start to become familiar with the magnitudes of the physical quantities involved in this word. Let us start with distances. In Fig. 1.1 we show schematically the inner structure of atoms. As we move inwards into atoms, thus probing smaller and smaller distances, we uncover a whole new world:

- The size of a typical atom is around $\simeq 10^{-10}$ meters.
- A atom is basically a lot of empty space, with a tiny *nucleus* of size $\simeq 10^{-14}$ meters surrounded by a cloud of *electrons* orbiting around.
- The atomic nucleus is composed by heavily packed *protons* and *neutrons*, each of them with size $\simeq 10^{-15}$ meters.
- Going further down, protons and neutrons are not elementary particles, but rather composed by other types of particles called *quarks*. More specifically, protons and neutrons are composed by two different types of quarks, the *up quark* u and the *down quark* d. The proton thus can be written in terms of quarks as a uud state, while the neutron can be written as a udd one.

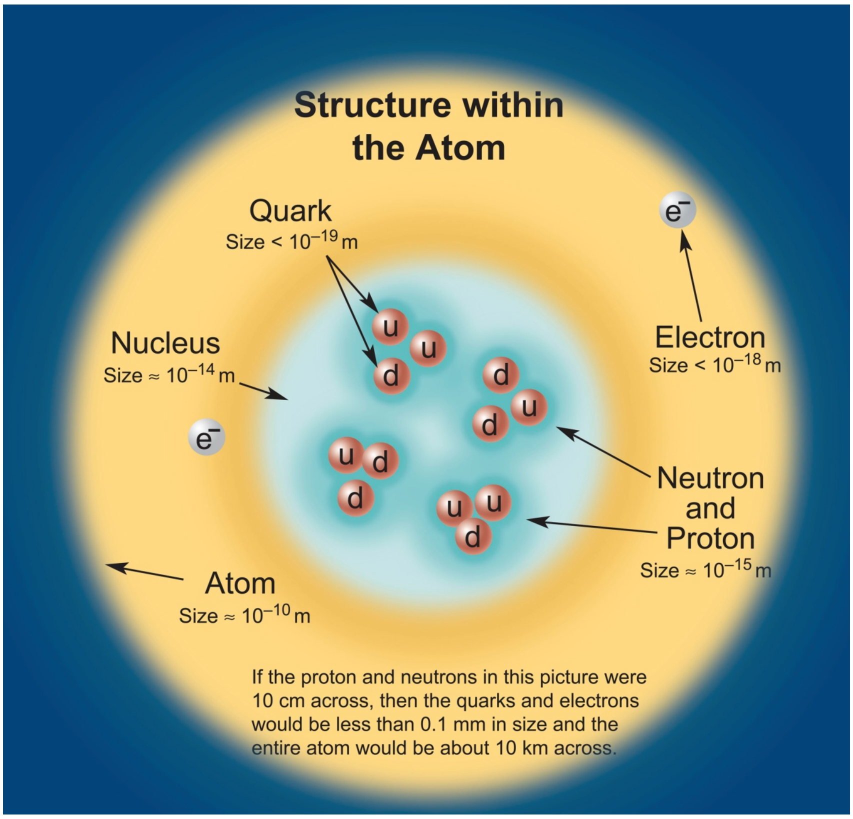

Figure 1.1. The inner structure of atoms. Atoms are composed by a tiny nucleus surrounded by a cloud of electrons. The atomic nucleus is composed by a tightly packed set of protons and neutrons. In turn, protons and neutrons are not fundamental particles themselves by are composed by quarks. As far as we know, quarks are fundamental particles, with the current upper limits on the quark radius being $r_q \lesssim 10^{-19}$ m. As discussed in the text, the ratio of the quark radius to the atomic radius is similar to the ratio of the Earth radius to the distance between the Earth and Proxima Centauri, about 9 orders of magnitude.

- We do not currently know if this Russian Doll scheme continues with electrons and quarks: current measurements indicates that their size is smaller than 10^{-18} (10^{-19}) meters for electrons (quarks), but it is conceivable that they could still exhibit further substructure for even smaller distances. Ongoing and future experiments might unveil then a further layer of reality when probing yet smaller distances.

It is important to emphasize that the atomic microcosm involves very steep changes of the relevant length scales. For instance, we see that going from the size of the atom to the current upper limits on the size of the quarks one spans around *9 orders of magnitude*.

To have an idea of what this change in distance scales involves, note that the Earth radius is $R_e \sim 6 \times 10^3$ km, while the distance between the Earth and the nearest start, Proxima Centauri, is around $d_{\rm PC} \sim 4 \times 10^{13}$ km. Therefore, the ratio of the Earth radius to the distance between the Earth and Proxima Centauri is $R_e/d_{\rm PC} \sim 10^{-10}$, to be compared with the ratio of the quark radius r_q to the atomic radius r_A, $r_q/r_A \lesssim 10^{-9}$. In other words, going from quarks to atoms involves an increase in typical lengths comparable to going *from the Earth to its closest neighboring star* (other than the Sun of course).

Is therefore clear that using units such as meters or kilometers is not really suitable to study the microcosm. Therefore, in this course we will use more frequently the following distance units:

- The *nanometer* (nm): one nanometer corresponds to 10^{-9} meters.
- The *Angstrom* (A): one Angstrom corresponds to 10^{-10} meters.
- The *picometer* (pm): one picometer corresponds to 10^{-12} meters.
- The *femtometer* (fm): one femtometer corresponds to 10^{-15} meters.

By comparing with Fig. 1.1, we see that for instance one Angstrom is the typical size of atoms, while one femtometer is the typical size of protons and neutrons. So this units are more suitable to describe phenomena in the world of elementary particles.

In a similar way as for the distances, we can also trace our journey towards the innards of the microcosm by mapping the relevant energy scales at the various steps. As in the distance case, we will find that the energy scales that we are used in everyday life are not really suited for the world of elementary particles. Consider for example the kinetic energy carried by a football of mass $m = 0.5$ kg traveling at $v = 25$ m/s: we know that it is given by

$$E_{\rm kin} = \frac{1}{2}mv^2 \simeq 150\,{\rm J}\,, \tag{1.1}$$

while the corresponding kinetic energy of an electron (whose mass is $m_e = 9.11 \times 10^{-31}$ kg) moving at $v = 3 \times 10^6$ m/s (which is about 1% of the speed of light) is instead

$$E_{\rm kin} = \frac{1}{2}mv^2 \simeq 4 \times 10^{-18}\,J\,, \tag{1.2}$$

so using Joules is not a particularly useful units when we are dealing with elementary particles. For this reason, in this course we will mostly use the *electronvolt* (eV), which is given by

$$1\,{\rm eV} = 1.602 \times 10^{-19}\,{\rm J}\,, \tag{1.3}$$

and which corresponds to the kinematic energy acquired by an electron which is accelerated from rest by an electric potential of magnitude $\Delta V = 1$ V. We then see that the above electron traveling at 1% of the speed of light has a kinetic energy of around $\simeq 25$ eV.

With this motivation, the energy scales that will be most frequently find in this course to describe elementary particles are given in eV by:

- The energy needed to strip the electron to an hydrogen atom is known as the *Rydberg constant*, and amounts to ~ 13 eV.
- The most energetic electronic transitions in heavier elements involve expelling an electron lying in the most inner layers. The typical energies involved here are of the order of a few keV (kiloelectronvolts, 1 keV $= 10^3$ eV).
- The transitions between different atomic nuclear levels, such as those involved in radioactivity (see below), are of the order of a few MeV (megaelectronvolts, 1 MeV $= 10^6$ eV).
- This is also true for the processes involved in nuclear fusion, which powers the Sun and all the stars. For example, the fusion of deuterium with tritium releases around 18 MeV in terms of kinetic energy.
- In high-energy particle reactions, the entire mass of the particles is converted into energy. For example, when a proton and an anti-proton annihilate with each other, they release an energy of around 2 GeV (Gigaelectronvolts, 1 GeV $= 10^9$ eV).
- At the most powerful particle accelerators built by mankind, protons can reach up to several TeV (Teraelectronvolts, 1 TeV $= 10^{12}$ eV) of energy.

We also mention here that as we will show in Sect. 2.3 in quantum theory one can establish a correspondence between distances and energies, where smaller distances correspond to larger energies and vice-versa.

The particles shown in Fig. 2.2 describe the virtual totality of everyday matter: everything from cars to computers, from our brain to trees, can be ultimately described in terms of *only quarks and electrons*. Indeed, the elementary particle picture of Nature is rather economical: it only takes two particles (the up and down quarks) to build up protons and neutrons, which in turn constitute all atomic nucleus. Now only add some electrons in the recipe, and now you can build all known chemical elements. While now we are rather familiar with this picture of Nature, it was established only in relatively recent times. As late as in the early 20th century, the very same existence of atoms was doubted by many scientist. But following a series of breakthroughs ranging between 1897 (with the discovery of electrons) to 1973 (with the discovery of quarks), our current framework of the structure of matter became solidly established.

Natural units. The units you are most familiar with correspond to the *International System of Units* (or SI from its French acronym). The basic ingredients of this system are the *ampere, kelvin, second, metre, kilogram, candela and mole*, plus other derivate units such as the Newton or the Pascal. However, when discussing the physics of elementary particles it is sometimes advantageous to work using *natural units*:

Natural units are physical units of measurement based only on fundamental physical constants, and therefore do not require any prototype (such as a reference atomic clock for the second or the kilogram). In natural units, we have that the speed of light c, Newton's gravitational constant G as well as other constants are all set to one, $G = c = 1$. In natural units, a length of 1.97×10^{-7} meters corresponds to 1 eV^{-1}. This allows to easily relate lengths with energies in the microcosm. For example, the typical energy released in nuclear transitions is ~ 1 MeV, which corresponds to a distance of

$$1\,\mathrm{MeV} \times \frac{10^{-6}\,\mathrm{eV}^{-1}}{1\,\mathrm{MeV}^{-1}} \times \frac{1.97 \times 10^{-7}\,\mathrm{m}}{1\,\mathrm{eV}^{-1}} \sim 2 \times 10^{-13}\,\mathrm{m}\,, \tag{1.4}$$

which is not too different with respect to the typical size of atomic nuclei.

The main advantage of natural units is that it allows to easily relate magnitudes that have an intrinsic physics connection. For example, energy can be measured in eV and mass in eV/c^2, with c being the speed of light. But as we will show in Sect. 2.1, in special relativity mass and energy are just two faces of the same coin, so using $c = 1$ allows to naturally relate mass and energy, since they are now measured in the same units. In this course we will try to stick to SI units, but the reader is warned that (just out of habit) equations written in natural units might pop out in some places of these lecture notes.

1.2 Accessing the microcosm

At this point it is useful to take a step back and ask ourselves how can we achieve all this information about the microcosm. After all, if now we look around ourselves matter does not look at all as composed by individual atoms! Actually, the concept of the atomic structure of matter is so counterintuitive that the American physicist and pop icon Richard Feynman said once:

> *If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis that all things are made of atoms little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.*

So let us review here some of the experimental evidence that we have to be confident that the atomic picture of matter sketched in Fig. 2.2 is actually the correct one.

How do we know that matter is made of atoms? There are several ways to answer this question. For instance, if we illuminate a crystal with light of wavelength $\lambda \sim 10^{-10}$ m, we see the onset of *interference patterns*, such as the one shown in Fig. 1.2. This behaviour can only be explained if the crystal is built in terms of point-like structures with typical separations among them of $\sim 10^{-10}$ m. A more direct example is provided by the fact that we can use powerful *electron microscopes* to actually *see* individual atoms. Remarkable, we can also use specific microscopes to move the atoms around the surface of a material. In Fig. 1.2 we show the logo of the IBM company built upon individual atoms (each of the cone-like structures in the image corresponds to a single atom) put together using the *Atomic Force Microscope*. The structure

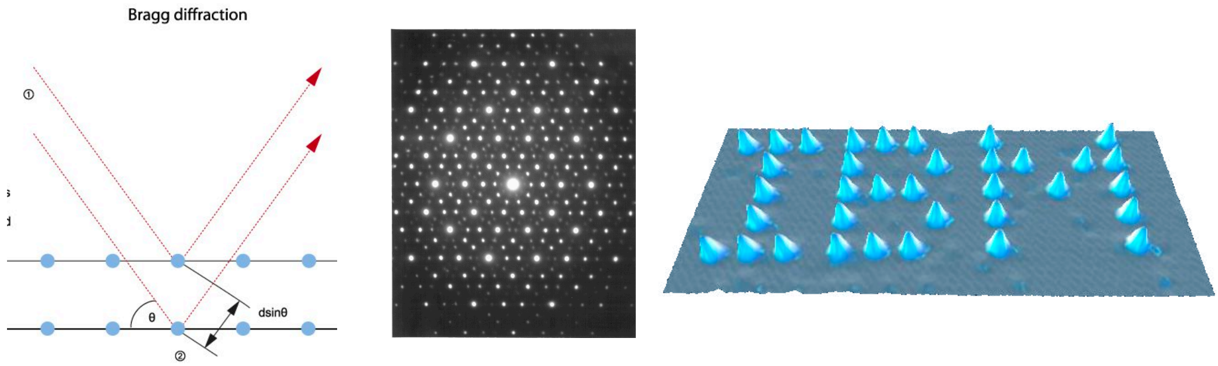

Figure 1.2. When interacting with a lattice of atoms (left), light of the appropriate wavelength λ produces a characteristic interference pattern (center). Right: atoms can be directly visualized with powerful microscopes, and even moved to *sculpt* your favorite logo.

of the periodic table was also of course historically of great importance to validate the atomistic picture of matter.

How do we know that atoms are composed by electrons and nuclei? Even after having established that matter was made of atoms, it was still far from obvious what their structure was. From the historic point of view, a first important fact that was ascertained about matter atoms is that they contained negatively electrically charged particles, that were later identified with the electrons. As shown in Fig. 1.3 the electron was discovered by J. J. Thomson towards the end of the 19th century in cathodic rays experiments as the one shown here. These experiments showed that corpuscles with negative electric charge can be extracted from neutral matter, and that propagate as if their were a particle. By measuring the displacement on the detector screen the charge-to-mass ratio q/m of these particles could be measured, showing that they were characterized by rather different properties than all known atoms. This result provided important evidence that atoms were not static and indestructible, but that were composed by even smaller components such as the electron.

Having established that the atoms contained electrons, the next question is how the positive electric charge was distributed. Since atoms are electrically neutral, the total negative charge of the electrons had to be compensated by the same amount of positive electric charge, and there were a number of theories about the latter. In Fig. 1.4 we show two possible models of the atom. In the first one, the electrons are distributed among a homogeneous distribution of positive electric charge. In the second (and correct) model, all the positive electric charge of the atom is concentrated on the tiny atomic nucleus, which is surrounded by a cloud of electrons. By launching energetic particles to atoms and subsequently analyzing how they scatter, the validity of the second model was demonstrated. Specifically, it was found that some of the incident particles scattered with very large deflection angles, which could be understood only if all the positive charge of the atoms is concentrated into a tiny nucleus. The measurements established that the ratio between the atomic radius r_A and the nuclear radius r_N was about $r_A/r_N \simeq 10^5$. Recall that the sun radius is $R_s \simeq 7 \times 10^5$ km, while the distance from the sun to Earth is $r_E \simeq 1.5 \times 10^8$ km, so $r_E/R_s \simeq 200$: the atom is like a extremely

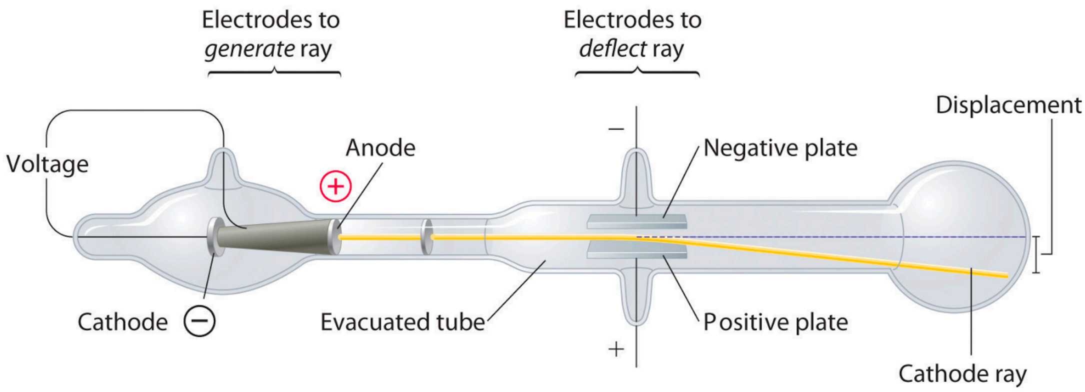

Figure 1.3. The electron was discovered by J. J. Thomson towards the end of the 19th century in cathodic rays experiments as the one shown here. These experiments showed that corpuscles with negative electric charge can be extracted from neutral matter, and that propagate as if their were a particle. By measuring the displacement on the detector screen the charge-to-mass ratio q/m of these particles could be measured, showing that they were characterized by rather different properties than all known atoms.

diluted version of the solar system.

Let us mention also here that this method of using energetic particles to bombard other particles and gain information about theory properties from the results of the scattering is still the most powerful method we have to analyze the properties of Nature at the smallest possible distances. Recall that as discussed above, scattering with particles of increasing energy corresponds to probing smaller and smaller distances.

How do we know that protons are composed by quarks? Fine, we know that atoms are composed by a packed nucleus of protons and neutrons surrounded by a cloud of electrons, such as a miniature solar system. But how do we know that protons themselves are composed by *quarks*? Perhaps the positive charge of protons is evenly spread within its volume? As shown in Fig. 1.5, the internal structure of protons can be studied by bombarding them with energetic electrons. These electrons will interact with one of the quarks in the proton and scatter inelastically. By analyzing the scattering pattern, we can infer that quarks are indeed point-like particles that add up to the totality of the electric charge of the proton. Remarkably, these experiments found that quarks have *fractional electric charge*, for example, the up quark has $Q_u = e/3$, with $-e$ being the electric charge of the electron. Therefore, the same idea that allowed to determine the internal structure of atomics in terms of electrons and nuclei also allows to probe the internal structure of protons in terms of quarks.

1.3 A first visit to the particle zoo

As discussed above, the particles shown in Fig. 2.2 describe the vast totality of everyday matter. Indeed, all chemical elements are built upon electrons, protons, and neutrons, with the latter being composed themselves by up and down quarks. However, there exist several other elementary particles that also play a role in many important phenomena both at low and at high energies. As we will see during this course, the particle zoo is actually a rather crowded area. Here we just list these additional elementary particles and mention briefly how they interact among them.

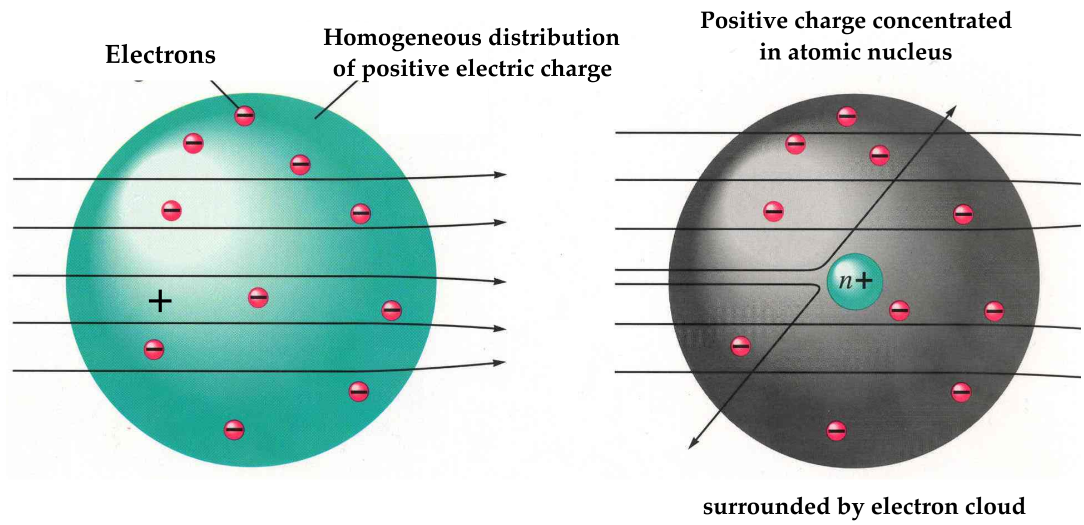

Figure 1.4. Two possible models of the atom. In the first one, the electrons are distributed among a homogeneous distribution of positive electric charge. In the second (and correct) model, all the positive electric charge of the atom is concentrated on the tiny atomic nucleus, which is surrounded by a cloud of electrons. By launching energetic particles to atoms of analyzing how they scatter, the validity of the second model was demonstrated.

Quarks. In addition to the up and down quarks, that compose protons and neutrons, there exist (at least) four other quarks, called *strange*, *charm*, *bottom*, and *top*. As we will discuss in Sect. 3, these extra quarks share the same properties as the up and down ones with the main difference of having a bigger mass.

Hadrons. Just as the up and down quarks form protons and neutrons, quarks can be combined in various other way to form many particles, that are known by the generic name of *hadrons*, and that interact by means of the strong nuclear force. Protons and neutrons are hadrons, as well as other particles that we will see in this course such as *pions* and *kaons*.

The strong interaction. Quarks interact among them by means of the *strong interaction*, also known as the *strong nuclear force*. This strong interaction is transmitted by a new type of particle called *gluons*. Hadrons also interact among them via the strong force. While both quarks and hadrons interact via the strong force, only the former are elementary particles, while the latter are *composite particles*. Gluons are charged under the strong force and thus can interact among themselves.

Leptons. By *leptons* we denote those elementary particles that do not experience the strong interaction. Some leptons are electrically charged and thus are affected by *electromagnetism*. The electron is one of these charged lepton, and it has two heavier siblings: the *muon* and the *tauon*, with identical physical properties other than its mass.

Photons. In the world of elementary particles, the electromagnetic interaction arises from the exchange of particles known as *photons*. These photons, which are themselves electrically neutral, mediate the interactions of any particles that are electrically charged. Quarks themselves are also electrically charged, and thus also experience the electromagnetic force.

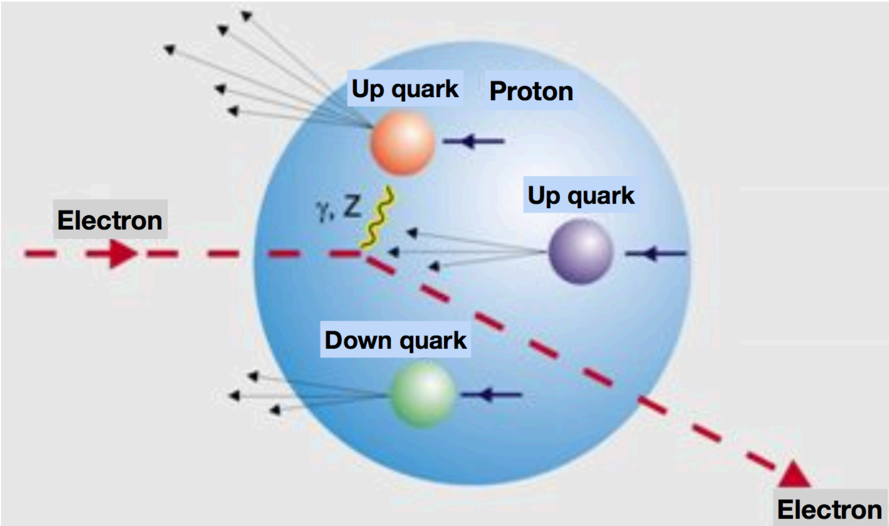

Figure 1.5. The internal structure of protons can be studied by bombarding them with energetic electrons. These electrons will interact with one of the quarks in the proton and scatter inelastically. By analyzing the scattering pattern, we can infer that quarks are indeed point-like particles that add up to the totality of the electric charge of the proton.

Neutrinos. In addition to the charged leptons (remember, leptons are those particles that are not affected by the strong force) there exist also neutral leptons, which are called *neutrinos*. These particles interact only via the weak interaction, and thus is very difficult to detect them. And not because there are few of them, every second about 100 trillion neutrinos from the Sun cross your body. Neutrinos also play a crucial rule in many nuclear reactions such as radioactivity.

The weak interaction. The weak interaction, also known as the *weak nuclear force*, is the third of the fundamental forces after electromagnetism and the strong interaction. All known fundamental particles experience the weak interaction, which is mediated by three new particles known as the *W and Z bosons*. While the carriers of the electromagnetic and strong forces (photons and gluons respectively) do not have mass, the *W and Z bosons* are very massive, around 80 and 90 times the proton mass respectively.

Antiparticles. For all the particles listed above, the combination of quantum theory and special relativity implies that each particle will have a kind of *mirror partner*, sharing many properties such as the mass but with all charges with opposite sign. For instance, the antiparticle of the electron is called the *positron*. One can even assemble atoms of anti-hydrogen or anti-helium. When antimatter particles interact with normal matter, they both annihilate into a big burst of energy.

The Higgs boson. The last missing element of the particle zoo is known as the *Higgs boson*. The Higgs particle, only discovered as recently as 2012, is responsible among other things of giving mass to the quarks and charged leptons, as well as to the W and Z bosons. The Higgs particle is the excitation of the *Higgs field*, which permeates the whole universe.

Gravity. The fourth and last of the fundamental interactions is actually the first one which was discovered, and whose effects are more manifest in everyday lives. Anything that has mass (actually anything that car-

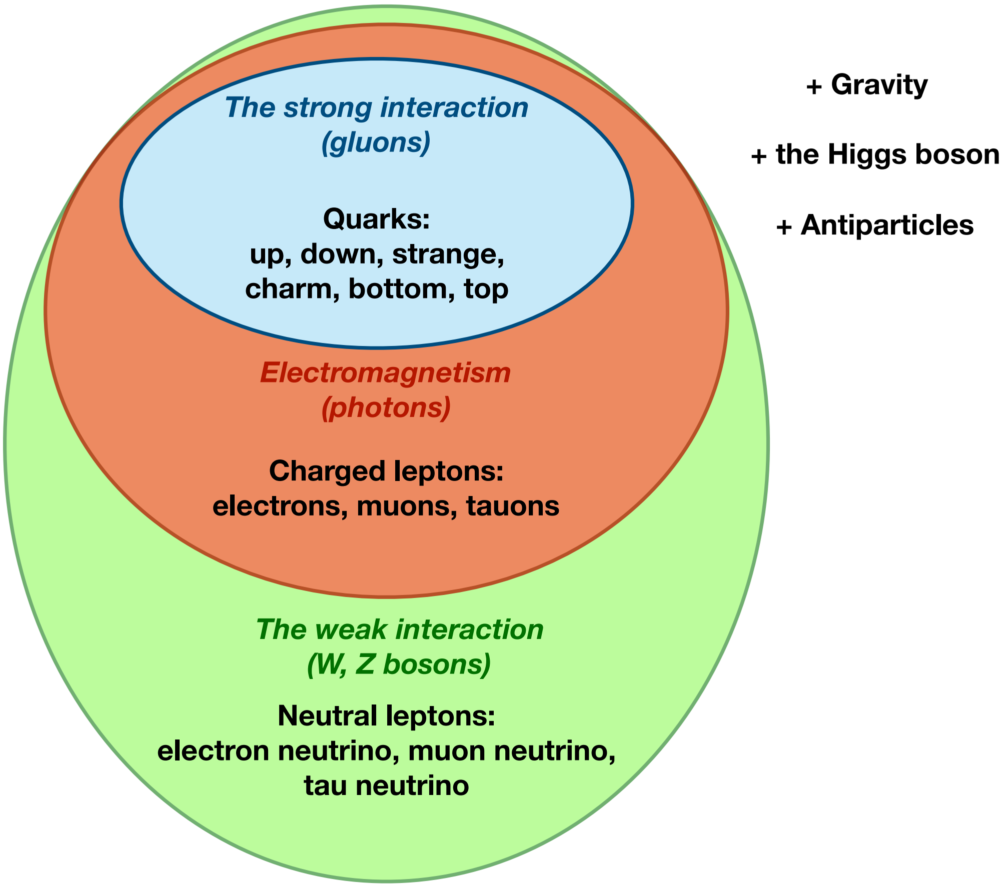

Figure 1.6. Schematic representation of the fundamental particles and interactions. The strong interactions, mediated by gluons, affects only the quarks. Quarks are also affected by electromagnetism, mediated by photons, same as for charged leptons such as electrons or muons. All known elementary particles interact also through the weak interactions, mediated by the W and Z bosons. Specifically, the neutrinos only interact via the weak force. In addition to these particles (and their anti-particles), a further particle, the Higgs boson, also exist and is responsible for their masses. While all elementary particles are affected by the gravitational interactions, its effects can be neglected for all practical purposes.

ries either mass or energy) will be affected by the gravitational interaction. However, gravity is a extremely weak force, and therefore in all calculations involving elementary particles it can be safely neglected.

If all these new names confuse you, that is perfectly normal: it takes some time and practice to get used to the particle zoo, and that is precisely what we will carry out during the rest of the course. In Fig. 1.6 we show an schematic representation of these fundamental particles and interactions. The strong interactions, mediated by gluons, affects only the quarks. Quarks are also affected by electromagnetism, mediated by photons, same as for charged leptons such as electrons or muons. All known elementary particles interact also through the weak interactions, mediated by the W and Z bosons. Specifically, the neutrinos only interact via the weak force. In addition to these particles (and their anti-particles), a further particle, the Higgs boson, also exist and is responsible for their masses. While all elementary particles are affected by the gravitational interactions, its effects can be neglected for all practical purposes.

Fig. 1.6 provides an overview of the world of elementary particles and fundamental interactions that we will study in this course. But before that, we need to equip ourselves with the mathematical language required to safely travel along the microcosm: Special Relatively and Quantum Theory, which we introduce next.

2 Special relativity and quantum physics for elementary particles

In order to quantitatively describe the world of elementary particles, we will need first to introduce some aspects of two of the founding theories of modern physics: Einstein's *special relativity* on the one hand, and *quantum mechanics* on the other hand. No previous knowledge of either topic will be assumed here, where we aim to provide a gentle, self-contained introduction to those features of special relativity and quantum theory that are necessary for the quantitative description of phenomena that involve elementary particles. We will also present here an introduction to two closely related topics: antimatter (which follows from special relativity) and nuclear radioactivity (which follows from quantum theory). No attempt is provided to give formal derivations or historical background: the interested student is encouraged to consult the "further reading" references pointed out in the Course Guide.

We also note here that this part of the course is supplemented by a set of slides, videos, and animations. This supplementary material is available from the *Brightspace* page of the course.

Learning goals of this lecture:

(a) Understand that when particles move close to the speed of light c, Newton's mechanics stops being reliable, and we need to adopt Einstein's special relativity formalism.

(b) Perform basic calculations using the special relativity formalism including four-vectors.

(c) Grasp the concept of antimatter and antiparticles, and their interactions with ordinary matter.

(d) Become familiar with the basic concepts of the quantum formalism, in particular that some important aspects that are present in the quantum world do not have a classical counterpart.

(e) Compute the outcome of simple radioactive processes.

2.1 A crash course on special relativity

You should by now be familiar with Newton's mechanics, in particular his three Laws of Motion. In Newton's mechanics, also known as *classical mechanics*, space and time are absolute concepts, well defined without reference to anything else. However, when objects are moving at very high speeds, meaning speeds that are comparable to the speed of light $c \simeq 3 \cdot 10^8$ m/s, the laws of motion become different, as we now explain. Since the world of elementary particle physics involves particles that move at speeds close to c, it is important that we review these updated equations of motion, dubbed *special relativity*, and formulated by Albert Einstein in 1905.

We do not have time here to provide a full-fledged introduction to special relativity, even less to present the relevant historical developments that lead to it, so here I will present in a succinct way the aspects of special relativity that are needed for this course, and the interested reader can find further reading material in the course references.

Recap of Newtonian mechanics. Without loss of generality, let us consider a single spatial dimension x. A particle moving in this one-dimensional space will be defined by two coordinates, a spatial one x and

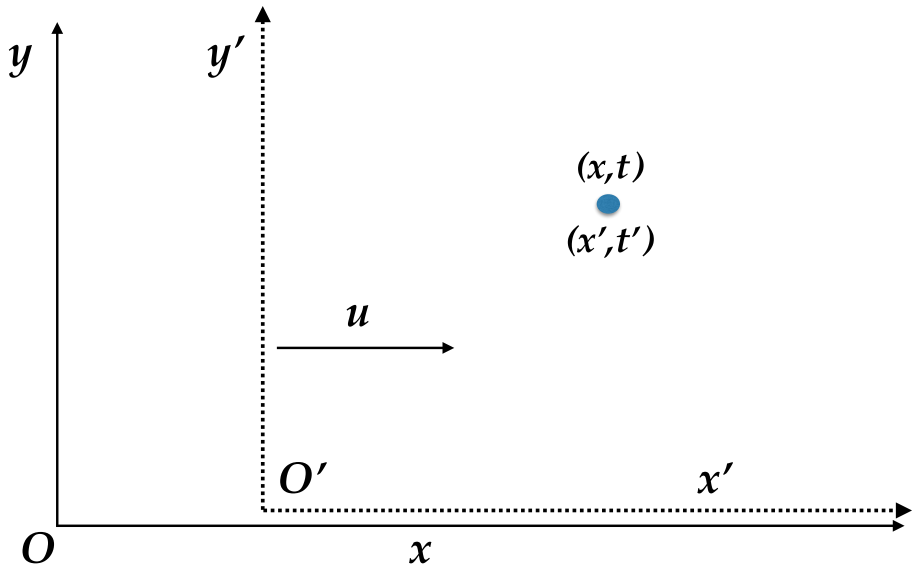

Figure 2.1. In this scheme we represent two reference frames, denotes by O and O' respectively, where O' is moving with velocity u with respect to O in the x direction. A particle moving along the x direction will be defined by space-time coordinates (x, t) in O and (x', t') in O'. As explained in the text, the transformation laws between O and O' are rather different in classical mechanics, Eq. (2.1), and in special relativity, Eq. (2.4).

a time one t, denoted by (x, t). In Newton's mechanics the relation is purely parametric, $x = x(t)$, but in special relativity we need to be more general. Let's now consider now a *change of reference frame*. We have two reference frames O and O', where O' moves with respect to O with velocity u, as indicated in Fig. 2.1. In the reference frame O', we denote by (x', t') the spatial and time coordinates of our particle.

In Newton's mechanics, time is universal, so $t' = t$. Moreover, the transformation law for the spatial coordinate equally simple, $x' = x - ut$. So putting it together we have the following set of transformation laws between the coordinates of two reference frames O and O':

$$\begin{aligned} t' &= t \\ x' &= x - ut\,. \end{aligned} \tag{2.1}$$

For instance, if I am moving with positive velocity u in the original reference frame O, I would be at rest in O'. The corresponding transformation laws for the velocities is given by

$$v' \equiv \frac{dx'}{dt'} = \frac{dx'}{dt} = \frac{dx}{dt} - u = v - u\,. \tag{2.2}$$

showing that, as mentioned above, if a particle is moving with speed $v = u$ in reference frame O then this particle will be stationary in the moving reference frame O' (since $v' = 0$ there). Note however that the acceleration is still the *same* in the two reference frames, since from Eq. (2.2) one has

$$a = \frac{dv}{dt^2} = \frac{dv'}{d(t')^2} = a'\,, \tag{2.3}$$

since u is constant. Therefore, in classical mechanics the acceleration measured by an observed is always the same for all *co-moving reference frames*, meaning all reference frames moving with a constant velocity with respect to each other.

Main postulate of relativistic mechanics. In *special relativity* we have a postulate which dramatically changes the simple transformation laws of classical mechanics. This postulate states that:

The speed of light c is the same in every reference frame.

This postulate is clearly inconsistent with classical mechanics, as shown by the transformation law Eq. (2.2). Indeed, in Newtonian mechanics the speed of light $c' = c-u$ in the two reference frames O and O' is different. It can be shown that the consistent application of this postulate leads to a different set of transformation rules between the coordinates of different reference frames, known as the *Lorentz transformations*. Therefore, the relationship between (x,t) and (x',t'), instead of being given by Eq. (2.1) and in Newton's mechanics are now given by:

$$\begin{aligned} t' &= \gamma\left(t - \frac{ux}{c^2}\right) \,, \\ x' &= \gamma\,(x - ut) \,, \end{aligned} \tag{2.4}$$

where the *Lorentz factor* γ is defined as $\gamma \equiv \left(1-u^2/c^2\right)^{-1/2}$, with c is again the speed of light. Sometimes the notation $\beta = u/c$ is used, which coincides with the velocity in the *natural units system* where $c = 1$, but here we will keep all factors of the speed of light c explicit.

Now, for Einstein's relativity to have any sense, we should demand that at very least for not too large velocities we recover the standard Newton's equations of motion, which are perfectly adequate in that regime. And indeed, we can see that if $u, v \ll c$, we can expand the Lorentz factor using a Taylor series to get

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \simeq 1 + \frac{\beta^2}{2} + \frac{3\beta^4}{8} + \ldots \tag{2.5}$$

so that the Lorentz transformation rules Eq. (2.4) reduce to

$$\begin{aligned} t' &\simeq t \,, \\ x' &\simeq (x - ut) \,, \end{aligned} \tag{2.6}$$

just as in classical mechanics, Eq. (2.1). So reassuringly we recover Newton's mechanics when all velocities involved are much smaller than the speed of light.

The postulate that the speed of light is always the same in all reference frames has a number of important consequences. In particular, it can be shown that:

Nothing can move faster than the speed of light, in any reference frame.

To be more precise, this means that *nothing carrying physical information or causal connections* can propagate faster than the speed of light. Let us show explicitly how the Lorentz transformations Eq. (2.4) satisfy this requirement. Say that we have a particle moving at speed $v < c$. Now we move to a reference system O' moving with velocity $-u$ (with $u < c$) with respect to O. In classical mechanics the velocity in O' would be, see Eq. (2.2),

$$v' = v - (-u) = v + u \,, \tag{2.7}$$

which in principle can be greater than c (for example in the case where $v = u = 3c/4$).

In special relativity, no matter which values u and v take, v' will always be smaller than c. To see this, we need to derive the velocity addition rule in special relativity, that is, the analog of Eq. (2.2) from classical mechanics. To do this, start with the Lorentz transformation or an infinitesimal time and spatial translation,

$$\begin{aligned} dt' &= \gamma\left(dt + \frac{udx}{c^2}\right) , \\ dx' &= \gamma\left(dx + udt\right) , \end{aligned} \tag{2.8}$$

taking into account the direction of u, so we see that

$$v' = \frac{dx'}{dt'} = \frac{dx + udt}{dt + udx/c^2} = \frac{dx/dt + u}{1 + (u/c^2)dx/dt} = \frac{v + u}{1 + uv/c^2} \tag{2.9}$$

which looks like the Newton law of velocity addition but with an extra factor for the denominator, which ensures that speeds are always kept smaller than the speed on light. Following with the previous example, if $u = v = 3c/4$ we now get

$$v' = \frac{3c/4 + 3c/4}{1 + 9/16} = 0.96c < c\,, \tag{2.10}$$

as required from the postulates of general relativity. You can easily convince yourselves that Eq. (2.9) implies that $v' < c$, no matter the values of v and u.

Energy and linear momentum. In special relativity, the energy E and the linear momentum $\vec{p}$ of a particle with mass m are defined in a different way as compared to Newtonian mechanics. Recall that in classical mechanics the kinetic energy and the linear momentum of a particle with mass m moving with velocity v are given by (again for the one-dimensional case),

$$\begin{aligned} E_{\rm kin} &= \frac{1}{2}mv^2\,, \\ p_x &= mv\,. \end{aligned} \tag{2.11}$$

On the other hand, in special relativity the energy and linear momentum are instead given by

$$\begin{aligned} E &= \widetilde{\gamma}mc^2\,, \\ p_x &= \widetilde{\gamma}mv\,. \end{aligned} \tag{2.12}$$

where we have defined

$$\widetilde{\gamma} \equiv \frac{1}{\sqrt{1 - v^2/c^2}}\,. \tag{2.13}$$

Now, what happens if velocities are small? In the limit $v \ll c$ one can use the expansion Eq. (2.5) to find

$$\begin{aligned} E &\simeq \left(1 + \frac{v^2}{2c^2}\right)mc^2 \simeq mc^2 + \frac{1}{2}mv^2 + \mathcal{O}(v^2/c^2)\,, \\ p_x &\simeq \left(1 + \frac{v^2}{2c^2}\right)mv \simeq mv + \mathcal{O}(v^2/c^2)\,, \end{aligned} \tag{2.14}$$

up to terms that can be neglected when $v \ll c$. Therefore, we recover the classical expression for p_x. For the energy, we see that in special relativity E is always the sum of two contributions: the *rest mass energy*, given by Einstein's iconic relation

$$E_{\rm rest} = mc^2 \,, \tag{2.15}$$

and what remains is called the *kinetic energy*. The leading term in the v/c expansion of the kinetic energy corresponds to the classical expression. More precisely, we have that in special relativity

$$E = E_{\rm rest} + E_{\rm kin} = mc^2 + (\widetilde{\gamma} - 1)\, mc^2 \,, \tag{2.16}$$

so the kinetic energy is $E_{\rm kin} = (\widetilde{\gamma} - 1)mc^2$.

From this discussion we can also glimpse another of the basic principles of special relativity, namely that

Energy and mass are ultimately equivalent, different manifestations of same underlying concept.

Indeed, special relativity contains a deep relation between *mass* and *energy*: these are not two different concepts, but actually two faces of the same physical phenomenon. In particular, it can be shown that upon a Lorentz transformation of the form shown in Fig. (2.1) one has the following transformation laws of energy and momentum between two reference frames:

$$\begin{aligned} p'_x &= \gamma \left(p_x - \frac{uE}{c^2} \right) \,, \\ E' &= \gamma \left(E - u p_x \right) \,. \end{aligned} \tag{2.17}$$

There we see that just as space and time mix, also energy and momentum (and thus mass) mix under Lorentz transformations.

From the relativistic definitions of Eq. (2.12) we see that energy and linear momentum satisfy the following relation

$$\frac{E^2}{c^2} - p_x^2 = \widetilde{\gamma}^2 m^2 c^2 - \widetilde{\gamma}^2 m^2 v^2 = m^2 c^2 \widetilde{\gamma}^2 \left(1 - \frac{v^2}{c^2} \right) = m^2 c^2 \,, \tag{2.18}$$

where we have used the definition of the Lorentz factor Eq. (2.13). Therefore, the $E^2/c^2 - p_x^2$ combination is directly related to the *rest mass* of the particle. For precisely, we can express the result of Eq. (2.18) as follows

$$m = \sqrt{E^2/c^2 - p_x^2} \Big/ c \,, \tag{2.19}$$

an expression that is known as the *mass-shell* condition, since it gives the rest mass of a particle in terms of its energy and its linear momentum. Since as we will show below this specific combination of E and p_x is the same in all reference frames, we find that the rest mass m of a given particle is independent of the reference frame.

Four-vectors and Lorentz invariants. As you can read from Eq. (2.4), the transformations between reference frames in special relativity mix time and space. This is the same for other quantities, such as the energy E and the linear momentum p_x, which are also mixed under Lorentz transformations, see Eq. (2.18). To better deal with this mixing of physical quantities it becomes convenient to introduce the mathematical concept of *four-vectors*. The space-time four-vector (for three spatial dimensions) is defined as

$$x^\mu = (ct, x, y, z)\,, \quad \mu = 0, 1, 2, 3\,. \tag{2.20}$$

Likewise, we can define a *energy-momentum four-vector* as

$$p^\mu = (E/c, p_x, p_y, p_z)\,, \quad \mu = 0, 1, 2, 3\,. \tag{2.21}$$

By convention, Greek indices like μ, ν label four-vectors, while Latin indices like i, j label normal (spatial) vectors. To compute the *norm* of a generic four-vector $A^\mu = (A^0, A^1, A^2, A^3)$ one needs to use the following rule:

$$|A| \equiv \left(\left(A^0\right)^2 - \left(A^1\right)^2 - \left(A^2\right)^2 - \left(A^3\right)^2\right)^{1/2}\,, \tag{2.22}$$

which is the analog of the *scalar product* in the three-dimensional case, where a vector $\vec{A} = (A^1, A^2, A^3)$ has norm $|A| = \sqrt{\vec{A}\cdot\vec{A}}$. Note that in the formula for the norm of a four-vector, Eq. (2.22), the spatial components carry the opposite sign than the time component.

A crucial property of proper four-vectors is that their *norm* is invariant under Lorentz transformation. Let us show this explicitly with $|p|$, the norm of the energy momentum tensor. Using the transformation rules Eq. (2.18), we see that

$$\begin{aligned} |p'|^2 &= (E')^2/c^2 - p_x^2 \\ &= \gamma^2 \left(E - up_x\right)^2/c^2 - \gamma^2\left(p_x - \frac{uE}{c^2}\right)^2 \\ &= \gamma^2\left(\frac{E^2}{c^2}\left(1 - \frac{u^2}{c^2}\right) - p_x^2\left(1 - \frac{u^2}{c^2}\right)\right) \\ &= \left(\frac{E^2}{c^2} - p_x^2\right) = |p|^2\,, \end{aligned} \tag{2.23}$$

where we have used that the Lorentz factor is given by $\gamma^2 = (1-u^2/c^2)^{-1}$. So indeed the norm of the energy-momentum four vector is invariant under Lorentz transformations, so the mass-shell condition Eq. (2.19) will be valid in any reference frame. The same property holds for all other proper four-vectors, and is very useful to solve problems in special relativity. For example, we can switch reference frames into one where a given calculation is easier, and then compute Lorentz invariants since we know that these are the same in any reference frame. In particular, it is frequently useful to work in the *center-of-mass* reference frame, defined by the frame where the total linear momentum vanishes, $\vec{p}_{\rm tot} = 0$.

Consider an unstable particle A at rest which decays into two lighter particles B and C, $A \to B + C$. Let us show how if we know the masses of the three particles involved in the decay, m_A, m_B, m_C, we can determine the velocities β_B and β_C of the two outgoing particles.
Let us write the four-momenta of the three particles involved in the decay, assuming that the decay takes place along the z axis:

$$\begin{aligned} p_A &= (m_A c, 0, 0, 0) \\ p_B &= (E_B/c, 0, 0, p_z) \\ p_C &= (E_C/c, 0, 0, -p_z) \end{aligned} \tag{2.24}$$

where we have already imposed linear momentum conservation. By using the relativistic expression of the three-momentum, Eq. (2.12,) we see that

$$p_z = \frac{\beta_B}{\sqrt{1-\beta_B^2}} m_B c = \frac{\beta_C}{\sqrt{1-\beta_C^2}} m_C c \,, \tag{2.25}$$

so to determine β_B and β_C we first need to determine p_z, which we can do using energy conservation complemented by the mass-shell relations Eq. (2.19) for B and C, namely

$$m_A c = E_B/c + E_C/c = \sqrt{m_B^2 c^2 + p_z^2} + \sqrt{m_C^2 c^2 + p_z^2} \,. \tag{2.26}$$

This equation can be easily solved in specific cases. For instance if $m_A \simeq m_B + m_C$, the linear momentum will be small compared with the rest masses and we can approximate

$$m_A \simeq m_B + m_C + \frac{p_z^2}{2c^2}\left(\frac{1}{m_B} + \frac{1}{m_C}\right) \to p_z/c = \left(2(m_A - m_B - m_C)/\left(\frac{1}{m_B} + \frac{1}{m_C}\right)\right)^{1/2} \tag{2.27}$$

and then from Eq. (2.25) we can determine the velocities β_B and β_C of the two outgoing particles. So we see that for a decay of the form $1 \to 2$, the entire kinematics of the final state are fixed by the mass of the three particles involved.

Time dilation and length contraction (*).[1] Another of the highly counter-intuitive consequences of special relativity is that the *time interval between two different events* is different depending on my reference frame. That is, the concept of *absolute time difference between two events* is not well defined in special relativity. For the same reason, whether or not two events are *simultaneous* will depend on the specific choice of reference frame. A related consequence holds for measurements of lengths and distances: The length that I will measure of an specific object will be different if I am at rest with respect to this object or if I am moving relative to it. These two phenomena are known as *time dilation* and *length contraction*.

Let us illustrate the time dilation phenomenon with a specific example. Assume that we have a clock with period Δt, as measured by an observer at rest in the same reference frame of the clock. Now, an observer

[1] Along these lecture notes, sections marked with (*) indicate that this is additional material, not required nor for the final exam nor to carry out the course problems.

moving with velocity u with respect to the clock will measure instead a period $\Delta t'$ given by

$$\Delta t' = \gamma \Delta t = \frac{\Delta t}{\sqrt{1 - v^2/c^2}} > \Delta t \,, \tag{2.28}$$

which is larger than the period measured by an observer at rest with respect to the clock, hence the name *time dilation*.

This property can be straightforwardly derived from the Lorentz transformation equations, Eq. (2.4). If one has two events happening at the space-time points (x_1, t_1) and (x_2, t_2) in the reference frame O, and defining $\Delta t = t_2 - t_1$ and $\Delta x = x_2 - x_1$, we can see that in the new reference frame O' one has

$$\Delta t' = \gamma \left(t_2 - \frac{u x_2}{c^2} \right) - \gamma \left(t_1 - \frac{u x_1}{c^2} \right) \,, \tag{2.29}$$

and since in O the clock is stationary, $x_2 = x_1$, we find that

$$\Delta t' = \gamma (t_2 - t_1) = \gamma \Delta t \,, \tag{2.30}$$

consistent with Eq. (2.28).

Imagine that you are traveling in a spaceship at a velocity close to the speed of light, $\beta = 0.9$. According to the clock that carries the spaceship, one year has elapsed. In the Earth, which is moving at velocity $-\beta$ with respect to the spaceship, using Eq. (2.28) this corresponds to

$$\Delta t' = \frac{1 \text{ year}}{\sqrt{1 - \beta^2}} = 5.3 \text{ years} \,. \tag{2.31}$$

So one year in the spaceship actually corresponds to *5.3 years on Earth*. If the starship returns to Earth after two years at the same velocity (as measured from the spaceship clock), the astronauts will find that actually 10.6 years have elapsed on Earth. So the returning astronauts would be much younger as compared to what would have happened if they would have stayed on Earth!

The corresponding phenomena in terms of distances is known as *length contraction*, which means that that a moving object's length is measured to be shorter than its proper length, defined as the length as measured in the object's own rest frame. This means that

$$L = \frac{L}{\gamma(v)} = L_0 \sqrt{1 - \frac{v^2}{c^2}} < L_0 \,, \tag{2.32}$$

This property can also be derived from Lorentz transformation, as before, since

$$\Delta x' = \gamma \left(\Delta x - u \Delta t \right) = \gamma \Delta x \,, \tag{2.33}$$

since $\Delta t = 0$ (the length is measured instantaneously). Therefore, if $\Delta x' = L_0$, then $\Delta x = L$ and the above relation follows.

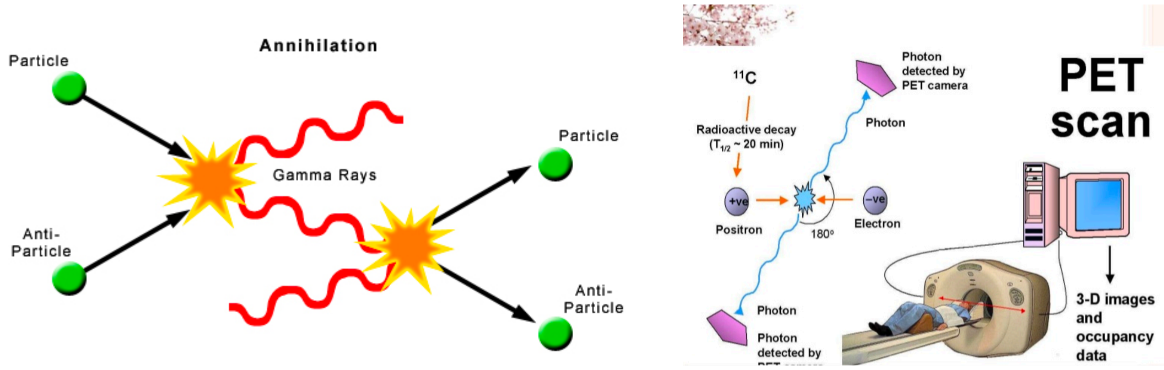

Figure 2.2. Left: schematic representation of a particle-anti-particle annihilation into two high-energy photons. The inverse process is also possible, where a pair of photons produce a particle-anti-particle pair. Right: in Positron Emission Tomography (PET), a radioactive isotope is injected into a patient. This isotopes emit positrons, which upon annihilation with nearby electrons generate two high energy photons that are recorded by a camera, indicating the position of the radioactive isotope within the patient.

2.2 Antimatter

The consistent combination of quantum theory (see below) with special relativity has associated a rather surprising consequence:

> *All particles in nature have partner particles that have some properties identical (such as the mass) but the opposite charges (such as the electric charge).*

For instance, the electron e^- has associated an antimatter particle called the *positron* e^+. The two particles have identical mass $m_{e^-} = m_{e^+}$ but opposite charges, $Q_{e^+} = -Q_{e^-}$. Likewise, the antimatter partner of the proton p is the *anti-proton* $\bar{p}$, with identical mass but opposite charge, $Q_{\bar{p}} = -Q_p$. So the anti-proton is *negatively charged*, in other words, the sign of its electric charge is the same as for the electron. We emphasize that there is nothing *mysterious* in antimatter particles: they are just the same type of matter as everyday particles, *just* with the opposite charges.[2]

If antimatter is just like normal matter but with opposite charges, why we don't see positively-charged positrons and negatively-charged protons around? The reason is that as soon as a matter particle finds its corresponding antimatter particle, they annihilate into energy (into photons, the quanta of light, for example). In Fig. 2.2 we show the schematic representation of a particle-anti-particle annihilation into two high-energy photons, namely the reaction

$$e^+ + e^- \to \gamma + \gamma \,. \tag{2.34}$$

The inverse process is also possible, where a pair of photons produce a particle-anti-particle pair. In this respect, the fact that matter and antimatter particles have the opposite charges means that they can annihilate into energy without violating any conservation law, such as the conservation of the electric charge. For example, in Eq. (2.34) photons do not carry electric charge, $Q_\gamma = 0$, therefore to conserve electric charge the electron and the position must have *exactly* opposite charges (which they do).

[2] The real conundrum is that our best understanding of the laws of elementary particles do not provide a mechanism to create the matter-antimatter asymmetry that we find in the Universe. We will briefly touch on this in the final lecture, see also Sect. 5.

Einstein's equation relating rest mass with energy, Eq. (2.15), indicate that the annihilation of matter and antimatter particles releases *a lot of energy*. Consider for example an electron moving at 10% the speed of light. This is very fast electron of course, moving at almost $v \simeq 10^9$ km/h!. What is therefore the ratio of kinematic energy over rest mass energy for this very fast electron? From Eq. (2.16) we see that

$$\frac{E_{\rm kin}}{E_{\rm rest}} = \frac{(\widetilde{\gamma}-1)\,mc^2}{mc^2} = (\widetilde{\gamma}-1) = \left(1-0.1^2\right)^{-1/2} - 1 \simeq 0.01\,, \tag{2.35}$$

so the kinematic energy that this very fast electron carries is just 1% of the energy contained in its rest mass. This result illustrates that most of the energy carried by particles is accounted for by their rest mass, unless they travel close to the speed of light. Indeed, for the kinetic energy to be comparable to the rest mass energy, $E_{\rm kin} \simeq E_{\rm rest}$, one requires $\widetilde{\gamma} \simeq 2$ which means a velocity of $\beta \simeq 0.87$.

Note that anti-particles experience the same types of interactions as the "regular" particles. In particular, it is easy to see that an anti-proton $\bar{p}$ and a positron e^+ can combine to create an *anti-hydrogen* atom. The properties of *anti-hydrogen* are found indeed to be *identical* to those of normal hydrogen with very high precision. This same argument extends to other more complicated elements, for example *anti-helium* has also been synthesized. Right now at CERN one can produce thousands of anti-hydrogen atoms per day, providing very stringent tests of the laws of nature.

The fact that matter and antimatter particles annihilate all their rest mass into energy suggest that they could represent the ultimately efficient fuel (or the ultimate weapon as well). But before this idea can be realized, first one needs to produce large amounts of antimatter and store it. This is easier said that done, since if antimatter touches a single matter particle it will annihilate into energy. Therefore, one needs to use techniques such as *magnetic traps*, which are able to store antimatter atoms preventing them of interacting (and thus annihilating) with normal matter.

In Dan Brown's bestseller *Angels and Demons*, the perverse *Illuminati* want to steal antimatter from CERN to build a bomb to blow up the Vatican. This bomb is supposed to carry 0.25 grams of antimatter. How much energy will be released when all this antimatter is annihilated in the contact with normal matter? We can easily compute it using special relativity:

$$E = mc^2 = \left(2\times 2.5\times 10^{-4}\,\text{kg}\right)\times\left(3\times 10^{8}\,\text{m/s}\right)^2 \simeq 5\times 10^{13}\,\text{J} = 50\,\text{TJ}\,, \tag{2.36}$$

where the factor 2 arises from the contribution of the 0.25 g of normal matter which annihilate with the antimatter in the bomb. This is about the same energy released by the atomic bomb dropped on Hiroshima ($\simeq$ 63 TJ). So indeed antimatter seems to be a very powerful weapon!
The only downside of this malicious program is that producing antimatter is *very slow*. Even using all the accelerator complex from CERN[a], at most one can produce 10^{-12} grams of antimatter per year, meaning that it would take around *one billion years* to produce that much antimatter as required by the *Illuminati* dark master-plan. Of course, the same limitation affect proposals to power interstellar spaceships with antimatter engines.

[a] http://angelsanddemons.web.cern.ch/antimatter/making-antimatter

Going back to reality, the remarkable properties of antimatter have crucial implications for everyday life, for example in modern medical imaging techniques. In the right plot of Fig. 2.2 we show a schematic representation of the Positron Emission Tomography (PET) method.

In this technique, a radioactive isotope is injected into a patient. This isotopes emit positrons, which upon annihilation with nearby electrons generate two high energy photons that are recorded by a camera, indicating the position of the radioactive isotope within the patient.

2.3 A crash course on quantum physics

Following this introduction to special relativity, we continue now by presenting a self-contained introduction to *quantum physics*, the mathematical language relevant for the description of small objects such as atoms and electrons. We will simply state the principles of quantum theory as well established facts, and refer the references for a list of the wealth of experimental analysis that confirm their validity. As in the case of the introduction to special relativity, we do not have the time to cover the historical developments that lead to the formulation of quantum theory.

Without further ado, we present the basic physical principles that underly quantum theory. For further motivation and historical background for each of this, the interested student is encouraged to consult the additional reading material. Most of the concepts presented here were already introduced in the first lecture of the course.[3]

Wave-particle duality. Every object, from a plane to a proton, will under specific conditions behave as a *wave*: this concept is known as *wave-particle duality*. And will thus experience behaviour usually associated to waves, such as diffraction when crossing a narrow slit and interference patters. This of course sounds very anti-intuitive: we don't see every day cars interfering between them or diffracting as they cross a tunnel. The reason is that their wave character becomes apparent only when the length scales involved are of the same order or smaller than their *De Broglie wavelength*, defined as

$$\lambda \equiv \frac{h}{p} = \frac{h}{mv}\,, \tag{2.37}$$

where $h = 6.26 \times 10^{-36}$ $\mathrm{m^2 kg/s}$ is a numerical constant known as *Planck's constant*, ubiquitous in the quantum world, and $p = mv$ is the linear momentum of the object. Planck's constant is one of the defining features of quantum theory, and in particular it determines when genuinely quantum effects will be important.

[3]See the slides here: http://pcteserver.mi.infn.it/~nnpdf/TUDelft/rojo-ElementaryParticles-Lecture1.pdf.

For example, let us consider an electron that travels at 1% of the speed of light. Special relativity tells us that this electron will have a momentum of $p = m \cdot v = 2.73 \times 10^{-24}\,\mathrm{kg\,m/s}$, and therefore its De Broglie wavelength, that is, the distance scale for which an electron should exhibit a wave-like character, is

$$\lambda = \frac{h}{p} \simeq 0.5 \times 10^{-10}\,\mathrm{m}\,, \tag{2.38}$$

which is the atomic scale. So for instance one expects the wave character of electrons to be apparent when crossing a crystalline lattice. On the other hand, for a car of mass $m = 1000$ kg moving at a velocity of $v = 100$ km/h, such that $p = 2.8 \times 10^4$ kg m/s, we can associate a De Broglie wavelength of

$$\lambda = \frac{h}{p} \simeq 2 \times 10^{-38}\,\mathrm{m}\,, \tag{2.39}$$

implying that the wave character of a car is unobservable: a car would diffract only when crossing a tunnel of width $\sim \lambda$!

Therefore, in quantum theory, when particles interact with objects of length similar or smaller than their de Broglie wavelength, their wave persona will become apparent.

The remarkable wave-particle behaviour of quantum theory is illustrated in Fig. 2.3: in classical theory, the behaviour of waves and particles in the double slit experiment is very different: waves lead to an interference pattern, while particles lead to just two blobs in the detector screen (upper diagrams). In quantum particle, the situation is very different (lower diagrams): if the number of incident particles is large enough, a clear interference patter also appears, just as if the particles behaved as waves.

The quantum wave-function. All the *information* about a given quantum system is encoded in its so-called *wave-function* $\phi(x)$. This wave function obeys a dynamical equation of motion known as the *Schroedinger equation*. The quantum wave-function $\phi(x)$ (or more precisely its square $|\phi(x)|^2$) represent the *probability density* of finding the particle in a specific region of space and at given time upon measurement. Knowledge of the wave-function specifies *all* the physical information that we can access for an specific quantum system.

Actually the notation $\phi(x)$ corresponds to the wave function in an specific *representation*, in this case the *position representation*. More in general, the wave function is an abstract quantity living in a vector space called the *Hilbert* space, and we denote it by $|\phi\rangle$. Its complex conjugate is denotes by $(|\phi\rangle)^* = \langle\phi|$, and their inner (scalar) product is written as $\langle\phi||\phi\rangle$. Since the probability of finding a particle *somewhere* must be one, the wave function of a quantum system must be normalized in a wave that $\langle\phi||\phi\rangle = 1$.

The Schroedinger equation. In classical physics, the motion of a free particle is described by the second of Newton's equations, namely

$$ma = m\frac{d^2x(t)}{dt} = F_{\rm tot}\,, \tag{2.40}$$

with a being the particle acceleration, and $F_{\rm tot}$ is the total force acting on the particle. For a *free particle* $F_{\rm tot} = 0$ and the solution is simply

$$x(t) = x_0 + v(t - t_0)\,, \tag{2.41}$$

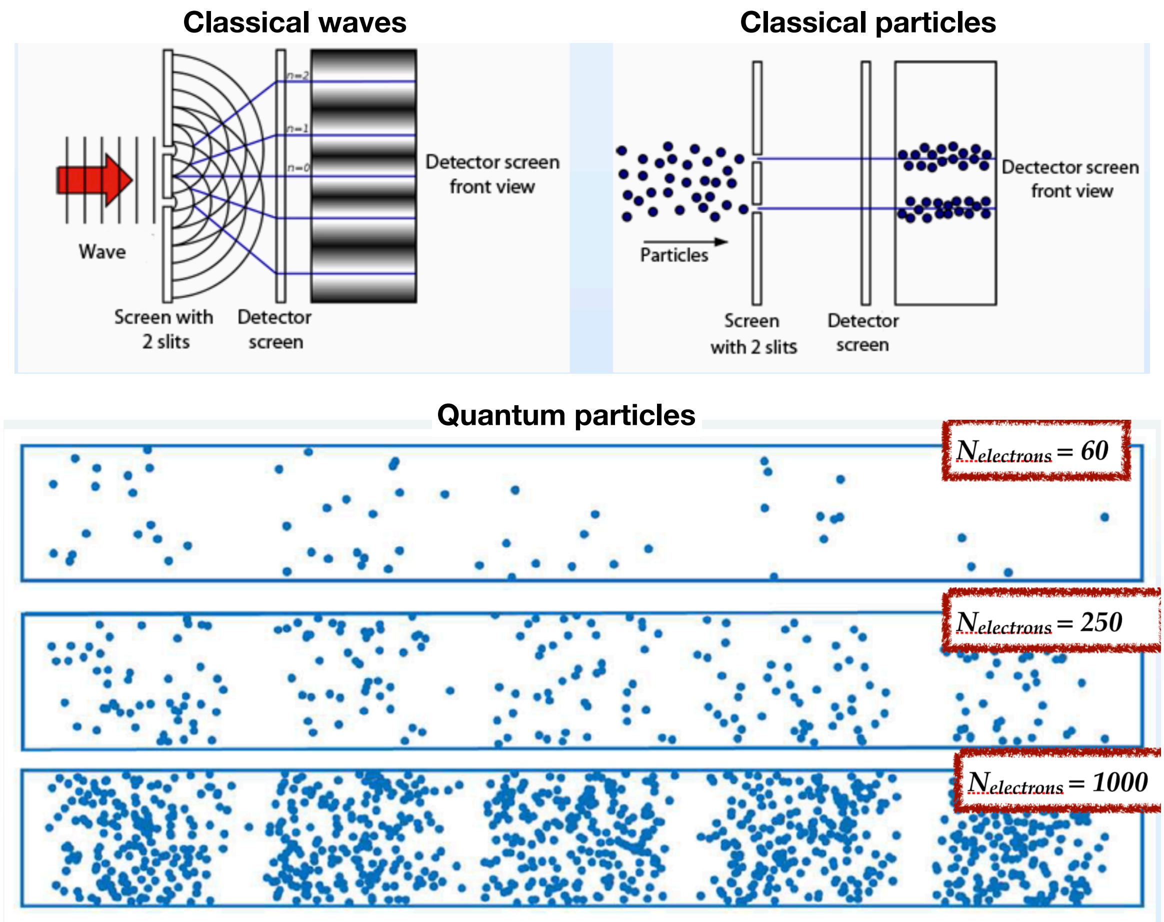

Figure 2.3. In classical theory, the behaviour of waves and particles in the double slit experiment is very different: waves lead to an interference pattern, while particles lead to just two blobs in the detector screen (upper diagrams). In quantum particle, the situation is very different (lower diagrams): if the number of incident particles is large enough, a clear interference patter also appears, just as if the particles behaved as waves.

where x_0 and v_0 are the initial conditions.

The corresponding dynamical equation in quantum theory is the *Schroedinger equation*, and in the simplest case of motion in one dimension reads in general

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)\,, \tag{2.42}$$

where $V(x)$ is the potential where the particle is moving and E its energy. For a free particle $V(x) = 0$ and therefore

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)\,, \tag{2.43}$$

whose solution is of the form

$$\psi(x) = A\cos\left(\sqrt{\frac{2mE}{\hbar}}x\right) + B\sin\left(\sqrt{\frac{2mE}{\hbar}}x\right)\,, \tag{2.44}$$

which is nothing but the mathematical form of a *plane wave*. So under exactly the same conditions, a particle propagates very differently in classical theory, Eq. (2.40) and in quantum theory, Eq. (2.44). The coefficients A and B are fixed by the specific boundary conditions of the problem.

Quantum states with wave function $\psi(x)$ and energies E that satisfy the Schroedinger equation Eq. (2.42) are physically allowed in quantum theory. However, as we will discuss below, there exist physically allowed quantum states which are not the solution of Eq. (2.42), but rather built up as a linear combination of the solutions of the Schroedinger equation.

Heisenberg's uncertainty principle. In quantum theory, certain pairs of physical variables, named *conjugated variables*, cannot be measured simultaneously with arbitrarily good precision. This is so as a matter of principle: this statement is true even for an ideal measurement apparatus. This condition restricts the amount of information that we can access for a specific quantum system, and is known as the *Heisenberg's uncertainty principle* .

For instance, if we denote by Δx the uncertainty associated to the position of a given particle, and Δp the uncertainty associated to its momentum, quantum theory tells us that both Δx and Δp cannot be arbitrarily small at the same time, but that they need to satisfy the following inequality

$$\Delta x \cdot \Delta p \geq \hbar \,, \tag{2.45}$$

where we have defined $\hbar \equiv h/2\pi$. This means for example that if we aim to measure the momentum of a particle with resolution Δp, then we will not be able to determine its position with a resolution better than

$$\Delta x \geq \frac{h}{2\pi}\frac{1}{\Delta p} \,. \tag{2.46}$$

We emphasize that this is not a restriction due to the imprecision of our experimental measurement apparatus: this is a fundamental limitation about the information that can be extracted from a quantum system even with a perfect measurement. The same principle holds for other variables, such as energy and time, where we have that

$$\Delta E \cdot \Delta t \geq \hbar \,. \tag{2.47}$$

This inequality implies that in order to measure the energy of a particle with perfect precision, the measurement would take infinite time, since $\Delta t \geq (\hbar/\Delta E)$.

How come then that classically we seem to be able to measure the position and momentum of any given object to any precision that we want? Because the restriction imposed by Heisenberg's uncertainty principle is only relevant when either distances or momenta are very small, else it is inconsequential. For example, let's consider the same car as in the previous example, with $m = 1000$ kg and $v = 100$ km/h, so that $p = 27.8 \times 10^3\,\mathrm{kg} \cdot \mathrm{m/s}$. So if one measures this momentum with precision better than *one part in a billion*, with $\Delta p = 10^{-12} \times p$, then Heisenberg's uncertainty principle implies that the uncertainty in the position of the car will be

$$\Delta x = \hbar/\Delta p = \left(1.05 \times 10^{-34}\,\mathrm{m^2 kg/s}\right) / \left(27.8 \times 10^{-8}\,\mathrm{kg} \cdot \mathrm{m/s}\right) = 3.8 \times 10^{-28}\,\mathrm{m}\,, \tag{2.48}$$

which certainly is irrelevant from the practical point of view.

The superposition principle. If we solve Schroedinger's equation for a given quantum system, we will end up with a list of possible solutions, that is, states in which we can find our system upon measurement. For instance, assuming that we have n solutions, we can represent them as

$$|k\rangle\,, \quad k = 1, \ldots, n\,, \tag{2.49}$$

reflecting that they are elements of a vector space. The superposition principle tells us that *any linear superposition* of possible states of a quantum system is also a well-defined quantum state itself. In the above case, this means that a general quantum state of this system will be given by

$$|\Psi\rangle = \sum_{k=1}^{n} a_k |k\rangle\,, \tag{2.50}$$

with $\{a_k\}$ arbitrary numerical coefficients.

Quantum evolution and coherence. A quantum state unperturbed from external influences evolves by means of the evolution operator $U(t)$ which is given in terms of the state Hamiltonian. An important feature of this evolution is its *coherence*, meaning that the overall evolution of the quantum state can be determined from the evolution of the individual basis states, in other words

$$|\Psi\rangle(t) = U(t, t_0)|\Psi\rangle(t = 0) = \sum_{k=1}^{n} a_k U(t, t_0)|k\rangle\,. \tag{2.51}$$

In the case of external perturbations such as a measurement, the quantum wave function $|\Psi\rangle(t)$ would collapse to one of the basis states $|k\rangle$ and therefore coherency would be lost. As we will see in this course, the superposition principle of quantum theory also plays a crucial role in the world of elementary particles. For instance, the finite mass of the neutrinos implies that the neutrinos that we measure in our detectors are actually a quantum superposition of different types of neutrinos, and the probability of finding a specific type is a function of parameters such as the energy and the distance traveled.

Quantum entanglement. A remarkable consequence of the basic principles of quantum theory is that there exists systems of many particles which *cannot be expressed as a combination of single-particle states*. This is at odds with the classical view of the world, where any system, no matter how complex it is, can always be decomposed in all of its components, each one obeying well-defined dynamical equations. In quantum theory this is not necessarily true, and some states (actually the wide majority) do not admit a description in terms of quantum states of individual particles. Those states are said to be *entangled*.

Let us consider the case in which a particle can occupy only two states, so our basis vectors will be $|0\rangle$ and $|1\rangle$. We will write the state composed by two of such particles as

$$|ab\rangle \equiv |a\rangle_1 |b\rangle_2 \,, \tag{2.52}$$

so the state $|ab\rangle$ is such that particle one is in the state $|a\rangle$ and particle 2 in the state $|b\rangle$. Is clear that some of the possible states admit a description in terms of single-particle states, for example

$$|\Psi\rangle = |01\rangle = |0\rangle_1 |1\rangle_2 \,, \tag{2.53}$$

but however other states do not admit such description, for example

$$|\Phi\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle \right) \,. \tag{2.54}$$

To demonstrate this, note that if the quantum state $|\Phi\rangle$ could be decomposed in terms of single particle states, one should be able to write it as

$$|\Phi\rangle = \left(a|0\rangle_1 + b|1\rangle_1 \right) \left(c|0\rangle_2 + d|1\rangle_2 \right) = ac|00\rangle + ad|01\rangle + bc|10\rangle + db|11\rangle \,, \tag{2.55}$$

but you can easily convince yourself that Eq. (2.54) does not admit a decomposition of the form of Eq. (2.55). Therefore, in the quantum state Eq. (2.54) it is not possible to describe separately the individual particles: they are fundamentally *entangled* with each other.

There exist many ways of quantifying entanglement in a quantum system. For example, our two-particle quantum state can be generically written as

$$|\phi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \,, \tag{2.56}$$

where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$ due to normalization. Now, we can define a measure of entanglement in terms of its *concurrence* as follows

$$\Delta \equiv 2 \left| \alpha\delta - \beta\gamma \right| \,, \tag{2.57}$$

By construction, $0 \le \Delta \le 1$, where the extreme cases $\Delta = 0$ (1) correspond to a product (maximally entangled) state. You can convince yourselves that Eq. (2.54) is not only an entangled state, but actually it corresponds to a *maximally entangled* state with $\Delta = 1$. Likewise, any state that can be decomposed into single particle states such as Eq. (2.55) has $\Delta = 0$, so exactly zero entanglement.

Entanglement is a very powerful feature of quantum theory. For instance, the exponential speed up in computational efficiency that *quantum computers* exhibit over the classical ones arises from the manipulation and exploitation of entanglement. Entanglement also plays a crucial role in many theories of elementary particle physics.

2.4 Radioactivity

The concept of *radioactivity* involves the spontaneous emission of fast energetic particles from *unstable atoms*. This phenomenon is a direct consequence of quantum theory. Radioactivity is an very important phenomenon in nuclear and particle physics, which has lead to many applications such as nuclear energy, and well as to very important medical imaging applications.

There are three main types of radioactive processes:

- α *radiation*: the emission of a He nucleus (a bound state of two protons and two neutrons).

  This radioactive decay process is mediated by the *strong interaction*. If the initial unstable element is ${}^{A}_{Z}X$, where Z is the *atomic number* (number of protons) and A is the *mass number* (numbers of protons plus neutrons), then the daughter element will be ${}^{A-4}_{Z-2}Y$, given the proton and neutron content of an α particle

- β *radiation*: the emission of an electron (or its anti-particle, the positron), together with a *neutrino*.

  This radioactive decay process is mediated by the *weak interaction*. If the initial unstable element is ${}^{A}_{Z}X$, for this process the final element will be ${}^{A}_{Z\pm1}Y$, depending on whether an electron or a positron has been emitted.

- γ radiation: the emission of a high energy photon γ.

  This radioactive decay process is mediated by the *electromagnetic interaction*. In this case, the values of the atomic Z and mass A numbers will be the same for the initial and final element: actually this transition consist of the relaxation of an excited atomic state to a state with less energy.

Therefore, as can be seen, each of these three radioactive processes is mediated by a different fundamental interaction. The mean features of these three kinds of radioactive processes are summarized in Fig. 2.4.

You might ask yourselves: what causes a specific atom to decay? And how does exactly the atom *know* when it has to decay? Since radioactive decays are determined by *quantum theory*, they are intrinsically *probabilistic* in nature: we can predict when *on average* a large collection of atoms will decay, but we cannot know when a given specific atom will decay.

If the number of atoms in a given radioactive sample is $N(t)$, and we have $N_0 = N(t=0)$ atoms to begin with, the infinitesimal variation dN in a time interval dt will be proportional to the number of atoms themselves, so it will be given by $-\lambda N(t)dt$, with λ known as the *decay constant* of this specific radioactive process. Solving the associated differential equation we see that the number $N(t)$ of radioactive atoms at any given time will be

$$dN = -\lambda N(t)dt \quad \rightarrow \quad N(t) = N_0 e^{-t/\tau}\,, \tag{2.58}$$

with $\tau = 1/\lambda$ representing the *average lifetime* of the atoms, that is, the average time they take to decay. To validate this interpretation, note that the average of a stochastic variable t with probability density $N(t)$ is given by

$$\langle t\rangle = \frac{\int_0^\infty dt\, t\, N(t)}{\int_0^\infty dt\, N(t)} = \tau\,. \tag{2.59}$$

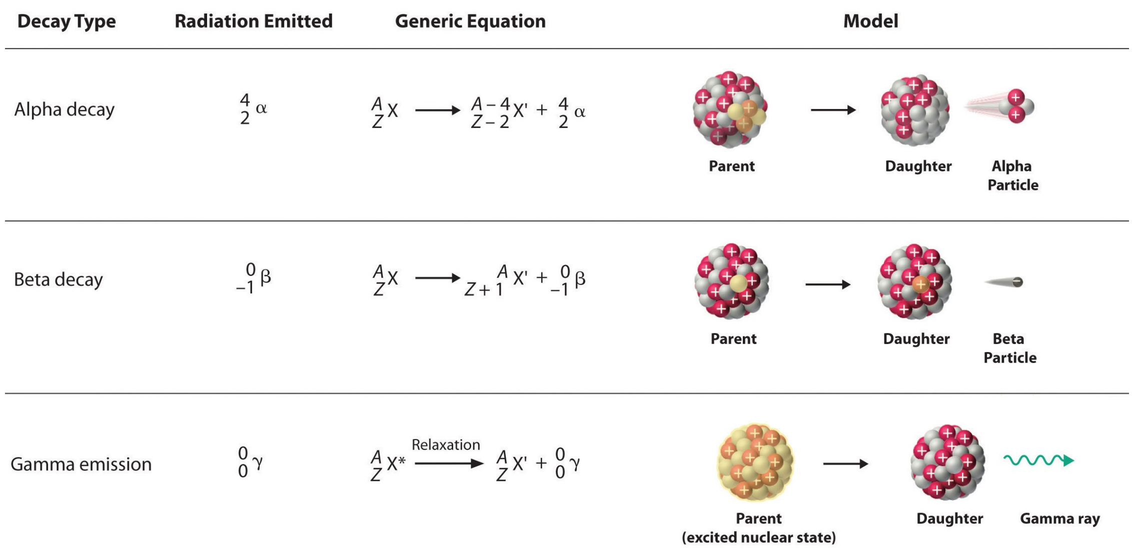

Figure 2.4. The three main types of radioactive decay processes. In alphas decay, mediated by the strong interaction, a He nucleus (2 protons and 2 neutrons) is emitted. In beta decay, mediated by the weak interaction, either an electron or a positron are emitted (together with a neutrino). In gamma decay, mediated by electromagnetism, a high energy photon is emitted.

Therefore, we see from Eq. (2.58) that the number of nuclei of a given sample that undergo radioactive decays decreases exponentially with time. One sometimes also defines the *activity* of a radioactive sample, defined as (minus) the rate of change of $N(t)$ per unit time,

$$A(t) \equiv -\frac{dN(t)}{dt} = \frac{N_0}{\tau} \exp\left(-\frac{t}{\tau}\right) . \tag{2.60}$$

The higher the value of the activity $A(t)$, the faster the radioactive decay process takes place.

The value of the decay constant λ is different for each atomic element and isotope. This provides among other things a rather useful method to determine the age of objects, the so-called *radiocarbon dating method*. This method is based on the fact that one of the isotopes of carbon, ^{14}C, has a decay constant of a few thousand years, while the more frequent isotope ^{12}C is stable. This means that if we know somehow the initial abundance of ^{14}C in a given sample, and measure the current abundance, since we know τ we can determine t from Eq. (2.58). In other words, we can determine in a rather reliable way the age of this specific sample. The original abundance of ^{14}C can be determined for instance by comparing with the current one of ^{12}C, since the ratio between the abundances of the two isotopes in various systems is rather well understood.

Let us illustrate the radiocarbon dating method with an example. The mean-life of the unstable isotope of Carbon, ^{14}C, is $\tau = 8267$ years. If we assume that a given sample had the same ratio $^{14}C/^{12}C$ as the one we have now in the atmosphere, we can compute the value N_0 of ^{14}C atoms in the original sample. By comparing with the current value of ^{14}C atoms in the sample, $N(t)$, we can then determine t.

For instance, assume that we find that a sample with only 10% of ^{14}C atoms as compared to its original abundance. By applying Eq. (2.58), we have that

$$0.1\,N_0 = N_0 e^{-t/(8267\,\mathrm{yr})} \qquad \to \qquad t = 19000\,\mathrm{yr}\,, \tag{2.61}$$

illustrating how radioactive dating can be used to determine the age of a given object.

A quantity of frequent interest is the amount of energy which is *released* in a given radioactive process. For a decay process of the form $A \to B + C + D + \ldots$, the amount of (kinetic) energy released is called Q and is given by the following expression:

$$Q = c^2 \left(m_A - m_B - m_c - m_D - \ldots \right) \,, \tag{2.62}$$

which follows from energy conservation: the initial energy is $m_A c^2$ (rest mass energy) and the final energy is $\sum_n \left(m_n c^2 + E_{\rm kin}^{(n)} \right)$, where n runs over all the particles in the final state. Therefore, we see that $Q = \sum_n E_{\rm kin}^{(n)}$. For a process to be physically allowed one should of course have $Q \geq 0$, since the kinetic energy is a positive-definite quantity.

An example of α process is provided by the following Uranium into Thorium decay:

$$^{238}_{92}U \;\to\; ^{234}_{90}Th \;+\; ^{4}_{2}He\,. \tag{2.63}$$

Our aim is to compute the kinetic energy Q released in the collision, which will be shared between the Th and the He. Using Eq. (2.62), we have that

$$Q = c^2 \left(m_U - m_{Th} - m_{He} \right) \,, \tag{2.64}$$

and given that the atomic weight of $^{238}_{92}U$ is $238.0507u$, that of $^{234}_{90}Th$ is $234.0436u$, and that of the α particle is $4.0015u$, with the atomic mass unit $u = 1.66 \times 10^{-27}$ kg, we find that the mass difference is $\Delta m = 0.0056u$ and thus the energy released, using the conversion factor 1 $u = 931.5$ MeV, is

$$Q \simeq 5.21\,\mathrm{MeV}\,. \tag{2.65}$$

This energy Q will be shared as the kinetic energy of the two decay products of the reaction.

We will revisit in the following lectures the concept of nuclear radioactive decays, once we have become more familiar with the world of elementary particles and theory interactions.

3 Elementary particles and their properties

Armed with the tools of special relativity and quantum theory, we continue our journey into the world of elementary particles by discussing the various types of particles that exist in Nature, emphasizing the essential role that symmetries play in determining their properties. We will discuss that point particles are characterized by a special kind of *internal rotation* called *spin*, which determines several of their properties. We will enumerate the different possible types of elementary particles, such as *fermions* and *bosons*, and *leptons* and *quarks*. We also explain how hadrons (particles that experience the strong interaction) such as protons are composed by quarks, emphasizing again the crucial role of symmetry principles to describe hadron structure. We will show how symmetry requirements impose very tight constraints on which hadrons are physically allowed, in excellent agreement with all experimental observations.

The **learning goals** of the lecture are:

(a) To become familiar with the concept of spin, and with its implications to classify elementary particles as either fermions or bosons.

(b) To be able to apply symmetries to explain the properties of elementary particles.

(c) To explain the observed hadron structure in terms of different arrangements of its constituent quarks as dictated by symmetry.

(d) To apply special relativity to decays and scattering reactions involving elementary particles.

3.1 Spin

In classical mechanics, objects spinning around a given rotation axis can have associated a a physical quantity called *angular momentum*, which is the analog of linear momentum in the case of rotational motion. For instance, in the case of a sphere rotating around an axis that goes through its center, we have that its angular momentum L will be given by

$$L = \left(\frac{2}{5}MR^2\right)\omega\,, \tag{3.1}$$

where M is its mass, R its radius, and ω the angular velocity of the sphere's rotation. So we see that the angular momentum is bigger the bigger the mass, radius, and rotation velocity of the sphere. Angular momentum is an important quantity since under some circumstances, specifically in a configuration with rotational symmetry, it will be *conserved*, just like linear momentum of a system is conserved in the absence of external forces.

Another of the remarkable consequences of quantum theory is that even fundamental particles (that is, point particles without internal structure, meaning vanishing *radius*) do have associated *angular momentum*. This *intrinsic angular momentum*, which does not have a classical counterpart, is known as *spin*. Unlike the classical angular momentum, *e.g.* Eq. (3.1) that can take any value, the spin of a particle can only take a restricted set of values (we thus say that the spin of a particle is *quantized*). Specifically, the spin of a particle can be either zero of an integer multiple of $\hbar/2$. In other words, just as several other properties of the quantum world, the spin of a fundamental particle is *quantized*: it can only take a finite set of values. In

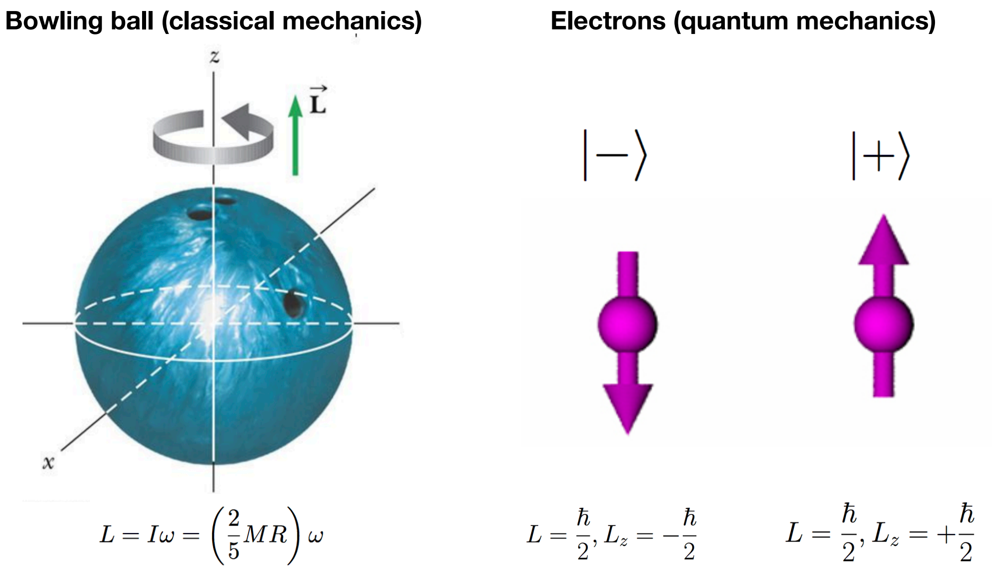

Figure 3.1. Left: in classical theory, objects rotating round an axis have associated a quantity known as angular momentum L. Right: in quantum theory, elementary particles have associated a new type of intrinsic angular momentum known as spin. The values that spin can take for a given particle are quantized. For instance, in the case of electrons, their total spin is $\hbar/2$, and therefore electrons exist in states with positive spin around a given axis (say the z axis), $L_z = +\hbar/2$ and in states with negative spin around the same axis, $L_z = -\hbar/2$.

this respect, $\hbar/2$ is the *quantum of spin*, the minimal amount of spin (other than zero) that a given particle can have.

To further illustrate this analogy between classical and quantum versions of angular momentum, in Fig. 3.1 we show how in in classical theory, objects rotating round an axis have associated a quantity known as angular momentum L, and that in quantum theory, elementary particles have associated a new type of intrinsic angular momentum known as spin. In the case of electrons, their total spin is $\hbar/2$, and therefore electrons exist in states with positive spin with respect a given axis, say the z axis, $L_z = +\hbar/2$ and in states with negative spin with respect to the same axis, $L_z = -\hbar/2$. Interestingly, the superposition principle of quantum mechanics (see Sect. 2.3) implies that an electron can also exist in a superposition of *up* and *down* spin states, for example with the wave function

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right) . \tag{3.2}$$

The concept of spin is essential for the description of elementary particles. As we will show next, the specific value of the spin of a given particle specifies many of its properties.

It is easy to see that interpreting the spin of the electron from a classical point of view, that is, arising from the angular momentum of a tiny rotating sphere, is meaningless. To see this, try to equate the expression for the angular momentum of a rotating sphere, Eq. (3.1), with the value of the spin of an electron,

$$\frac{2}{5} M R \omega = \frac{\hbar}{2} , \tag{3.3}$$

and using in this expression the current upper bound on the electron radius, $R < 10^{-18}$ m, and its mass, $M = 9.11 \times 10^{-31}$ kg, we can determine its angular velocity,

$$\omega = \frac{5}{4} \frac{\hbar}{M R^2} = 9.1 \times 10^{32} \, \mathrm{rad/s} \tag{3.4}$$

which corresponds to a velocity of the electron's surface of $v = \omega R \sim 10^{15}$ m/s, more than a million times larger than the speed of light c, and thus inconsistent with special relativity.

3.2 Fermions and bosons: spin and statistics

In terms of their spin, all particles (either elementary, like quarks, or composite, like protons) can be divided into *two main classes*:

- *Fermions*: particles with *half-integer spin*: $\hbar/2$, $3\hbar/2$, ...
- *Bosons*: particles with *integer spin*: 0, $\hbar$, $2\hbar$, ...

For instance, all quarks and leptons are fermions, specifically with spin $\hbar/2$, while all force carries (photons, gluons, and W, Z bosons) are bosons with spin $\hbar$. On the other hand, the Higgs particle is the only known fundamental particle that is also a *scalar*, that is, a particle, with vanishing spin. Several hadrons have spin zero as well, such as the pion π and the kaon K.

This classification into fermions and bosons is summarized in Fig. 3.2, which shows how the elementary particles and the associated force carriers can be classified into two main classes in terms of its spin.[4] On one hand, all quarks and leptons have spin $\hbar/2$ and thus are classified as *fermions*. On the other hand, the Higgs boson and the force carries (photon, gluon, and W, Z particles) are all bosons, the first with spin 0 and the second with spin $\hbar$. The graviton, the hypothetical mediator of the quantum theory of gravity, is expected to have spin $2\hbar$.

The main practical difference between fermions and bosons is related to the fact of whether or not two identical particles can *occupy the same quantum state*. To illustrate this property, let us consider a simple system composed by two *identical particles*, one at position x_1 and the other at position x_2. As discussed in Sect. 2.3, all the information about the system is encoded in its wave function. For this system composed by two particles, the wave function can be written as

$$\psi_{\mathrm{tot}}(x_1, x_2) = \psi_1(x_1)\psi_2(x_2) . \tag{3.5}$$

[4] Interestingly, in theories with smaller number of space-time dimensions, such in special condensed matter systems, new types of quasi-particles may arise that are nor fermions nor bosons.

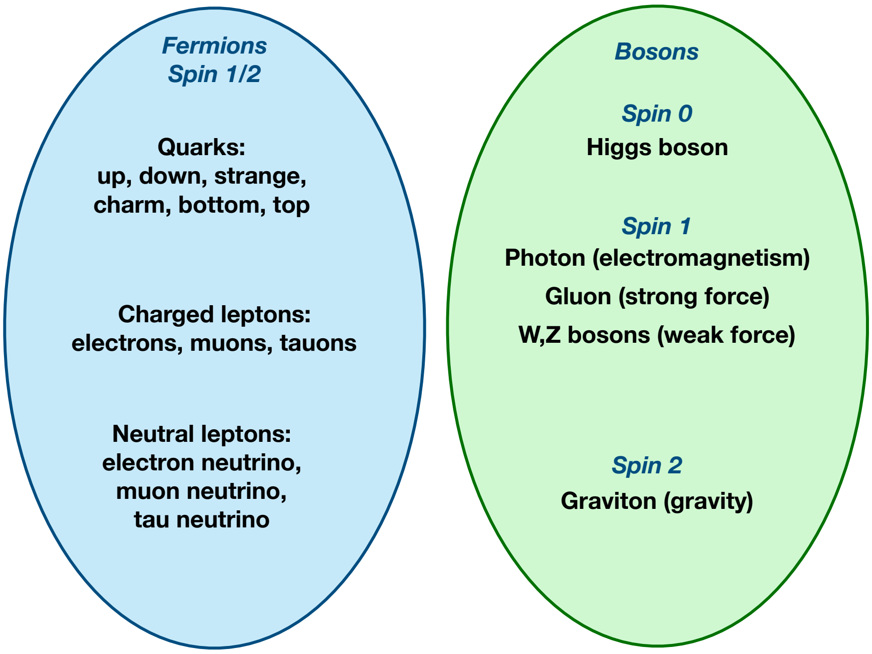

Figure 3.2. The elementary particles and the associated force carriers can be classified into two main classes in terms of its spin. On one hand, all quarks and leptons have spin $\hbar/2$ and thus are classified as *fermions*. On the other hand, the Higgs boson and the force carriers (photon, gluon, and W, Z particles) are all bosons, the first with spin 0 and the second with spin $\hbar$.

Now let us exchange the positions of the two particles $x_1 \to x_2$ and $x_2 \to x_1$. The new wave function of the system will be

$$\widetilde{\psi}_{\rm tot}(x_1, x_2) = \psi_1(x_2)\psi_2(x_1)\,. \tag{3.6}$$

However, since the particles are identical, all physical information that I can extract from the system should be the same as before exchanging their positions. This requirement implies square of the wave function (which determines the probability of finding the system in a given quantum state) should be unchanged, that is,

$$|\psi_{\rm tot}(x_1, x_2)|^2 = |\widetilde{\psi}_{\rm tot}(x_1, x_2)|^2\,, \tag{3.7}$$

which implies that the wave function itself can only vary up to a complex phase,

$$\widetilde{\psi}_{\rm tot}(x_1, x_2) = e^{i\phi}\psi_{\rm tot}(x_1, x_2)\,. \tag{3.8}$$

Moreover, if we exchange back again the positions of the two particles, one is back to the starting configuration Eq. (3.5), and then the wave function should be the same as before. This implies that

$$e^{2i\phi} = 1 \qquad \to \qquad e^{i\phi} = \pm 1\,. \tag{3.9}$$

This exercise tells us that identical quantum particles can behave only on two ways if they exchange positions (or more in general, if they exchange quantum states) within a system:

- either the wave function of the system remains unchanged,

- or else it get a minus sign.

We denote the first class of particles as *bosons*. Therefore, the wave function of a system composed by bosons satisfies upon the exchange of two particles:

$$\widetilde{\psi}_{\rm bosons}(x_1, x_2) = \psi_{\rm bosons}(x_1, x_2)\,, \tag{3.10}$$

while we denote the second class of particles as *fermions*, which behave as

$$\widetilde{\psi}_{\rm fermions}(x_1, x_2) = -\psi_{\rm fermions}(x_1, x_2)\,. \tag{3.11}$$

We emphasize that this is not just a curious esoterical property of quantum theory but that it has enormous practical implications. The most important of these is called the *Pauli exclusion principle*. Its derivation is rather straightforward: consider the wave function of a system composed by two identical fermions that occupy the same position (again, more in general, that they occupy the same quantum state). From their properties upon wave function exchange

$$\psi_{\rm fermions}(x_1, x_1) = -\psi_{\rm fermions}(x_1, x_1) = 0\,, \tag{3.12}$$

implying that:

Two fermions cannot occupy simultaneously the same quantum state

The stability of everyday matter is indebted in great part to the Pauli exclusion principle, which prevents for instance electrons piling on top of each other within atoms. On the other hand, bosons can occupy the same quantum state without no limitations. You can either pile a macroscopic amount of bosons all in the same ground state of a given system, creating a new state of matter called a *Bose-Einstein condensate*.

The fact that quarks and leptons are *fermions* has very important consequences for the theory of elementary particles, as we will illustrate in the following.

3.3 Radioactive decays revisited and neutrinos

As discussed in Sect. 2.4, atomic nuclei are not always stable. Under some circumstances, they will experience a *radioactive decay* process, leading to its transformation into a nucleus of a different element together with the emission of additional energetic particles. One of these radioactive decays is the α-decay, where the decaying nucleus emits a Helium nucleus $^4_2$H, known as an α-particle for historical reasons. This transition is governed by the strong force, and as we will discuss in Sect. 4, in this type of interaction the number of protons and neutrons is (separately) conserved. Therefore such transition takes the form, see also Fig. 3.3,

$$^A_ZX \,\to\, ^{A-4}_{Z-2}Y + {}^4_2H\,, \tag{3.13}$$

where Z is the number of protons (nuclear atomic number) and A the number of nucleons (nuclear mass number) of the original element X.

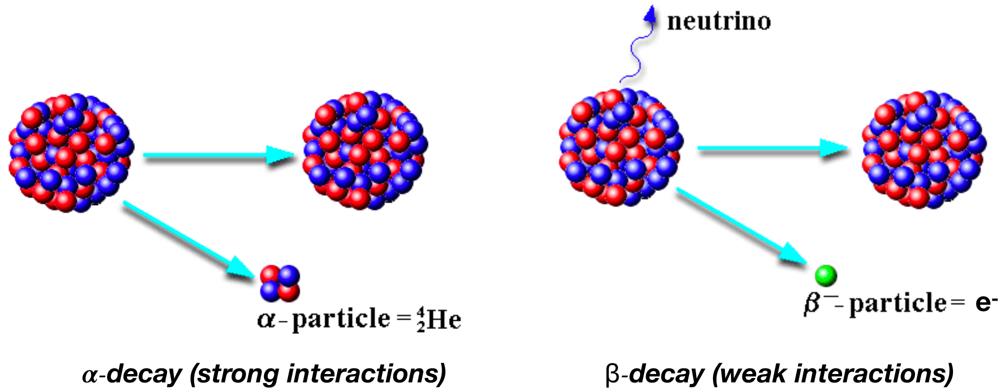

Figure 3.3. Schematic representation of two important types of nuclear radioactive decays. The left plot shows an α-decay, Eq. (3.13), mediated by the strong force and which results in the emission of a He nucleus. The left plots a β-decay, Eq. (3.21), mediated by the weak force, and that results in the emission of an electron (or positron) and a neutrino.

An important property of this type of decays is that the amount of energy that the α particle (the ${}^4_2H$ nucleus) gets is always the same for a given initial element X. To see this, let us put into practice the four-vector notation of special relativity introduced in Sect. 2.1, and write the four-momenta of the various particles involved in this decay in the rest frame where the nucleus X is not moving:

$$\begin{aligned} p_X^\mu &= \left(M_X c, \vec{0}\right) , \\ p_Y^\mu &= \left(E_Y/c, \vec{p}_Y\right) , \\ p_\alpha^\mu &= \left(E_\alpha/c, \vec{p}_\alpha\right) . \end{aligned} \tag{3.14}$$

Next, we apply the condition that *four-momentum must be conserved* in the decay, since there are no external forces acting on the system. Imposing this requirement, we find the following set of relations:

$$M_X c = E_Y/c + E_\alpha/c \,, \qquad \vec{p}_Y = -\vec{p}_\alpha \equiv \vec{p} \,. \tag{3.15}$$

In addition, we can impose the mass-shell conditions, Eq. (2.19), that relates the mass with the energy and the linear momentum of a given particle:

$$m^2c^2 = E^2/c^2 - \vec{p}^2 \,, \tag{3.16}$$

which in the case of the nucleus Y and the α particle implies

$$E_Y/c = \sqrt{m_Y^2 c^2 + \vec{p}^2} \,, \qquad E_\alpha/c = \sqrt{m_\alpha^2 c^2 + \vec{p}^2} \,. \tag{3.17}$$

Combining Eq. (3.15) with Eq. (3.17) and doing some algebra, we can determine E_α, the energy of the outgoing α-particle, in terms of the masses M_X, M_Y, and M_α, finding

$$E_\alpha = \frac{M_X^2 + M_\alpha^2 - M_Y^2}{2M_X} c^2 \,, \tag{3.18}$$

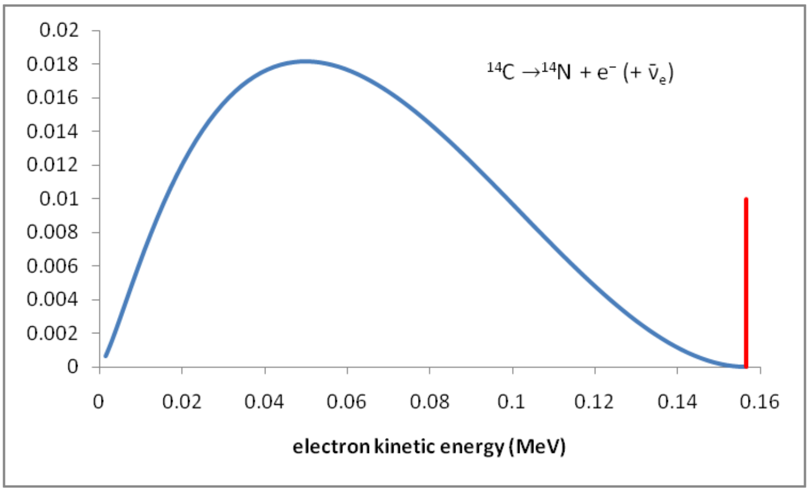

Figure 3.4. The distribution of energy of the electrons in the β-decay of Carbon, $^{14}C \to^{14} N + e^- + \bar{\nu}_e$. The red line indicates the result that would be expected if the neutrino were absent, since in that case the energy of the electron is fixed by Eq. (3.20).

which is *fixed* since once X is determined, then M_Y is also known from Eq. (3.13). This result has the important consequence that an experiment measuring the energies of the α particles from the decay of a nucleus X will always find the same value.[5] Note that E_α in Eq. (3.18) is the total energy understood in the special relativity sense, including the contribution both from the rest mass and from the kinetic energy.

As we also discussed in Sect. 2.4, another important class of radiative decays, are known as β-decays, see Fig. 3.3. These decays are characterized by the emission of an energetic electron. These β decays are governed by the weak interaction, where as we will see it is allowed for a neutron to decay into a *neutrino, a electron, and a proton.* However, before the neutrino was discovered, scientists thought that the underlying process for β decay was

$$ {}^A_Z X \to^A_{Z+1} Y + e^- \,, \tag{3.19} $$

where note that the atomic number A is conserved. Since this is a $1 \to 2$ decay, the kinematic derivation from the previous exercise also holds, and we should find that the electron carries always the *same fixed energy*, for a given initial element X. Specifically one should find that

$$ E_e \simeq \frac{M_X^2 - M_Y^2}{2M_X} \,, \tag{3.20} $$

since the small electron mass can be neglected as compared to the much heavier atomic masses. However, experimentally one finds what is shown in Fig. 3.4, the distribution of energy of the electrons in the β-decay of Carbon, $^{14}C \to^{14} N + e^- + \bar{\nu}_e$. We see that the electron energy can take any value between 0 and a maximum value, which corresponds precisely with the upper limit Eq. (3.18).

This conundrum was resolved by postulating that a new particle was also produced in the β-decay

[5] Note that this derivation has been based only on kinematics, without assuming anything about the details of the underlying interactions. Therefore, the same formula will also apply to any other particle decaying into two particles, the so-called $1 \to 2$ decay.

processes, which was called the *neutrino*. Once this particle was introduced, the modified β decay process

$$^{A}_{Z}X \to^{A}_{Z+1} Y + e^{-} + \bar{\nu}\,, \qquad (3.21)$$

reproduced nicely the experimental measurements shown in Fig. 3.4. However, during many years this ghostly neutrino remained undetected: its effects could only be observed in an indirect way. In order to explain this difficulty in measuring neutrinos, one had to posit the following properties for the neutrino:

- It has no electric charge $Q_\nu = 0$.
- It does not feel the strong atomic force.
- Normal matter is essentially transparent to neutrinos.
- It is massless or almost massless, with at least $m_\nu/m_e \le 5\times 10^{-5}$. This least property was derived from the E_e spectra in Fig. (3.4), which would be different if the neutrino has a larger mass.

These conditions explain why neutrinos are so difficult measure experimentally: they interact *very weakly with matter*. It is certainly not because there are no neutrinos around, since every second *trillions of neutrinos* emitted from the sun cross our bodies. It was only in 1956, around 25 years after the original prediction, that neutrinos were detected experimentally. As mentioned in Sect. 1, neutrinos come in *three different types*: the electron neutrino ν_e, the muon neutrino ν_μ, and the τ neutrino ν_τ. In addition, there corresponding antiparticles also exist, and are labelled as $\bar{\nu}_e$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$. Understanding whether or not neutrinos are their own antiparticles is an important open issue for modern particle physics.

Today we understand neutrinos rather, and in particular we have demonstrated that they have an *small but finite non-zero mass*, $m_\nu \neq 0$ (at least for two of the neutrino species). This follows from a very important discovery presented in the late 1990s, where it was found that neutrinos can spontaneously oscillate among different flavour types. For example, the following transitions were found to be possible

$$\nu_e \to \nu_\mu \quad \text{and} \quad \nu_e \to \nu_\tau\,. \qquad (3.22)$$

In quantum theory, this means that neutrinos with well-defined mass, which we will denote as ν_1, ν_2, and ν_3, are *quantum superpositions* of neutrinos with well-defined lepton number, for example, we have something like

$$\nu_1 = \frac{2}{3}\nu_e + \frac{1}{6}\nu_\mu + \frac{1}{6}\nu_\tau\,, \qquad (3.23)$$

and therefore, the flavour state of a neutrino will oscillate among the different flavour eigenstates.

3.4 Leptons

As discussed in Sect. 1, we denote as *leptons* all particles that *do not experience the strong interaction.* There are two types of leptons: those which have electric charge $Q \neq 0$ and thus also interact by means of electromagnetism (electrons, muons, and kaons) and those which are electrically neutral $Q = 0$ and therefore only interact via the weak force (the three neutrinos). We now discuss their properties in turn.

Charged leptons. There exist (as far as we are aware) three different charged leptons (plus the corresponding antiparticles): the electron e^-, the muon μ^-, and the tau lepton or tauon τ^-. The electron is the lighter lepton, with $m_e = 0.511$ eV.[6] The two big brothers of the electron share all of its properties except for its mass: the muon μ has a mass of $m_\mu = 107.7$ MeV (200 times heavier than the electron), and the tau lepton τ has a mass of $m_\tau = 1777$ GeV (3500 times heavier that the electron). So the tau lepton is even heavier than a proton. Note that these three charged leptons have the same electric charge, $Q_{e^-} = Q_{\mu^-} = Q_{\tau^-} = -e$, with e being the absolute value of the electron charge.[7]. The corresponding antiparticles have the opposite electric charges, and therefore $Q_{e^+} = Q_{\mu^+} = Q_{\tau^+} = +1$.

The fact that the muon has a larger mass than the electron has an important consequence. Since $m_\mu > m_e$, the following decay is kinematically allowed

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \ , \tag{3.24}$$

by means of the weak interaction, and where the tiny neutrino masses can be neglected. On the other hand, the conjugate process

$$e^- \to \mu^- + \bar{\nu}_\mu + \nu_e \ , \tag{3.25}$$

is not allowed since it would violate energy conservation (since $m_e < m_\mu$). This means that since the muon can decay into electrons, it is not a stable particle, but rather that it is *unstable* with a finite lifetime. Its average lifetime turns out to be rather short, 2.2×10^{-12} seconds, meaning that as soon as a muon is produced, it will decay almost instantaneously by means of process Eq. (3.24) The heavier tau lepton decays even sooner, with a lifetime of 3×10^{-13} seconds. On the other hand since Eq. (3.25) is kinematically forbidden, electrons cannot decay into muons or tau leptons. In principle they could decay into only neutrinos or only quarks, but as we will now show these other decay channels are forbidden and therefore the electron (and the positron) is an *absolutely stable* particle.[8]

Leptonic number. In quantum theory, we call a *quantum number* a label for specific properties of a particle or of a system of particles, such as the total spin or electric charge. Quantum numbers are particularly important because under specific circumstances they obey *conservation laws*. Leptons have associated a specific *quantum number* (a new type of charge, in other words), called the *leptonic number* L. In particular, the leptons that have leptonic number $L = +1$ are

$$\left(e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau \right) \ , \tag{3.26}$$

while the corresponding anti-particles, that have instead the opposite lepton number $L = -1$, are

$$\left(e^+, \mu^+, \tau^+, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau \right) \ . \tag{3.27}$$

Recall that the relation between particles and anti-particles is such that they have all quantum numbers and charges opposite between them. The leptonic number of a system is *additive*, meaning that for instance the

[6]Recall that $m_e = 9.11 \times 10^{-31}$ kg, which translates into $m_e = 0.511$ eV when using the more suitable natural units.

[7]In the rest of these notes, we will understand that all electric charges are measured in units of e, and therefore write that $Q_{e^-} = -1$ and so on.

[8]Which is quite a useful property, since otherwise normal matter would be unstable!

value of L for the $e^- + e^+$ system is $L = +1 + (-1) = 0$. All particles other than leptons, such as quarks or hadrons, have $L = 0$.

An important property of the leptonic number is that it is a *conserved quantity* in all production and decay processes involving leptons. Actually, this statement is even stronger: the *separate lepton numbers* of the first, second, and third generation of leptons must be conserved. For instance, only reactions where the *electron leptonic number* L_e is conserved are allowed, and the same property holds for the muon L_μ and tau L_τ leptonic numbers. This requirement is very useful to determine which processes involving leptons are physically allowed and which ones are not.

Therefore, taking into account this requirement, for reactions involving leptons in either the initial or the final state, only those that satisfy the following requirements will be physically allowed:

- *Energy conservation*, and specifically this implies that the sum of the masses of the particles in the final state cannot be larger than the sum of the particles in the initial state.
- *Electric charge conservation.*
- *Leptonic number conservation*, separately among the three *generations* of leptons: the electronic leptonic number L_e, muonic leptonic number L_μ, and tauonic leptonic number L_τ must be separately conserved. In other words, its value must the same for the initial and final parts of a given reaction.

Let us now now apply these requirements to determine whether or not the following processes involving leptons are physically allowed:

(a) $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$

This reaction is allowed since $m_\mu > m_e$ (and the neutrinos are effectively massless), electric charge is conserved (both $Q = -1$ in initial and final state) and leptonic numbers are conserved ($L_e = 0$ and $L_\mu = +1$).

(b) $\mu^- \to \tau^- + \nu_\mu + \bar{\nu}_\tau$

This reaction is not allowed due to energy conservation: $\mu_\mu < \mu_\tau$.

(c) $\mu^- \to e^- + \nu_\mu + e^+$

This reaction is not allowed, since the electric charge of the initial state ($Q = -1$) is different from that of the final state ($Q = 0$).

(d) $\mu^- \to e^- + \nu_\mu$

This reaction is not allowed either since the electron leptonic number of the initial state ($L_e = 0$) is different from that of the final state ($L_e = +1$). On the other hand, in this reaction the muon lepton number L_μ is conserved.

Exploiting these various requirements allows to determine when a scattering or decay process involving leptons is physically allowed. After this discussion of leptons, that do not experience the strong force. we move to discuss those particles that indeed interact strongly: hadrons and their components, the quarks.

3.5 Hadrons, quarks, and the strong interaction

We now turn to discuss those particles that are affected by the *strong interaction*. There are two types of particles that experience the strong force:

- *quarks*, which are elementary, and
- *hadrons*, which are composed by quarks.

The two most important hadrons are the *protons* p and *neutrons* n, that constitute the vast majority of the visible mass in our Universe. But there exist many other hadrons, not present in normal matter, that can be created in high energy collisions.

Before there existed man-made accelerators, scientists exploited the natural accelerators provided by Nature, namely the *cosmic rays* (high energy particles produced in astrophysical processes), in order to identify new particles. The analysis of such cosmic ray events revealed many new particles that experienced the strong nuclear force but that were clearly different from protons and neutrons. One of these particles was called the *pion* π^+, whose charge was measured to be the same as the proton, and which decays via the following chain

$$\pi^+ \to \mu^+ + \nu_\mu \to e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu \,. \tag{3.28}$$

The mass of this pion was found to be around $m_\pi \simeq 140$ MeV, not that different from that of muon with $m_\mu \simeq 105$ MeV. The main difference between these two particles is that the pion was found to interact much more frequently with normal matter than the muon, highlighting that the pion experienced the strong interaction but the muon not. Note that as required in the reaction Eq. (3.28) both the electron and muon leptonic numbers, L_e and L_μ, are conserved.

Actually there are two types of charged pions, π^+ and π^- as well as a neutral pion π^0. Despite their rather different masses ($m_\pi \simeq 140$ MeV, $m_{p,n} \simeq 940$ MeV) pions are close relatives of protons and neutrons, since they all feel the strong force. An important difference is that

- the pions π have *spin zero*, and therefore they are *bosons*,
- while protons and neutrons have *spin* $\hbar/2$, and therefore are *fermions* instead,

see the discussion in Sect. 3.2. Other hadrons, originally discovered in cosmic ray collisions, include the kaon K or the Λ particle. As we will show below, the differences between the properties of the various hadrons can be traced back to their different quark composition in each case. In terms of their spin, hadrons are divided into two main classes:

- *Baryons*: these are *hadrons* that have *spin one-half* (in units of $\hbar$), and thus are *fermions*. Baryons are composed by three quarks. The corresponding antiparticles, the anti-baryons, are instead composed by three anti-quarks
- *Mesons*: these are hadrons that have *spin zero*, and thus are *bosons*. They are composed by a quark-antiquark pair.

In addition to mesons and baryons, other more exotic hadrons exists, such as those composed by four quarks (*tetraquarks*) and by five quarks (*pentaquarks*).

The baryonic quantum number. In complete analogy with the leptonic quantum number, we can also introduce a *baryonic quantum number* B that satisfies the following rules:

- All baryons (such as protons and neutrons) have $B = +1$.
- All anti-baryons (such as anti-protons and anti-neutrons) have $B = -1$.
- Other hadrons, such as mesons and their antiparticles, have $B = 0$.
- Likewise, leptons do not carry baryon number, and thus have $B = 0$.

In the same way as with the leptonic number, also the baryon number is a *conserved quantity* in all reactions involving hadrons. This implies that only reactions and decay processes for which B is conserved (in other words, the baryon number of the initial state is the same as that of the final state) are physically allowed.

Let us take a look at what are the consequences of the requirement of baryon number B conservation for various reactions:

(a) $n \to p + e^- + \bar{v}_e$

This reaction is allowed, since the baryon number $B = +1$ both for the initial and final state.

(b) $p \to e^+ + \nu_e + \pi^0$

This reaction is not allowed, since the baryon quantum number of the initial state $B = +1$ is different from that of the final state $B = 0$.

(c) $\tau^+ \to p + \nu_\tau$

This reaction is also not allowed, since the baryon quantum number of the initial state $B = 0$ is different from that of the final state $B = +1$.

An important consequence of baryon number conservation is that protons, being the lightest of the baryons, are stable (since decaying to any particle would violate baryon number conservation). Recall that $m_p = 938.27$ MeV and $m_n = 939.47$ MeV. Since the neutron is slightly more massive than the proton, it can decay without violating baryon number conservation, as shown by the example above, by means of the reaction $n \to p + e^- + \bar{v}_e$. You can check that other conservation laws, such as energy and electric charge conservation, are also satisfied in this case. However, this is true for free neutrons but for neutrons bound in nuclei there are additional requirements that imply that under most configurations neutrons are stable.

Isospin symmetry. We now turn to discuss an *approximate* (as opposed to exact) symmetry that relates different hadrons among them, the so-called *isospin* symmetry. The original motivation to introduce this symmetry was that some hadrons seemed to be closely related between each other, in particular they had very similar masses. Such similarities often suggest that there is an underlying symmetry relating different hadrons.

Two examples of groups of hadrons that seem to be related among them are:

- The proton and neutron have almost the same mass, $m_p = 938.27$ MeV and $m_n = 939.47$ MeV.

- The two charged pions and the neutral pions have also very similar masses, $m_{\pi^+} = 139.57$ MeV, $m_{\pi^-} = 139.57$ MeV, $m_{\pi^0} = 134.98$ MeV. The masses of π^+ and π^- are actually identical since one is the anti-particle of the other.

These similarities can be accommodated by making the hypothesis that some hadrons have a property called *isospin* I that relates them. Following with the previous examples, we have that:

- The proton and neutron have both isospin $|I| = 1/2$. The difference is that one has $I_3 = +1/2$ and ther other $I_3 = -1/2$. This means that one can transform a proton into a neutron by a rotation in isospin space.[9]

- The three pions, π^+, π^-, and π^0, are all different realizations of the same underlying particle with isospin $|I| = 1$. Each of the three members of this multiplet has $I_3 = 1, 0, -1$. So we can *rotate* one pion into another by means of an isospin transformation.

As we will show shortly, isospin symmetry can be explained by the fact that the u and d quark have very similar masses, as well as to the fact that from the point of view of the strong force the u and d quarks undergo the same interactions (electromagnetic effects can be neglected at this level). Therefore, isospin symmetry can be explained by the fact that from the strong force point of view, a transformation of the type $u \to d$ should leave the hadron properties unaffected. On the other hand $m_u \simeq m_d$ but not quite identical identical, explaining why isospin is an *approximate* rather than an *exact* symmetry.

Strangeness. In addition to the hadrons introduced so far (remember that hadrons are particles that experience the strong interaction but that are not fundamental since they are composed by quarks) such as protons, neutrons, and pions, other hadrons were discovered that had different properties. Historically, this new hadrons were called *strange* particles, and include mesons such as the kaon K and baryons such as the Λ^+ baryon. As you might have guessed, we need now to introduce yet another quantum number associated to these specific hadrons, called *strangeness*. While introducing these various quantum numbers and their associated conservation laws might seem a bit confusing in the beginning, at the end of the day they make much easier to determine which processes are allowed and which ones are not.

This new quantum number, the strangeness S, depends on the specific type of mesons and baryons under consideration. We have for example that pions and kaons have

$$S(\pi^+) = S(\pi^0) = S(\pi^-) = 0\,, \tag{3.29}$$

$$S(K^+) = +1\,, \quad S(K^-) = -1\,, \tag{3.30}$$

and other baryons such as the Λ^0 and Σ^+ have

$$S(\Lambda^0) = S(\Sigma^+) = -1\,. \tag{3.31}$$

By definition, the value of S for a given antiparticle will be the same with opposite sign than for the original particle. As will be shown below, the strangeness S is a measure of how many strange quarks (and strange

[9] The mathematical formalism to deal with this transformation is called *group theory*. For the purposes of this course, you only need to know that specific symmetries can transform some particles into another.

antiquarks) a specific hadron contains. Therefore, any hadron that does not contain strange quarks will have $S = 0$.

As for the other quantum numbers, there are important conservation principles associated to strangeness. Specifically we have that:

- S is *conserved* in the electromagnetic and weak interactions.
- S is *not necessarily conserved* in the weak interactions.

These requirements imply that if we observe a physical processes where the strangeness quantum number S is not conserved, this means that this specific process is certainly mediated by the weak interaction. An example of a scattering process which conserves strangeness is

$$\pi^+ + \pi^- \to K^0 + \Lambda^0 \,, \tag{3.32}$$

since $S(\pi^+) = 0$, $S(\pi^-) = 0$, $S(K^0) = +1$, and $S(\Lambda^0) = -1$, so both the initial and final states have the same total net strangeness $S = 0$. A list of the values of S for a number of important hadrons is collected in Table 3.2.

The quark structure of hadrons. As mentioned above, all hadrons are composed by *different arrangements of quarks*. In particular, the hadrons listed so far can be explained by introducing just *three types of quarks* with the following quantum numbers:

- The *up quark* u, with electric charge $Q_u = +2/3$, strangeness $S_u = 0$, baryon quantum number $B_u = +1/3$, and mass $m_u \simeq 5$ MeV.
- The *down quark* d, with electric charge $Q_d = -1/3$, strangeness $S_d = 0$, baryon quantum number $B_d = +1/3$, and mass $m_d \simeq 5$ MeV.
- The *strange quark* s, with electric charge $Q_s = -1/3$, strangeness $S_s = -1$, baryon quantum number $B_s = +1/3$, and mass $m_s \simeq 100$ MeV.

Frequently one call *flavour* the type of quark. For example, one can say that the proton, with quark composition (uud), contains quarks of two different flavours. Note that the masses of these three quarks cannot be measured directly (since quarks are always confined inside of hadrons) so the information on their mass can only be obtained via indirect measurements or theoretical calculations. It is important to also emphasize that the quarks have *fractional electric charges*: they are the only known particles whose charge is not an integer multiple of the electron charge.

As discussed in Sect. 2.2, the consistency of quantum theory and special relativity requires that in addition to these quarks also the corresponding *antiquarks* exists, having the opposite quantum numbers (and everything else is left equal). Therefore, the quantum numbers of the antiquarks will be the following:

- The *anti-up quark* $\bar{u}$, with electric charge $Q_u = -2/3$, strangeness $S_u = 0$, baryon quantum number $B_u = -1/3$, and mass $m_u \simeq 5$ MeV.
- The *anti-down quark* $\bar{d}$, with electric charge $Q_d = +1/3$, strangeness $S_d = 0$, baryon quantum number $B_d = -1/3$, and mass $m_d \simeq 5$ MeV.

- The *anti-strange quark* $\bar{s}$, with electric charge $Q_s = +1/3$, strangeness $S_s = +1$, baryon quantum number $B_s = -1/3$, and mass $m_s \simeq 100$ MeV.

We can think of quarks as special Lego pieces that we can assemble together to construct the hadrons.

In most cases it is not too difficult to guess what are the corresponding quark structure of a given hadron. As an illustrative example, we can consider the proton p, and recall their quantum numbers:

- Electric charge: $Q_p = +1$
- Strangeness: $S_p = 0$
- Baryon number: $B_p = +1$

You can convince yourselves that the combination of quarks uud gives the right quantum numbers for the proton. Indeed, we see that by adding the quantum numbers of two up quarks and one down quark we reproduce the proton quantum numbers, since

$$Q_p = 2Q_u + Q_d \,, \quad B_p = 2B_u + B_d \,, \quad S_p = 2S_p + S_p \,. \tag{3.33}$$

Note however that not all possible combinations of quarks lead to physically acceptable hadrons. In particular, only the quark combinations that fulfill the following requirements are allowed:

- The electric charge of any hadron, Q_h, must take *integer values*: $Q_h = 0, \pm 1, \pm 2, \ldots$.
- The electric charge Q_h, strangeness S_h, and baryon number B_h of a given hadron hadron should correspond to the *sum* of the corresponding quantum numbers of the constituent quarks.
- The hadron spin and isospin should also be consistent with the corresponding properties of the constituents quarks. Note however that *spin*, being a vector quantity, is *not necessarily additive*. For example, three quarks of spin 1/2 can be combined into an hadron also of spin 1/2, as happens for protons and neutrons.

By combining the up, down, and strange quarks following the above rules we can generate a large number of possible hadrons, the majority of which have been experimentally observed.

Building hadrons from quarks. We now show how we can apply the above rules to determine the quark content of different hadrons. This is useful since it shows how a large number of particles can be constructed just in terms of three fundamental components, illustrating the reductionist paradigm that has been so successful to make progress in our understanding of elementary particles.

- If the quark composition of the K^+ meson is $(u\bar{s})$, what is the quark composition of the K^- meson (its anti-particle)?

  The quark composition of an antiparticle is simply given by taking the quark composition of the original particle but in terms of the corresponding antiquarks. So if the K^+ meson is composed by $(u\bar{s})$, then the K^- meson is composed by $(\bar{u}s)$. Note that the quantum numbers are consistent, for example the electric charge $Q_{K^-} = -1 = Q_{\bar{u}} + Q_s = -2/3 - 1/3$.

- What is the quark composition of the Δ^{++} baryon?

  As indicated by its symbol, this baryon (composed by three quarks) has electric charge $Q_{\Delta^{++}} = +2$*. The only way that we have with three quarks to combine consistently their electric charges is with three up quarks, so that* $Q_{\Delta^{++}} = 3 \times Q_u = 3 \times (+2/3) = +2$*. Therefore, we conclude that the* Δ^{++} *baryon has a quark composition of* (uuu)*.*

- What is the quark composition of an anti-neutron?

  The neutron n *is has* (udd) *as quark composition. Therefore, the quark composition of the anti-neutron will be* $\left(\bar{u}\bar{d}\bar{d}\right)$*. Note that as expected the electric charge of the anti-neutron is still zero, since* $Q_{\bar{u}} + 2 \times Q_{\bar{d}} = -2/3 + 2 \times 1/3 = 0$*. However, the neutron* n *and the anti-neutron* $\bar{n}$ *are still not the same particle: in particular they have the opposite baryon number, since* $B_n = +1$ *and* $B_{\bar{n}} = -1$*.*

- The so-called *cascade* baryon Ξ^- has strangeness $S = -2$, electric charge $Q = -1$, spin $\hbar/2$, and baryon number $B = +1$. What is therefore its quark composition? Does a particle Ξ^+ can exist and why?

  The fact that the Ξ^- *baryon has strangeness* $S = -2$ *means that it contains two strange quarks* s*, adding up to a electric charge of* $2 \times Q_s = -2/3$*. To construct a baryon with* $Q = -1$*, the only possibility is adding an up quark with charge* $Q_d = -1/3$*. Therefore, we can conclude that the quark content of* Ξ^- *is* (dss)*. Note that this is consistent with a baryon number of* $B = +1$*.*

  The antiparticle of the Ξ^- *baryon would have a quark composition of* $\left(\bar{d}\bar{s}\bar{s}\right)$*, with electric charge* $Q = +1$*. So indeed the* Ξ^+ *is expected to exist from basic theory considerations.*

Hadron decays. Once we have determined the quark composition of the hadrons involved in a specific process, for example in decay and scattering reactions, we can trace back how do they behave and transform, by connecting initial with final quarks. For instance, we can draw quark-level diagrams for the following scattering process:

$$\pi^0 + p \to n + \pi^+ \,, \tag{3.34}$$

which is represented in Fig. 3.5. Note that we can draw a line connecting a quark and an antiquark of the same flavour in the initial (or final) state. The reason for this will be discussed in more detail in the next section: it means that in some diagrams a quark and an antiquark of the same flavour can *annihilate* into a gluon, the mediator of the strong interaction, which in turn can produce another quark-antiquark pair (possibly of a different flavour).

If one considers different reactions among hadrons in terms of their quark composition, one will encounter two rather different situations:

- For some processes, the type (*flavour*) of the quarks is never changed. Quarks remain always of the same type. What can happen is that a pair of quark and antiquark *of the same flavour* appears from the vacuum or annihilates among them, leaving the overall quantum numbers of the system unchanged. As mentioned above, we will see that this behaviour is mediated by the gluons, the carries of the strong force. This behaviour is typical of processes mediated by the strong and electromagnetic interactions. The $\pi^0 + p \to n + \pi^+$ scattering process shown in Fig. 3.5 belongs to this class, and indeed it is mediated by the strong interactions.

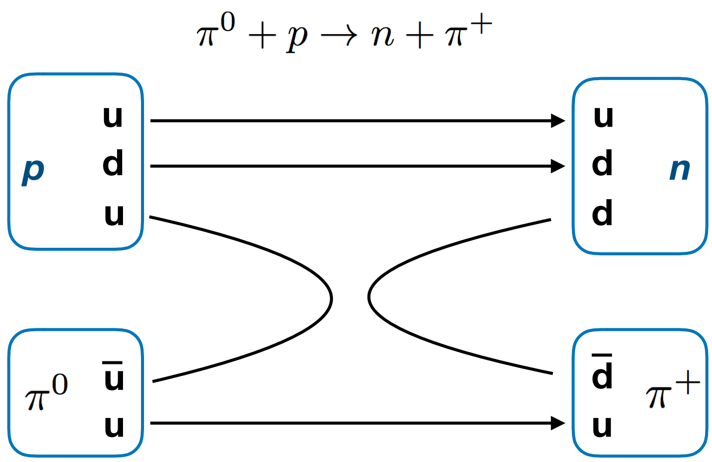

Figure 3.5. Schematic representation of the $\pi^0 + p \to n + \pi^+$ scattering process at the quark level.

- For other processes, one finds that one or more of the quarks involved in the reaction *changes flavour*, for example in some cases the strangeness quantum number S is not conserved. This kind of processes can only take place in the *weak interaction*, since in the strong and electromagnetic interactions strangeness is always a conserved quantity.

Let us take a closer look at the reaction $\pi^0 + p \to n + \pi^+$ shown in Fig. 3.5, taking into account the quark content of the hadrons involved. We have that $\pi^0 = (u\bar{u})$, $p = (uud)$, $n = (udd)$, and $\pi^+ = \left(u\bar{d}\right)$. Actually the neutral pion π^0 is a superposition of $(u\bar{u})$ and $\left(d\bar{d}\right)$, but any of the two possible choices works fine here. Therefore this scattering process at the quark level is given by

$$(u\bar{u}) + (uud) \to (udd) + \left(u\bar{d}\right) , \tag{3.35}$$

Note that the quark lines are never broken, and always connect either the same quarks on the initial and in the final state, or the a quark-antiquark pair either in the initial and final state. As will be discussed below, this feature is typical of the strong interactions, and indeed we can see the process $u\bar{u} \to d\bar{d}$ in Fig. 3.5 as mediated by a *gluon*, the mediator of the strong force.

The *color* of quarks and hadrons. One of the problems that physicists found when trying to make sense of the quark structure of hadrons was the following. Consider the Δ^{++} particle baryon. As have have shown above, this hadron is composed by three up quarks, (uuu), so its quantum wave function $|uuu\rangle$ is *fully symmetric* upon the exchange of any of the quarks. Moreover, the spin part of its wave function is

$$|s, s_z\rangle = \left|\frac{3}{2}, +\frac{3}{2}\right\rangle , \tag{3.36}$$

which means that the three up quarks all have spins pointing in the same direction, in other words, we have that $|\Psi\rangle_{\rm spin} = |\uparrow\uparrow\uparrow\rangle$. Therefore, also the spin part of hadron wave function is fully symmetric. This seems to be inconsistent with what we learned in Sect. 3.2, since:

- The Δ^{++} baryon has spin $s = 3/2$, therefore it is a *fermion*.

- We know that the wave function of a system composed by fermions should be *antisymmetric* with respect to the exchange of any of its components.
- However, this specific hadron has a wave function $|\Psi\rangle_{\rm quark} \otimes |\Psi\rangle_{\rm spin} = |uuu\rangle \otimes |\uparrow\uparrow\uparrow\rangle$ which is fully symmetric with respect the exchange of any quark.

To overcome this problem, physicists assumed the existence of yet another quantum number, which would be carried *only by quarks*. This quantum number was called *color* and can exist in three different types, *blue, red, green*.[10]. Then, if the color part of the wave function is composed by quarks of three different colors, $|rgb\rangle$ then Fermi statistics is rescued and everything is consistent. Indeed, the total wave function will be

$$|\Psi\rangle_{\Delta^{++}} = |\Psi\rangle_{\rm quark} \otimes |\Psi\rangle_{\rm spin} \otimes |\Psi\rangle_{\rm color} = |uuu\rangle \otimes |\uparrow\uparrow\uparrow\rangle \otimes |rgb\rangle \, , \quad (3.37)$$

which now is antisymmetric with respect to the exchange of any of the quarks, as required for a fermion.

As we will see in the next chapter, the color quantum number of quarks plays a similar role than the *electric charge in electromagnetism*. The theory that governs the interaction of particles with color charge is called the strong interaction or *Quantum Chromodynamics*. Just as electric charge is conserved in the electromagnetic interactions, also the color charge is conserved in the strong interactions.

A crucial consequence of the properties of the strong interaction is that quarks are never *observed isolated* in nature, but only confided into hadrons, which are color-singlet (that is, without net strong color charge) particles. The reason for this behaviour is that the properties of the strong force are such that if one tries to separate two quarks, their mutual attraction becomes larger. This behaviour is similar as in the case of a spring: if you displace the spring by an amount Δx with respect to its equilibrium position, the spring will experience a recovery force of the form

$$F = -k\Delta x \, , \quad (3.38)$$

so the bigger the separation, the bigger the force trying to restore the spring back to its equilibrium position. With this analogy, is clear that separating completely two quarks among them would require an infinite force F. To illustrate this point, in Fig. 3.6 we show the results of a calculation of the strong force potential $V(r)$ that determines the attraction between two quarks as a function of their separation r. The monotonic growth with r indicates that their mutual attraction increases as their separation increases.

Heavy quarks. So we have seen that many hadrons can be explained as built from different combinations of just three quarks: the *up*, *down*, and *strange* quarks. On the other hand, a number of additional hadrons were found that could not be fit in the same pattern, and that in all cases were characterized by much higher masses as compared to the other hadrons. These extra hadrons can be explained by the fact that there exist (at least) three more types of quarks: the *charm* quark c, the *bottom* quark b, and the *top* t quarks. As opposed to the first three quarks, which have masses much smaller than the proton mass $m_p \simeq 1$ GeV, these other three quarks have masses larger than the proton, and hence they are called *heavy quarks*.

Let us start with the *charm* quark c. Experimentally, it was found that this quark has *the same quantum numbers as the up quark* with two exceptions: its isospin is zero, and it has a new quantum number called *charmness* C. A charm quark has $C = +1$, while a charm anti-quark has $C = -1$. This new quantum

[10] Again, there is no relation with the everyday colors, we simply use a convenient name to reflect a new property of the world of elementary particles.

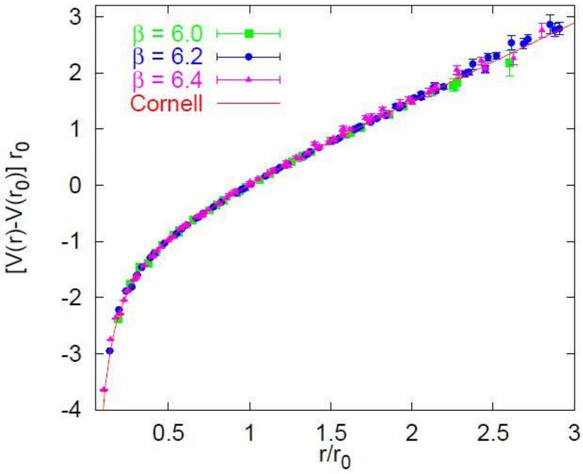

Figure 3.6. A calculation of the strong force potential $V(r)$ that determines the attraction between two quarks as a function of their separation r.

number is the analog of strangeness S characterizing the strange quarks. As such, charmness C is conserved in the strong and electromagnetic interactions, but it is not conserved in general in the weak interactions. The mass of the charm quark turns out to be around $m_c \sim 1.3$ GeV, so a bit bigger than the nucleon mass.

Next we have the bottom quark b, which seems to be a copy of the down and strange quarks, just with a larger mass, $m_b \simeq 4.5$ GeV. It also has a new quantum number, *bottomness* b (note the use of lower case to avoid confusion with the baryon quantum number B). A bottom quark has bottomness $b = -1$, while a bottom anti-quark has the opposite quantum number $b = +1$. The same properties as for strangeness and charmness hold for bottomness, in particular, b is conserved by the strong and electromagnetic interactions but is not conserved by the weak interactions. Therefore any reaction that involves a variation of S, C and b between the initial and final state can only be mediated by the weak interaction.

The heaviest quark of all is the top quark t, with a mass of $m_t \simeq 173$ GeV - it has the same mass of an atom with 180 nucleons. This quark seems a copy of the up and charm quark, just with much heavier mass. As opposed to all the other, lighter, quarks, the top quark is so heavy that it decays almost instantaneously, and therefore *it cannot form hadrons*. So while we can have hadrons composed by various combinations of up, down, strange, charm, and bottom quarks, we cannot have any hadron that includes top quarks in it, since once produced t decays very quickly into lighter quarks before an hadron can be found.

Therefore in total there exist (as far as we know) *six different flavours of quarks*, and in Table 3.1 we summarize their properties. In each case we indicate its mass, its electric charge, and the year of its discovery. There could very well exist even heavier quarks, and this possibility is being actively searched in high-energy colliders such as the LHC. For more details about the properties of quarks, you can check the PDG review, `http://pdg.lbl.gov/2017/tables/rpp2017-sum-quarks.pdf`. From this table is clear that the quarks are organized by *families*, also known as *generations*, where remarkable the second and third generation seem to be copies of the first one but with higher masses. The reason why elementary particles are organized in these *generations* is still unknown, we will briefly mention different speculative explanations in Sect. 5.

By combining the light quarks (u, d and s) with the heavy quarks (b and c) we can built many new mesons and baryons, as well as more exotic hadrons containing four quarks (called *tetraquarks*) and five quarks (called *pentaquarks*). For example, a bound state of charm quark and an anti-down quark, $(c\bar{d})$ is

	Flavour	Mass m	Electric charge Q	year of discovery
First family	up	~ 5 MeV	$+2/3$	-
	down	~ 10 MeV	$-1/3$	-
Second family	strange	~ 100 MeV	$+2/3$	~ 1950
	charm	1.3 GeV	$-1/3$	1973
Third family	top	175 GeV	$+2/3$	1995
	bottom/beauty	4.5 GeV	$-1/3$	1977

Table 3.1. Summary of the properties of the six types of quarks. In each case we indicate its mass, its electric charge, and the year of its discovery. The electric charge Q is given in units of the absolute value of the electron charge, e. Now how the quarks are organized by *families*, also known as *generations*, where second and third generation are copies of the first one only with higher masses. See text for more details.

known as a D^+ meson, while the bound state of a up quark and an anti-bottom quark, $\left(u\bar{b}\right)$ defines the so-called B^0 meson. In Tables 3.2 and 3.1 we list the properties of additional mesons and baryons composed by heavy quarks. Let us just mention here that the rules to construct mesons and baryons are exactly the same with light and with heavy quarks. This makes sense since as we will show in the next lecture, the strong interaction (responsible for binding the quarks into hadrons) is *blind* with respect the quark flavour, and treats exactly in the same way quarks with different flavour.

Hadron structure: a summary. In Table 3.2 we provide a detailed summary about the most important properties of hadrons that we have learned in this lecture. For each hadron (or group of hadrons) we indicate is mass in MeV, its spin in units of $\hbar$, and then its baryonic B, strangeness S, charmness C and bottomness b quantum numbers. This information, together with the quark combination rules described above, can be used to determine the quark content of any hadron, although in some cases there is not a unique choice for the quark content. Note that by convention the electric charge of an specific hadron is indicated already in its symbol. For instance, the Δ^{++} baryon has electric charge $Q = +2$, while the Λ^0 baryon has zero electric charge $Q = 0$.

Let us highlight some important patterns that we can observe from Table 3.2:

- All hadrons have *integer electric charge*: hadrons with non-integer electric charge are forbidden.
- The spin of all hadrons is always a *multiple of* $\hbar/2$. Therefore, hadrons are either fermions or bosons. Specifically, we see that all mesons are bosons, while all baryons are fermions.
- The baryon, strangeness, charmness, and bottomness quantum numbers always take integer values. Any hadron which has a non-integer value for one of these quantum numbers would be forbidden.
- Hadrons with charm quantum number $C \neq 0$ are heavier in general than those with $C = 0$, since the charm quark c is much heavier than the up, down, and strange quarks. The exception are those hadrons with $B \neq 0$, since these contain bottom quarks, which are about three times heavier than the charm quarks. This implies that the mass of an hadron provides useful information in order to determine its quark content.

Naam	Symbool	Massa (MeV/c^2)	Spin $(\hbar)$	Baryongetal B	Vreemdheid S	Charm c	Bottomness b
Pion	π^+	139.6	0	0	0	0	0
	π^0	135.0	0	0	0	0	0
Kaon	K^+	493.7	0	0	+1	0	0
	K^0	497.7	0	0	+1	0	0
Phi	Φ	1019.5	1	0	0	0	0
D-meson	D^+	1869.4	0	0	0	+1	0
	D^0	1864.5	0	0	0	+1	0
	D_s^+	1968	0	0	+1	+1	0
J/psi	J/ψ	3097	1	0	0	0	0
B-meson	B^+	5279	0	0	0	0	+1
	B^0	5279	0	0	0	0	+1
	B_s^0	5366	0	0	−1	0	+1
	B_c^+	6277	0	0	0	+1	+1
Upsilon	Υ	9460	1	0	0	0	0
Proton	p	938.3	1/2	1	0	0	0
Neutron	n	939.6	1/2	1	0	0	0
Delta	Δ^+	1232	3/2	1	0	0	0
	Δ^{++}	1232	3/2	1	0	0	0
Lambda	Λ^0	1116	1/2	1	−1	0	0
Sigma	Σ^+	1189	1/2	1	−1	0	0
	Σ^0	1193	1/2	1	−1	0	0
	Σ^-	1197	1/2	1	−1	0	0
Xi	Ξ^0	1315	1/2	1	−2	0	0
	Ξ^-	1321	1/2	1	−2	0	0
Omega	Ω^-	1673	3/2	1	−3	0	0
Lambda-c	Λ_c^+	2286	1/2	1	0	+1	0
Xi-c	Ξ_c^+	2468	1/2	1	−1	+1	0
	Ξ_c^0	2471	1/2	1	−1	+1	0
Omega-c	Ω_c^0	2695	1/2	1	−2	+1	0
Lambda-b	Λ_b^0	5620	1/2	1	0	0	−1
Xi-b	Ξ_b^-	5791	1/2	1	−1	0	−1
	Ξ_b^0	?	1/2	1	−1	0	−1
Omega-b	Ω_b^-	6071	1/2	1	−2	0	−1

Table 3.2. List of properties of a number of selected hadrons. From left to right we indicate the name and symbol of each hadron, its rest mass (in units of MeV/c^2), its spin in units of $\hbar$, its baryon number B, strangeness S, and its charm and bottom quantum numbers c and b respectively. Note that by convention the value of the electric charge of a given hadron is indicated as an extra label in its symbol. Recall that we use the convention in which a strange quark carries $S = -1$, a charm quark carries $C = +1$, and a bottom quark carries $b = -1$, namely with the same sign as their electric charge.

In Table 3.3 we show the quark content of a number of selected hadrons. The table is separated into mesons, and baryons. In turn, mesons and baryons are divided into light (containing only u, d, s quarks) and heavy (containing either c or b quarks) hadrons. The complete list of hadrons with up-to-date information on their properties can be found in the Particle Data Group website, `http://pdg.lbl.gov/`, we encourage the interested reader to peruse it. For the corresponding antiparticles, the quark content is obtained by transforming quarks into antiquarks and vice-versa.

Let us discuss some interesting highlights from Table 3.3. To begin with, note how all mesons are composed by a pair of quark and antiquark (thus $B = 0$), while all baryons are composed by three quarks (thus $B = 1$). Also, note that quarks of any flavour can be combined among them to create different hadrons, there is no restriction in this respect. We also see that the electric charge is always explicitly indicated in the symbol of each hadron: in this respect, it is an useful exercise to check that in all the cases listed in Table 3.3, adding up the electric charges of the constituent quarks gives the electric charge of the hadron.

The combination of the information contained in Tables 3.2 and 3.3 should be enough to understand the properties of the hadron spectrum as well as the structure of scattering and decay processes involving these hadrons. It is a useful exercise to verify that the information contained in the two tables is fully consistent.

Let us consider the D_s^+ meson. From Table 3.2 we see that it has $S = C = 1$ and $b = B = 0$. This means that it contains one anti-strange quark (to have $S = +1$) and one charm quark (to have $C = +1$). Therefore its quark content is $(c\bar{s})$. Note that with this assignment we get the correct value of the electric charge, since $Q_{D_s^+} = +1$ and $Q_c + Q_{\bar{s}} = +2/3 + 1/3 = +1$.
Let us do the same exercise with a baryon, Ω^-. This baryon has $S = -3$, indicating that it is composed by three strange quarks. So the quark composition is $\Omega^- = (sss)$. The electric charge is correct since $Q_{\Omega^-} = -1 = 3 \times Q_s$.
As a final example we take the Ξ_c^+ baryon. From Table 3.2 we see that it has $S = -1$ and $C = +1$, $b = 0$. This means that Ξ_c^+ contains an strange quark and a charm quark, but no bottom quarks. Since $Q_{\Xi_c^+} = +1$, the other quark of this baryon should be an up quark, since this arrangement leads to a consistent electric charge because $Q_u + Q_s + Q_c = +1$.

	Hadron	Quark content
Light mesons	π^+	$(u\bar{d})$
	π^-	$(d\bar{u})$
	π^0	$(u\bar{u})/\sqrt{2} - (d\bar{d})/\sqrt{2}$
	η	$(u\bar{u})/\sqrt{6} + (d\bar{d})/\sqrt{6} - s\bar{s}/\sqrt{3}$
	K^-	$(\bar{u}s)$
	K^+	$(u\bar{s})$
	K^0	$(d\bar{s})$
	$\overline{K}^0$	$(s\bar{d})$
heavy mesons	D^+	$(c\bar{d})$
	D^0	$(c\bar{u})$
	B^0	$(d\bar{b})$
	B^+	$(u\bar{b})$
Light baryons	p	(uud)
	n	(udd)
	Λ^{++}	(uuu)
	Λ^0	(uds)
	Σ^+	(uus)
	Σ^0	(uds)
	Σ^-	(dds)
	Δ^{++}	(uuu)
	Δ^+	(uud)
	Δ^0	(udd)
	Δ^-	(ddd)
Heavy baryons	Σ_c^{++}	(uuc)
	Σ_c^+	(udc)
	Λ_c^+	(udc)
	Λ_b^0	(udb)
	Σ_b^+	(uub)

Table 3.3. The quark content of a number of selected hadrons. For the corresponding antiparticles, the quark content is obtained by transforming quarks into antiquarks and vice-versa. The table is separated into mesons (upper part), and baryons (bottom part). In turn, mesons and baryons are divided into light (containing only u, d, s quarks) and heavy (containing either c or b quarks) hadrons. See text for more details. The complete list of hadrons with up-to-date information on their properties can be found in the Particle Data Group website, http://pdg.lbl.gov/.

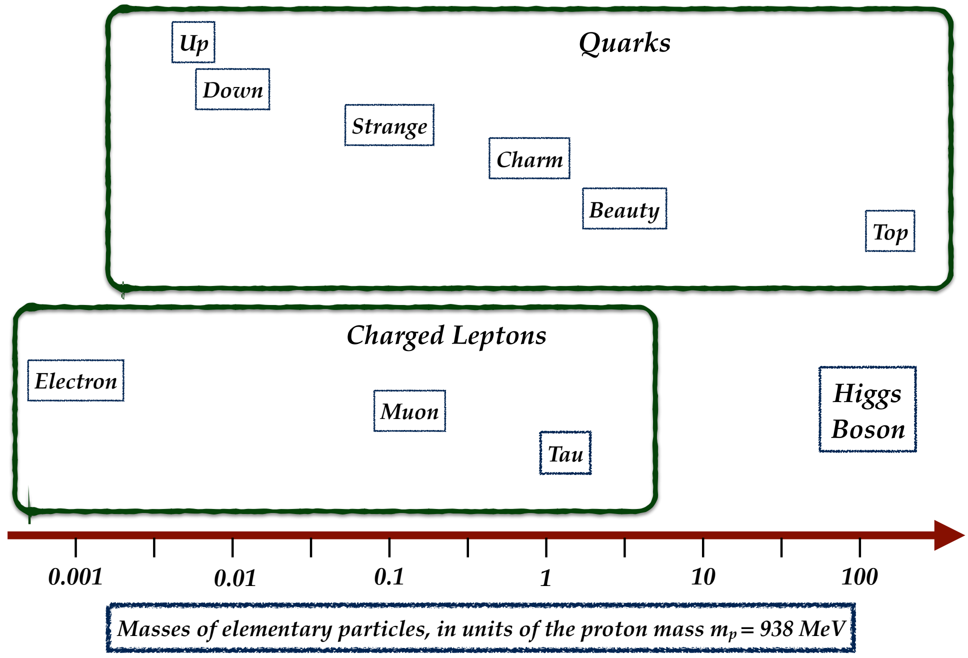

Table 3.4. Overview of the masses of the matter particles that we have studied in this part of the course, represented in units of the proton mass $m_p = 938$ MeV. In particular, we show the values of the masses of the six quarks, the three charged leptons, and of the Higgs boson. Note that the scale in the x axis is logarithmic.

3.6 Summary

As an general overview of the various types of elementary particles that we have studied in this part of the course, in Fig. 3.4 we represent the masses of the matter particles that we have studied in this part of the course in units of the proton mass $m_p = 938$ MeV. In particular, we show the values of the masses of the six quarks, the three charged leptons, and of the Higgs boson. Note that the scale in the x axis is logarithmic. In addition to the particles shown in Fig. 3.4, note that the corresponding antiparticles have exactly the same masses. We now know that the neutrinos also have mass, but this is much smaller than the electron mass: the current bounds on the heaviest neutrino mass are $\sum_k m_{\nu_k} \lesssim 0.3$ eV, where the sum runs over all neutrino species. You can compare this with the corresponding value for the electron, $m_e = 511$ eV.

We now summarize what we have learned in this part of the course. The main highlights are the following:

- Elementary particles can be divided into two main families: *bosons* and *fermions*, depending of their spin. Matter particles such as protons, electrons, and quarks, are all fermions with spin $s = \hbar/2$. Force carriers, like the photon, are bosons with spin $s = \hbar$.

- Particles can be divided also into *leptons* and *hadrons* depending whether or not their experience the strong interaction. Hadrons are composite particles, built up from quarks.

- The main properties of known hadrons can be understood in terms of the properties of its constituent quarks. In particular, the hadron quantum numbers can be derived from its quark content.

- There exist strict conservation rules that determine which reactions between elementary particles are physically allowed. For example, electric charge, baryon number, and the individual leptonic numbers must always be conserved, while strangeness, charmness, and bottomness are conserved by the electromagnetic and strong forces but can be violated by the weak interaction.

In the next part of the course we will study how the elementary particles that we have just introduced interact among each other, by describing the three fundamental interactions that are relevant in the world of subatomic particles: *electromagnetism*, the *strong nuclear force*, and the *weak nuclear force*.

4 The fundamental interactions between elementary particles

After this overview of the main types and properties of elementary particles, we now turn to study the fundamental interactions among them. We will show how these interactions can be understood as mediated by the exchange of the force carries: the photons (for electromagnetism), the gluons (for the strong interaction) and the W, Z bosons (for the weak interaction). We will introduce a powerful method to represent particle interactions, known as *Feynman diagrams*. As outlined in the previous section, we will highlight how conservation laws impose important constraints on which reactions between elementary particles are physically allowed.

The **learning goals** of the lecture are:

(a) To describe the interactions between elementary particles in terms of the exchange of force carriers, and to represent graphically these interactions.

(b) To apply conservation laws to determine when a specific reaction between elementary particles is allowed or not.

(c) To be able to draw Feynman diagrams to represent different types of scattering and decay processes of fundamental particles.

(d) To be able to identify the characteristic features of phenomena that are governed by the strong and the weak interactions.

Therefore, in this section we will discuss how the different types of elementary particles that we have discussed in Sect. 3 *talk to each other* by means of four fundamental interactions:

- *Gravity.*
- *Electromagnetism.*
- *The strong nuclear force.*
- *The weak nuclear force.*

In the following, we discuss the properties of each of these four fundamental interactions in turn, and explain how they determine which scattering or decay process involving elementary particles are possible.

4.1 Gravity

We begin with the most ubiquitous of the fundamental forces: gravity. Every object or particle that has *mass* m interacts with other massive objects by means of the *gravitational interaction*.[11] In classical physics, the gravitational attraction between two particles of mass m_1 and m_2 is given by Newton's force,

$$F = G\frac{m_1 m_2}{r^2}\,, \qquad (4.1)$$

[11] To be precise, either mass or energy seed the gravitational force, as described by Einstein's theory of General Relativity, and where gravity is understood as geometric deformations of the space-time continuum. General relativity is an extremely successful theory, with remarkable predictions such as the recently discovered *gravitational waves*: unlike usual waves that propagate on some medium, gravitational waves are oscillations of space-time itself.

with r being their separation and G is Newton's gravitational constant $G = 6.673 \times 10^{-11}$ Nm^{2}kg^{-2}. Therefore, the gravitational attraction between two particles is proportional to its mass and inversely proportional to the square of their separation. A crucial feature of the gravitational interaction is that it is *universally attractive*, *i.e.* there exist no such a thing a gravitational repulsion. This is why gravity is so pervasive in everyday life: *everything with mass attracts other masses, never repeals them.*

Despite common intuition, gravity is an *extremely weak* force as compared to any other interaction. To check this, one can compare the gravitational force between the proton and the electron in a hydrogen atom, given by

$$F_G = G\frac{m_e m_p}{r_A^2}\,, \tag{4.2}$$

with r_A the atomic radius, with the corresponding force from the electrostatic interaction

$$F_E = k\frac{Q_e Q_p}{r_A^2}\,, \tag{4.3}$$

with $k = 9 \times 10^9$ Nm^{2}C^{-2} being Coulomb's constant. Using the fact that $k/G \simeq 10^{20}$ kg^2/C^2, and the values of the electron and proton masses and charges (see App. A), one finds that $F_E \simeq 10^{39} \times F_G$. So the electric force is enormously bigger than the gravitational one.
You can also think it in other way: with a cheap toy magnet I can hold metallic objects. That is, a small magnet can counteract the gravitational pull that the *whole Earth* is exerting to push this object downwards!

This comparison illustrates why gravity is too weak to have any influence for elementary particles. Which is actually good news, since we still do not really know how to *combine gravity with quantum theory.* So in the rest of the course we can happily ignore any effects of gravity when discussing the interactions of elementary particles. Gravity is only relevant when there is a very large number of particles together, since then the small gravitational pulls from each particle add up into a much stronger interaction. We will come back briefly to discuss possible avenues to unify gravity and quantum theory in Sect. 5.

4.2 Electromagnetism

In classical electromagnetism the interaction between two static particles with electric charges Q_1 and Q_2 is governed by Coulomb's law:

$$F_E = k\frac{Q_1 Q_2}{r^2}\,, \tag{4.4}$$

with r being the distance between the two charges and $k = 9 \times 10^9$ Nm^{2}C^{-2} being Coulomb's constant. In this classical picture, each electric charge generates an *electrostatic field* that permeates space, and that the attraction or repealing of other electric charges. Note that this is a big difference with respect to gravity: we can have both *positively and negatively* charged particles, which in gravity all masses are *positive*, using a rather naive analogy.

In quantum theory, we have a different picture to describe the interactions between two electrically charged particles, such as two electrons: these interaction proceed via the *exchange* of *photons* γ among them. Photons are in this respect the *quantum* of the electromagnetic field, in other words, the minimal unit

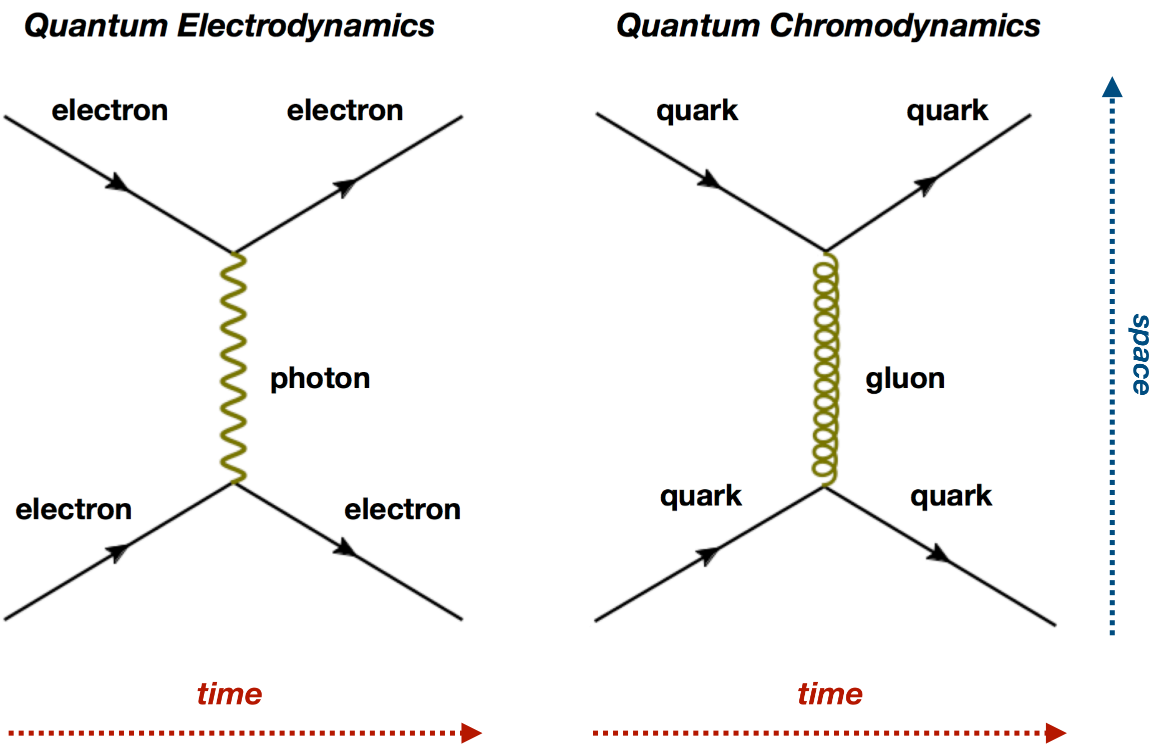

Figure 4.1. Feynman diagrams representing some of the possible interactions in the quantum theory of electromagnetism (Quantum Electrodynamics, left plot) and of the nuclear strong force (Quantum Chromodynamics, right plot). In these diagrams, going from left to right represents the time dimension, while the vertical axis indicates the spatial dimension.

of the electromagnetic interaction. This implies a radical change of point of view for the electromagnetic interactions: the classical concept of the electric field is now replaced with the *quantum concept of photon exchange*.

Feynman diagrams. The interactions between elementary particles can be represented in a rather elegant way by means of the so-called *Feynman diagrams*. As an illustration, the Feynman diagrams for the repulsive interaction between two electrons is shown in Fig. 4.1 (left). This diagram needs to be understood as follows: when two electrons come close to each other, they exchange a photon which results in a repulsive pull that brings the two electrons farther from each other. So the diagram needs to be read with *time* as the x axis and *space* in the y axis. We note that in the limit where the number of photons exchanged is very large, one recovers the classical electromagnetic field theory.

It is important to emphasize that Feynman diagrams are not only handy representation tools for interactions in quantum theory: they have associated very specific rules that indicate how to calculate the probability associated to each specific interaction between fundamental particles. While this is beyond the scope of this course, let us here only mention that by computing Feynman diagrams such as those in Fig. 4.1 we can compute the rates of decays and scattering processes between elementary particles to any given order in the perturbative expansion (starting from relatively simple diagrams and moving to more complicated ones).

The quantum version of electromagnetism is called *Quantum Electrodynamics* or QED for short. This theory is able to provide some of the most precise predictions that any scientific theory has ever provided.

The electromagnetism interaction plays a paramount role in most everyday phenomena, from making our cell phone work to the Internet. On the other hand, once we enter into the subatomic, two new interactions become important or event dominant: the *strong* and the *weak nuclear forces*, which we now discuss in turn.

4.3 The strong interactions

The first piece of evidence that we had that something called the *strong nuclear force* existed comes from the fact that everyday matter is *stable*. This observation is not so trivial when we realize that atomic nucleus are composed by protons (positively charged) and neutrons (electronically neutral). So if electromagnetism was the only interaction around, atomic nucleus would immediately disintegrate due to the repulsive force between the protons. Fortunately (for us), one of the properties of the *strong interaction* is binding the atomic nuclei together.

There are both crucial similarities as well as differences between the strong interaction and electromagnetism. The main difference is the following:

- In the electromagnetic interaction only a *single type of charge* exists, the only thing that can vary is the *sign* of the charge (positive vs negative) and its size.

- On the other hand, in the strong interaction instead there exists *three different types of charges*, denoted as *blue*, *red*, and *green*. As discussed in Sect. 3.5, this *color quantum number* is a feature of all particles (elementary or composite) that interact via the strong force. Therefore, a particle charged under the strong interaction can have a blue color charge, a red color charge, a green color charge, or any arbitrary combination of this (and the corresponding anti-charges).

The fact that there is a single type of electric charge but three different types of color (strong nuclear) charge leads to rather striking differences between electromagnetism and the strong interaction.

It can be shown that a combination of *three quarks each with a different color charge* leads to a particle with no color charge at all. This explains why the proton or the neutron themselves do not exhibit a color charge: they are *color neutral particles*. Particles composed by quarks but that have zero color charge are called *color singlets*: free particles with color charge are physically forbidden. Actually all known hadrons, presented in Sect. 3.5 are color singlets. This is the same for the combination of a quark and an antiquark, which leads to a meson.[12]

Another important difference between the strong and the electromagnetic interaction is the type of *force carrier* that mediates the interaction:

- The electromagnetic force is transmitted by the exchange of massless photons γ. These photons are *electrically neutral* themselves: a photon cannot interact with another photon.

[12] The fact that hadrons are color singlets does not mean that they do not experience the strong force: they still do, in the same way that a electric dipole (the combination of a positive $+Q$ and negative $-Q$ charges separated by a certain distance) still interact electromagnetically despite its vanishing total charge.

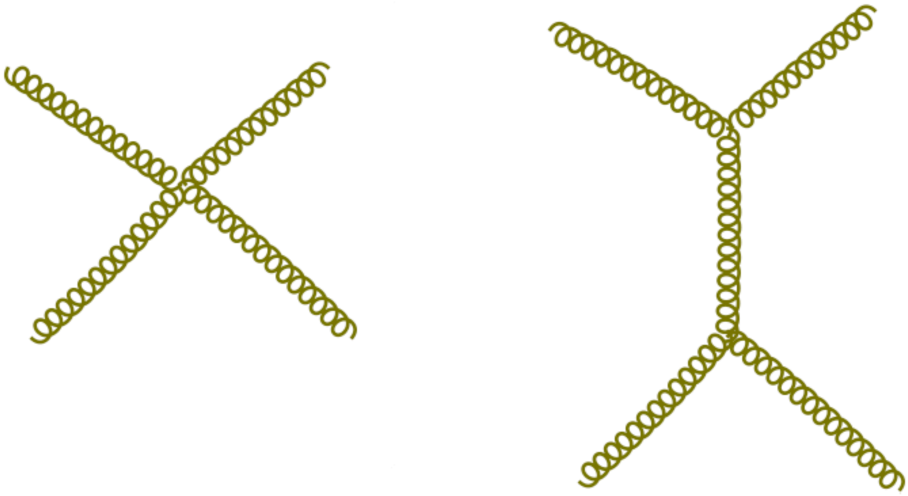

Figure 4.2. As opposed to the photon γ of electromagnetism, which is electrically neutral, the gluon g of the strong force is *charged under color*. This means that gluons can mediate not only interactions between quarks, as shown in Fig. 4.1, but they can also *mediate interactions between gluons*. As shown in this Feynman diagrams here, gluons can interact among themselves by means of a vertex with four gluons (left plot) or with three gluons (right plot).

- The strong interaction is transmitted by the exchange of particles known as *gluons g*. As photons, gluons are massless $m_g = 0$, but they *carry color charge* themselves: they can interact with other gluons, but also with themselves.

Therefore, as opposed to the photon of electromagnetism, which is electrically neutral, the carrier of the strong force (the gluon) is *charged under color*. This means that gluons can mediate not only interactions between quarks, as shown in Fig. 4.1, but they can also *mediate interactions between gluons*. As shown in the Feynman diagrams of Fig. 4.2, gluons can interact among themselves by means of a vertex with four gluons (left plot) or with three gluons (right plot). This fact leads to one of the most dramatic difference that we observe between the electromagnetic and the strong interactions:

> The strength of the strong interaction is not fixed, but depends on the specific energy of the scattering process. Specifically, this strength becomes large strong at low energies (below the proton mass), while at high energies the strong coupling constant becomes small, and the strong force behaves then in a similar way as electromagnetism.

This seemingly technical detail is responsible for the huge differences that we observe in the strong and electromagnetic interactions. In particular the self-interactions between gluons shown in Fig. 4.2 lead to the fact that the strength of the strong force between two quarks grows very steeply when we try to separate them, explaining why quarks are confined into color-neutral hadrons such as protons and neutrons (recall Fig. 3.6 and the corresponding discussion). This property is further illustrated in Fig. 4.3, where the strength of the strong interaction, as quantified by the value of its coupling constant $\alpha_s \sim g^2$ as a function of the energy E. For energies much above the proton mass $E \gg m_p$, the value of the coupling is reasonably small and we can use the Feynman diagram expansion. For energies $E \lesssim m_p$, the coupling constant becomes very large,

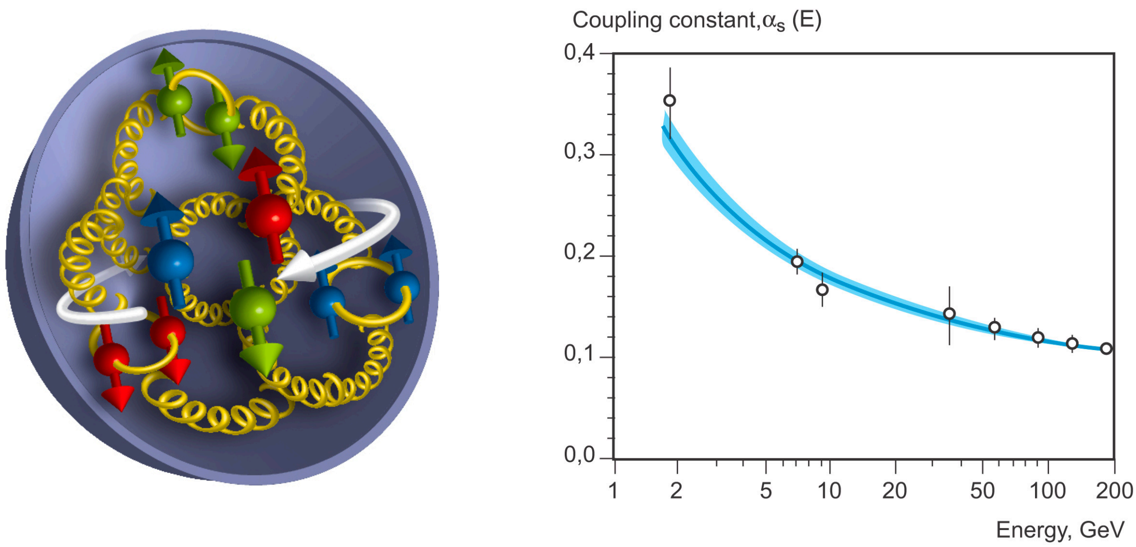

Figure 4.3. Left plot: schematic representation of a proton, where the three valence quarks are surrounded by quark-antiquark pairs, and where quarks are kept together by the gluons. Right plot: the strength of the strong interaction, as quantified by the value of its coupling constant $\alpha_s \sim g^2$ as a function of the energy E. For energies much above the proton mass $E \gg m_p$, the value of the coupling is reasonably small and we can use the Feynman diagram expansion. For energies $E \lesssim m_p$, the coupling constant becomes very large.

and calculations becomes more difficult. This is the opposite than in electromagnetism, see Eq. (4.4), where the attraction or repulsion between two charges decreases quadratically with their separation, irrespective of the particle energy or charge.

The fact that the strong force becomes large at low energies, $E \lesssim 1$ GeV, explains why quarks are bounded within the color-singlet hadrons. In Fig. 4.3 we show an schematic representation of a proton, where the three valence quarks are surrounded by quark-antiquark pairs, and where quarks are kept together by the gluons. The main role of the gluons is keeping the quarks *glued* within a given hadron, ensuring that there is no free color charge. In technical jargon, we say that quarks are *confined* within hadrons.

In analogy with QED, the quantum theory of the strong interactions is called *Quantum Chromodynamics* (QCD). This is a rather more complex theory than QED, and only at high scales it can be treated with the same calculational techniques as those used for electromagnetism.

In terms of conservation laws, any process such as a scattering reaction or a decay that is mediated by the strong interaction has to fulfill the following conditions:

- conservation of electric charge Q;
- conservation of baryon number B and individual leptonic numbers L_e, L_μ, L_τ;
- conservation of strangeness S, charmness C and bottomness b;

in addition to the usual conservation of energy and momentum. Specifically, if we see that a process violates S, C, or b, it cannot be driven by the strong interaction (nor by electromagnetism). The first of these two

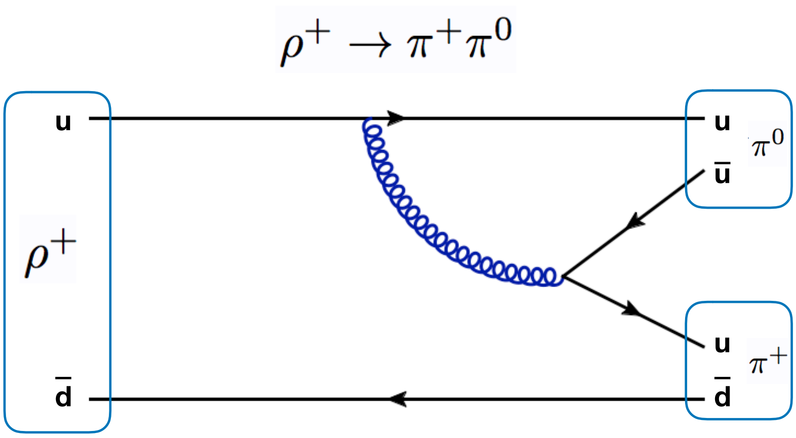

Figure 4.4. Feynman diagram representing the decay of a ρ^+ meson into a $\pi^+ + \pi^0$ pair. This decay process is mediated by a gluon, the force carrier of the strong interaction.

requirements, as we will see, are also satisfied by the weak interactions.

Let us illustrate how the strong interaction mediates the decays of specific hadrons, in particular of the ρ^+ meson. Let us consider the decay $\rho^+ \to \pi^+\pi^0$. The quark composition of ρ^+ is $(u\bar{d})$, while that of the pions is (see Table 3.3) $\pi^+ = (u\bar{d})$ and $\pi^0 = (u\bar{u})$ (using $(d\bar{d})$ for the π^0 would also work). So at the quark level this process is given by

$$(u\bar{d}) \to (u\bar{d}) + (u\bar{u}) \ . \tag{4.5}$$

The corresponding Feynman diagram for this process is shown in Fig. 4.4. We see how the u quark from the ρ^+ meson emits a gluon, which in turn splits into a $u\bar{u}$ quark anti-quark pair to create two pions. Note that in the strong interactions, quark flavour is conserved. What can happen is that a a gluon can emit a quark-antiquark pair of the same flavour. Alternatively, this quark-antiquark pair can also annihilate into a gluon, *e.g*, $u + \bar{u} \to g$.

4.4 The weak interaction

We complete this description of the fundamental forces between elementary particles by discussing the weak interaction, also known as the *weak nuclear force*. The effects of this interaction manifest themselves mostly in nuclear radiative processes and other nuclear reactions such as nuclear fission. For example, all the energy that we can extract in nuclear power plants, as well as all the energy emitted by the Sun, is made possible by the weak interactions. So even if its effects are hidden, their impact of the weak force on our everyday life is rather huge.

What are the differences and similarities of the weak interaction as compared to the nuclear and electromagnetic forces? To begin with, both the electromagnetic and the strong interactions conserve the *flavour quantum numbers* such as strangeness S, charmness C, and bottomness b. In other words, in reactions mediated by either the strong or the electromagnetic interactions, the number of say strange quarks minus that of anti-strange quarks $N_s - N_{\bar{s}}$ in the initial and final state of the reaction should be the same.[13] On the other hand, in the weak interactions these *flavour quantum numbers are not necessarily conserved*. This means that we can have for example reactions in which $N_s - N_{\bar{s}}$ or $N_c - N_{\bar{c}}$ varies between the initial and final state. There are however some restrictions about this, as we now explain.

Let us provide here two examples of *hadron decays* that are mediated by the weak interaction:

- $D^+ \to \overline{K^0} + e^+ + \nu_e$

  This reaction represents the decay of a charmed D-meson into a neutral kaon, a positron and a neutrino. If we look at the underlying quark content of this reaction, we find that

  $$\left(c\bar{d}\right) \to \left(s\bar{d}\right) + e^+ + \nu_e \tag{4.6}$$

  which means that both the conservation of strangeness and of charmness are violated: in the initial state $S = 0$ and $C = +1$, while in the final state $S = -1$ and $C = 0$.

  As we will show below, the weak interaction induces a transition $c \to s + e^+ + \nu_e$ by means of the exchange of one of the *carriers of the weak interaction*, the *W boson*, such that $c \to s + W^+$, which then decays into $e^+ + \nu_e$. The W boson is the analogous of the photon in electromagnetism with two main differences: it has a large mass and it is charged under both the strong and electromagnetic forces.

- $\overline{K^0} \to \pi^+ + \pi^-$

  This process represents the decay of a neutral kaon into a pair of charged pions. Again, looking at the underlying quark content of this reaction we find that

  $$\left(s\bar{d}\right) \to \left(u\bar{d}\right) + \left(\bar{u}d\right) \tag{4.7}$$

  so also here the strangeness quantum number conservation is violated, since $S = 1$ in the initial state but $S = 0$ in the final state. We will show below how this reaction can also be understood by means of the exchange of a W boson, in this specific case we have $s \to W^- + u$, followed by the decay $W^- \to +\bar{u}d$.

Other important examples of decay processes mediated by the weak interaction are the *muon decay* and the *neutron decay* by means of the β-decay process, the latter discussed in Sects. 2.4 and 3.3. To begin with, the muon decay proceeds as

$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e \,, \tag{4.8}$$

where we note that both electric charge and the electron L_e and muon L_μ leptonic quantum numbers are separately conserved. This reflects the fact that while the weak interaction does not conserve the quark flavour quantum numbers, it does still conserve the leptonic quantum numbers, separately for each of the three families.

Concerning the β-decay of a neutron, as we have discussed above, it proceeds by means of the following reaction:

$$n \to p + e^- + \bar{\nu}_e \,, \tag{4.9}$$

[13] Please note that S can be conserved even if N_s is not: for example, a gluon splitting $g \to s + \bar{s}$ mediated by the strong force would change N_s, but S itself would be invariant.

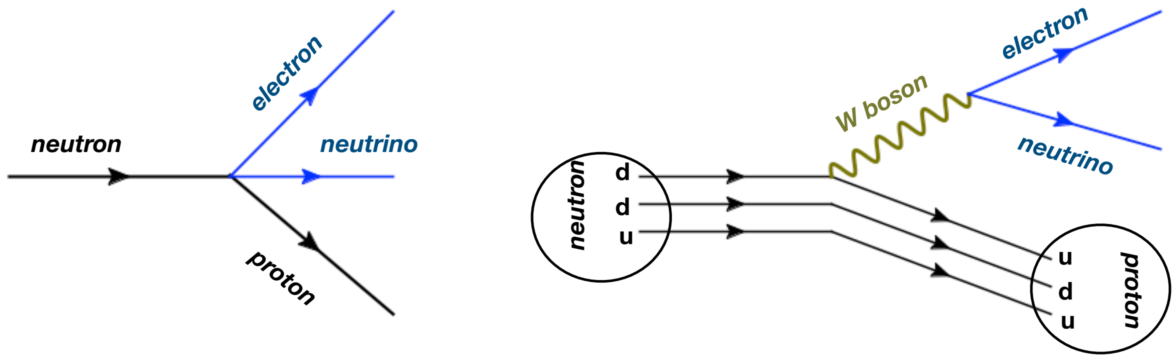

Figure 4.5. The β-decay of neutrons results into a proton, and electron and a neutrino (left plot). This process is mediated by the weak interaction, where the emission of a W boson transforms a down quark into an up quark, thus turning the neutron into a proton (right plot). The W boson then decays into the experimentally observed electron and neutrino.

and writing the quark content of the neutron and the proton, we see that the underlying reaction is

$$(udd) \to (uud) + e^- + \bar{\nu}_e \,, \qquad (4.10)$$

in other words, the weak interaction induces a transition $d \to u + e^- + \bar{\nu}_e$. As illustrated schematically in Fig. 4.5, this reaction is also mediated by a W boson, the counterpart of the photon and the gluon in the electromagnetic and strong interactions respectively. We see that the emission of a W boson transforms a down quark into an up quark, thus turning the neutron into a proton. The W boson then decays into the experimentally observed electron and anti-neutrino.

Note that in Fig. 4.5 the W boson is charged negatively, so we use the notation W^-. This condition can be readily derived from the fact that electric charge is conserved by the weak interaction, so for example in the reaction $d \to W + u$, since $Q_d = -1/3$ and $Q_u = +2/3$ then clearly ones requires $Q_{W^+} = -1$ to ensure charge conservation.

The charged weak bosons $W^\pm$. Let us now discuss in some more detail the properties of these charged weak bosons, W^+ and W^-. Their two main properties are:

- As opposed to the photon γ and the gluon g, the W bosons are not massless but have instead a large mass, $m_W \simeq 80$ GeV, that is, around 80 times the proton mass.

- As in the case of the gluon (but not the photon), they are charged themselves, both with weak and electric charge. Therefore, the weak W bosons will undergo *self-interactions*, as the similar way as the gluon in Fig. 4.2.

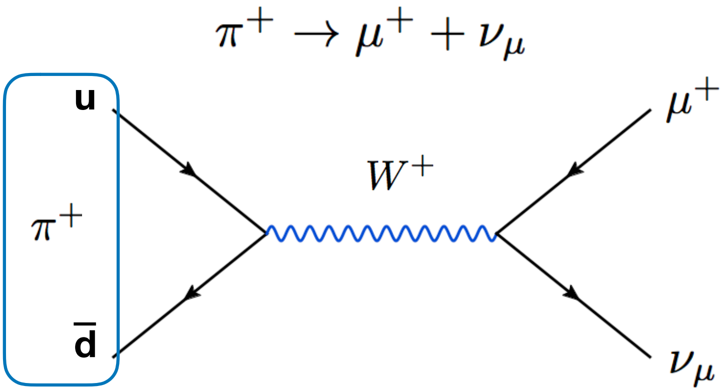

Figure 4.6. Feynman diagram for the decay of the charged pion π^+ into a $\mu^+ + \nu_\mu$ pair. This diagram is mediated by the weak interaction by means of the exchange of a W^+ weak boson.

Since they have electric charge $Q_W = \pm 1$, a W boson acting on a quark will change both its charge and its flavour. Some of the reactions involving W bosons and quarks are the following:

$$
\begin{aligned}
& u + W^- \to d\,, \quad u + W^- \to s\,, \quad d + W^+ \to u\,, \quad s + W^+ \to u\,, \\
& \bar{u} + W^+ \to \bar{d}\,, \quad \bar{u} + W^+ \to \bar{s}\,, \quad \bar{d} + W^- \to \bar{u}\,, \quad \bar{s} + W^- \to \bar{u}\,, \\
& W^+ \to u + \bar{d}\,, \quad W^+ \to u + \bar{s}\,, \quad W^- \to d + \bar{u}\,, \quad W^- \to s + \bar{u}\,,
\end{aligned}
\tag{4.11}
$$

where note that electric charge is always conserved in these transitions. In the reactions above, you can always exchange a down-type quark (d, s, and b) by a different down-type quark, and you can always exchange an up-type quark (u, c, and t) by a different up-type quark. For example, since $W^+ \to u + \bar{d}$ is physically allowed, also $W^+ \to c + \bar{d}$ or $W^+ \to u + \bar{b}$ are possible decay modes of the W^+ boson.

The $W^\pm$ bosons also couple to leptons and neutrinos. Since the W boson is charged, it will involve transforming a lepton into its corresponding neutrino and vice-versa. The analog reactions to Eq. (4.11) involving $W^\pm$ bosons but now in terms of charged and neutral leptons are the following:

$$
\begin{aligned}
& e^+ + W^- \to \bar{\nu}_e\,, \quad e^- + W^+ \to \nu_e\,, \quad \nu_e + W^+ \to e^-\,, \quad \bar{\nu}_e + W^+ \to e^+\,, \\
& W^+ \to e^+ + \nu_e\,, \quad W^- \to e^- + \bar{\nu}_e\,, \quad e^+ + \nu_e \to W^+\,, \quad e^- + \bar{\nu}_e \to W^-\,,
\end{aligned}
\tag{4.12}
$$

where note that both electric charge and the individual leptonic numbers L_e, L_μ and L_τ are conserved. So unlike the case of quarks, where W bosons mediate interactions between *different generations*, for leptons the W bosons can only relate members of the *same generation*. In Eq. (4.12) you can replace the first generation lepton by the corresponding leptons from either the second or third generation. For instance, since $W^+ \to e^+ + \nu_e$ is possible, also $W^+ \to \mu^+ + \nu_\mu$ and $W^+ \to \tau^+ + \nu_\tau$ are physically allowed. And since the reaction $e^- + W^+ \to \nu_e$ is possible, also the reactions $\mu^- + W^+ \to \nu_\mu$ and $\tau^- + W^+ \to \nu_\tau$ are allowed.

Let us illustrate how we can draw a Feynman diagram for a relatively simple process involving the weak interactions and the W boson, specifically the decay of the positively charged pion, $\pi^+ \to \mu^+ + \nu_\mu$. This decay process involves a neutrino in the final state, so it is certainly driven by the weak interaction. Moreover, it involves the annihilation of an up quark with a down antiquark, and this can only be mediated by a W^+ boson by means of the subprocess $u + \bar{d} \to W^+$, taking into account electric charge conservation. And indeed a W^+ boson can also decay into a $\mu^+ + \nu_\mu$ pair, see Eq. (4.12). Therefore at the quark level this process reads as

$$\left(u\bar{d}\right) \to W^+ \to \mu^+ + \nu_\mu \,, \tag{4.13}$$

and the corresponding Feynman diagram for this process is drawn in Fig. 4.6.

Heavy meson decays. One of the caveats concerning the previous discussion about reactions involving the weak bosons is that, in the case of quarks, these W bosons couple *more strongly* between quarks of the same *generation*, recall Table 3.1. For instance, the rate for the decay $W^+ \to c + \bar{s}$ is much larger than the rate for the decay $W^+ \to c + \bar{d}$, since both the charm and strange quarks belong to the *second generation*, but the down quark belongs instead to the *first generation*.

To further illustrate this property, let us consider the decay of the B^0 meson. The quark content of this meson (see Table 3.3) is $\left(d\bar{b}\right)$. We observe experimentally that the following decay takes place

$$B^0 \to D^- + \mu^+ + \nu_\mu \,, \tag{4.14}$$

and using the fact that the D^- meson is composed by a $(d\bar{c})$ pair, we see that at the quark level the underlying reaction is

$$\left(d\bar{b}\right) \to (d\bar{c}) + W^+ \to (d\bar{c}) + \mu^+ + \nu_\mu \,, \tag{4.15}$$

In other words, by means of the weak interaction the $\bar{b}$ quarks transforms into a $\bar{c}$ quarks by emitting a W^+ boson ($\bar{b} \to \bar{c} + W^+$), which then decays into a muon and a muonic neutrino.

As mentioned above, the W boson couples rather less strongly between quarks of different generation (as is the case between bottom and charm quarks) than between quarks of the same generation. For this reason, the effective coupling of this interaction is rather small, so the corresponding lifetime of this decay will be rather large.[14] Another possible decay of the B^0 meson is the following

$$B^0 \to \pi^- + \mu^+ + \nu_\mu \,, \tag{4.16}$$

which at the quark level reads

$$\left(d\bar{b}\right) \to (d\bar{u}) + W^+ \to (d\bar{c}) + \mu^+ + \nu_\mu \,, \tag{4.17}$$

so therefore involves a reaction of the form $\bar{b} \to \bar{u} + W^+$, where the W boson couples two quarks from different generations (in this case the third and the first generation). For this reason, the decay of B^0 mesons into

[14] As a general rule, the smaller the value of the coupling that mediates a given decay process, the longer the corresponding life-time of this decay mode will be.

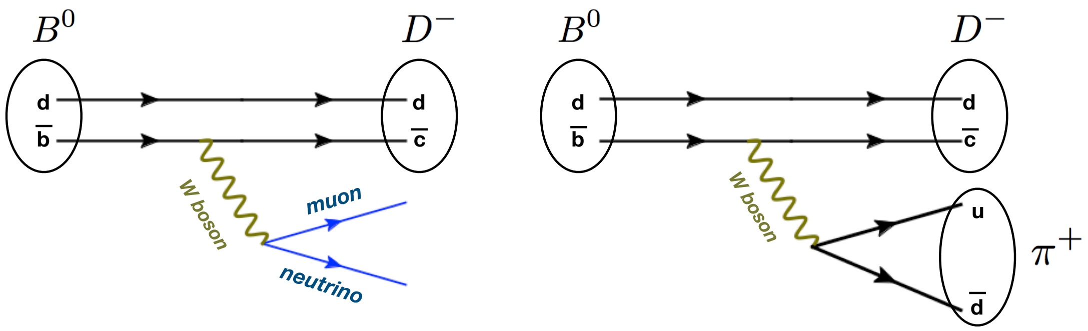

Figure 4.7. Feynman diagrams representing two possible decays of the B_0 meson, the first into $D^- + \mu^+ + \nu_\mu$ (left plot) and the second into $D^- + \pi^+$ (right plot). Note that despite the different final state, the underlying process at the quark level is the same, with the only difference being that in one case the W^+ boson decays into $\mu^+ + \nu_\mu$ and the other into a $u + \bar{d}$ pair, which then hadronizes into a π^+.

$\pi^- + \mu^+ + \nu_\mu$ is significantly suppressed as compared to the decay into the $D^- + \mu^+ + \nu_\mu$ final state.

There exist other decay modes of the B^0 meson that are closely related to the ones described above. For example, the following decay mode

$$B^0 \to D^- + \pi^+ \,, \tag{4.18}$$

does not involve any leptons in the final state, so it seems rather different than Eq. (4.14). However the underlying process at the quark level is rather similar: the only difference is that the W^+ boson decays into an up quark and a down-antiquark, see Eq. (4.11), which then combine to create a positively charged pion π^+, since it is composed precisely by a $(u\bar{d})$ pair (see Table 3.3).

The Feynman diagrams for these two possible decay modes of the B^0 meson are represented in Fig. 4.7, and allow to see more clearly the similarities and differences between these two decay modes. Since Eq. (4.18) only involves the coupling of the W boson with either quarks or leptons of the same generation, the likelihood of this process will be similar to that of Eq. (4.14).

The processes in Fig. 4.7 can only be induced by the weak interaction, which is the only one of the fundamental forces that can induce a change in the quark flavour. On the other hand, as emphasized above, the weak interactions do still *conserve* other quantum numbers such as the electric charge, color, the baryon number, and the separate electron, muon, and tau lepton numbers. So the tell-tale sign of the weak interaction is the violation of the quark flavour quantum numbers such as strangeness S, charmness C and bottomness b.

Weak interactions and the Z boson. In order to make sense of the quantum theory of the weak interactions, it was realized that it was not enough with the W boson: there should be another particle, called the Z boson, that was also able to mediate the weak interactions. As opposed to the two charged W bosons, W^+ and W^-, the Z boson is electrically neutral, $Q_Z = 0$, so it couples only to particles with weak charge. The mass of this Z boson was found to be $m_Z \simeq 91$ GeV, so around 90 times bigger than the proton mass, and thus slightly heavier than the W boson which has $m_W \simeq 80$ GeV.

Since it is electrically neutral, the Z boson is a kind of a *heavy photon*, and indeed most reactions involving the exchange of a photon should also include the effects of the exchange of a Z boson. From the practical point of view, these effects can be neglected unless we are dealing with very high energy reactions, due to

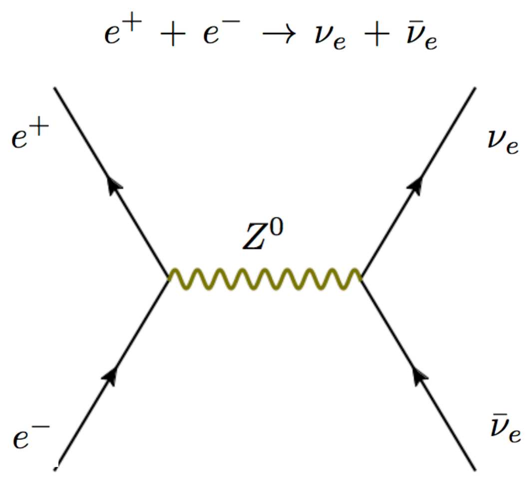

Figure 4.8. Feynman diagrams for the scattering of an electron and a positron into a neutrino-antineutrino pair. This reaction is mediated by the weak interaction by means of the neutral weak gauge boson Z^0.

the large value of its mass m_Z. But other than that, one can sometimes replace a photon γ that appears in the intermediate part of an Feynman diagram by a Z boson. The exception are those diagrams involving neutrinos: since $Q_\nu = 0$, the neutrino does not interact electromagnetically and thus a coupling with the photon γ is not allowed.

One example of a process mediated by the Z boson is electron-positron annihilation into neutrinos:

$$e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e \,. \tag{4.19}$$

This process can only be mediated by the weak force, since it involves neutrinos in the final state and recall that neutrinos do not couple to neither the strong nor to the electromagnetic interaction. But one notices that the total electric charge of both the initial and final state is $Q = 0$, so this process needs to be mediated by a neutral gauge boson that couples also to neutrinos: the Z^0 boson. The corresponding Feynman diagram for this process is shown in Fig. 4.8. This process is particularly interesting since the measurement of its cross-section allows determining the number of neutrino generations, at least those with masses $2m_\nu \lesssim M_Z$.

The range of the weak interaction. From the discussion in this section we can see that the weak interaction shares many similarities with electromagnetism. For example, the Z^0 boson is from many points of view the heavier sibling of the photon γ. So why the effects of the weak interaction are confined to inside the atomic nucleus? The reason is the large mass of the W and Z bosons. In general, the *range* of an interaction, meaning the distances for which this interaction can have an effect, is roughly given as the inverse of the mass of the mediating gauge boson,

$$\text{interaction range} \sim \frac{1}{\text{mass of boson mediator}} \,. \tag{4.20}$$

In this respect, we say that electromagnetism is an interaction with *infinite range*, since the photon is massless. And indeed a photon emitted from the other side of the universe will have a certain effect once it impacts a charged particle here on Earth.

On the other hand, for the weak interaction the range is

$$\text{interaction range} \simeq \frac{1}{M_W} = \frac{1}{80\ \text{GeV}} \times \frac{1\ \text{GeV}}{5.06 \times 10^{15}\ \text{m}^{-1}} \simeq 2.5 \times 10^{-18}\,\text{m}\,. \tag{4.21}$$

where we have used the conversion factor between GeV and meters, see App. A. Therefore, the range of the weak interaction is limited to distances smaller than the proton and neutron radius, implying that it is a very short range interaction. As mentioned above, this does not mean that its effects are only important at the microcosm, but that we really need to go to the microcosm to study its fundamental properties.

4.5 The Higgs boson

With the various elementary particles, and the corresponding force carriers, that we have presented so far we have *almost* completed the *Standard Model of elementary particles*, the unified theory of fundamental particles and their interactions. But we are still missing a very important part: the Higgs boson, which was only recently discovered at the LHC experiments at CERN.[15] The Higgs is important because the symmetries of the Standard Model require that not only all particles are massless, but that also the W and Z bosons are massless. But we know that this is very far from the truth! The Higgs mechanism allows fundamental particles to acquire a mass without breaking its symmetries.

Indeed, there are two main reasons for which the picture of elementary particles and their interactions introduced up to now is incomplete:

- To begin with, the calculation of certain processes in electroweak theory leads to a violation of *unitarity*, which in a nutshell implies that the probability of this process becomes *larger than one*, which is certainly unphysical. As shown in Fig. 4.9, one needs to introduce a further particle, in this case a *fundamental scalar* (of spin zero, thus a boson) to obtain a sensible behaviour of the scattering amplitudes.

- In the quantum theory of the weak interactions, the W and Z should in principle be *massless* in order to respect some of the symmetries of the theory. Moreover, this also holds true for other particles that experience the weak interaction, such as the quarks and the charged leptons: they should have $m = 0$. The underlying reason is that the weak interaction has a built-in asymmetry between *left and right* spatial directions, which requires particles involved to be massless.

The way to bypass these two limitations is to break the relevant symmetry *spontaneously*, rather than directly, by introducing the Higgs boson by means of the *Higgs mechanism*. Schematically, the Higgs mechanism works as follows. In addition to the various quarks, leptons, and force carriers that the Standard Model contains, we introduce a new particle, called the *Higgs boson*, denoted by ϕ. This particle is different from all other SM particles in that it is an *scalar*, that is, its spin is zero, and thus is a boson. Recall that we have seen other spin zero particles before, such as the pions $\pi^\pm$ and π^0, but those particles are not fundamental but rather composite ones.

This new particle, the Higgs boson, is subject to a *potential energy* of the following form:

$$V(\phi^2) \equiv -\mu\,\phi^2 + \lambda\,\phi^4\,, \tag{4.22}$$

[15] To be precise, its discovery was announced on the 4rth of July 2012.

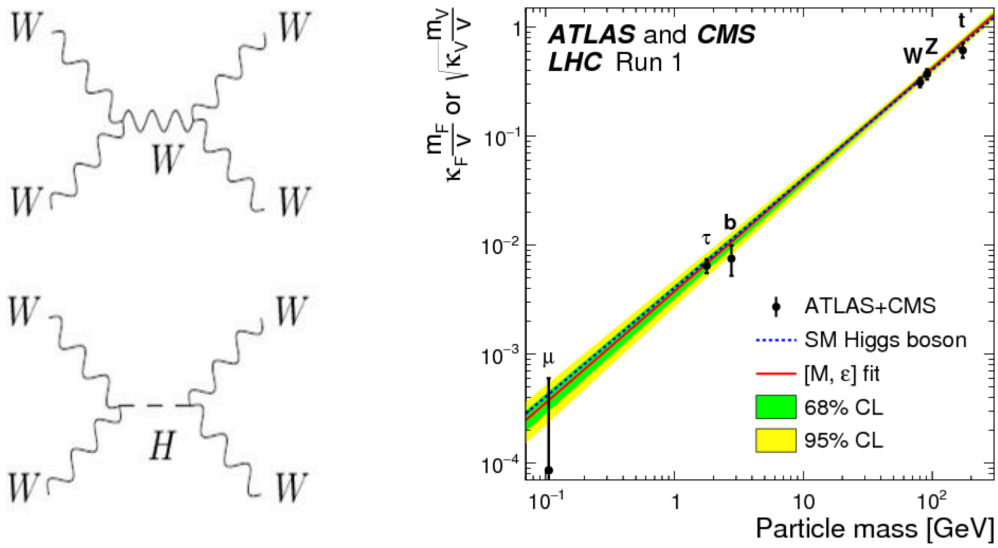

Figure 4.9. Left plot: some processes in the electroweak theory, such as the scattering of the W bosons, give unphysical results (probabilities greater than 1) unless a new scalar particle (the Higgs boson) is introduced. Right plot: The Higgs is responsible for giving mass to all elementary particles, including the W, Z bosons. The mass of each particle is then proportional to the strength of its interaction with the Higgs boson, as demonstrated by the measurements of the Higgs properties by the ATLAS and CMS experiments at the CERN's Large Hadron Collider.

with μ, λ being in principle *free parameters* of the model. In Fig. 4.10 we represent the Higgs potential Eq. (4.22) for two different possibilities for the sign of μ, either positive or negative. We see that the shape of the potential is quite different depending on the sign, in particular for $\mu > 0$ the classical minimum of the potential corresponds to a non-zero value of the scalar field ϕ. This particular feature of the classical potential is crucial in the Higgs mechanism: the state with lowest energy of the theory is such that the Higgs particle is non-zero. On the other hand, for $\mu \leq 0$ the minimum value of the potential (which corresponds to the *vacuum state* of the theory) is the one where the scalar field ϕ vanishes.

Classically, we know that the vacuum of the theory (that is, the state with the smallest total energy) will be the one for which the potential $V(\phi)$ has a minimum. Then, imposing this condition

$$\frac{\partial V(\phi)}{\partial \phi} = 0 \, , \tag{4.23}$$

we find two possibilities for the vacuum state of our theory:

- For $\mu < 0$, we find that the state of minimum energy of the theory is that where the field ϕ vanishes, $\langle \phi \rangle = 0$. In this case, the resulting theory is the standard classical electrodynamics with a massless photon coupled to a charged scalar particle.

- for $\mu > 0$ instead, the state with minimum energy is such that $\langle \phi \rangle \neq 0$, and the scalar field will acquire a vacuum expectation value (VEV) $\langle \phi \rangle = \sqrt{\mu^2/2\lambda} \equiv v/\sqrt{2}$. In this case gauge symmetry will be *spontaneously broken*, due to the fact that the vacuum (preferred configuration) is not invariant under a gauge transformation. To see these, note that either $\langle \phi \rangle = +\sqrt{\mu^2/2\lambda}$ or $\langle \phi \rangle = -\sqrt{\mu^2/2\lambda}$ are equally good solutions for Eq. (4.23) (as follows from the $U(1)$ rotational invariant of the theory), however only one of the two options can actually be implemented in nature, breaking thus the original gauge

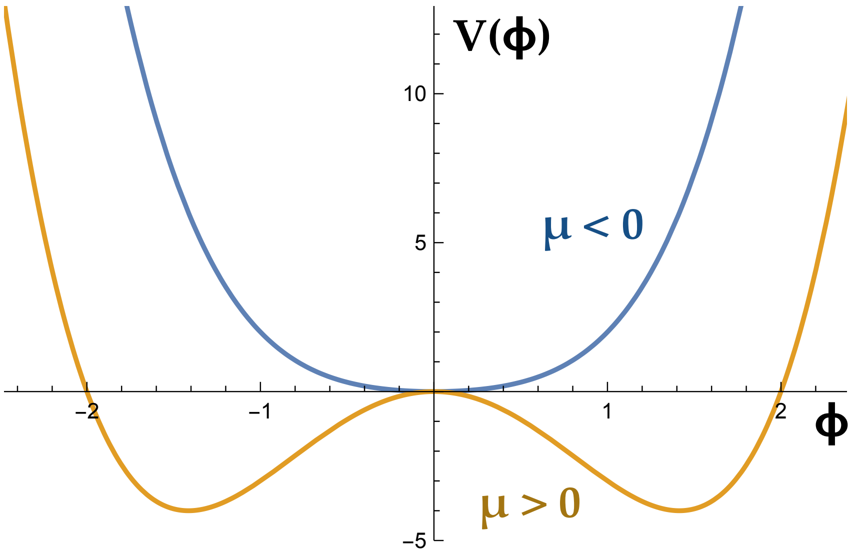

Figure 4.10. The Higgs potential Eq. (4.22) for two different possibilities for the sign of μ, either positive or negative.

invariance.

Therefore, we see that the key ingredient of the Higgs mechanism is that the state with lowest energy of the theory is *not* the state where the Higgs field takes a zero value, but rather than the Higgs field is non-vanishing. In other words, the Higgs field permeates all space, so that particles moving in this field acquire mass by coupling to it. In this context, the Higgs particle represents the *quantum excitations* of the Higgs field. One of the direct consequences of the Higgs mechanism is that the mass of each particle should then be proportional to the strength of its interaction with the Higgs boson. This fundamental prediction has been verified measurements of the Higgs properties by the ATLAS and CMS experiments at the CERN's Large Hadron Collider, as shown in the right panel of Fig. 4.9.

While the Higgs boson is responsible for giving mass to all elementary particles, it is *not* responsible for most of the mass of ordinary matter. The reason is that the up and down quark masses ($m_{u,d} \simeq 10$ MeV) are only a rather small fraction of the proton and neutron masses ($m_{p,n} \simeq 1$ GeV). The majority of the proton mass is thus provided by the strong interaction, in particular by the gluons that bind the quarks among them within the nucleon. So while the Higgs boson is certainly crucial in the theory of elementary particles, the statement that is often heard that the Higgs boson is responsible for most of the visible mass in the universe is certainly not correct.

Counting Higgs bosons. The Higgs boson was predicted in the early 60s, but only detected experimentally much later, in 2012. Why finding the Higgs boson was so difficult? Because producing a Higgs boson at the LHC is a *rare* phenomenon: for instance, you need to *collide protons around* 10^8 *times* to produce a single Higgs particle. The Higgs boson can be produced in many ways, for example in collisions involving high-energy protons such as those taking currently place at CERN's LHC. The likelihood of two protons colliding and producing a Higgs boson is given by their *interaction cross-section* σ, which has units of *area*.

The cross-section is a measure of how many Higgs bosons will be produced per proton-proton collision. At the Large Hadron Collider, protons are collided against each other at a center-of-mass energy of $\sqrt{s} = 13$ TeV, that is, at energies around 1.3×10^4 times higher than the proton mass $m_p \simeq 1$ GeV. At these energies, the cross-section to produce Higgs bosons is around 40 picobarns (pb), where a pb has units of *area* and is related to meters squared by the following relation

$$1\,\text{picobarn} \, = \, 10^{-40}\, m^2 \,, \tag{4.24}$$

where the tiny value of the cross-section when expressed in everyday units (m^2) reflects that it is quite difficult to produce the Higgs boson.

In order to be able to predict how many Higgs bosons will the LHC produce, we need to know not only the probability that when two protons collider, they will produce a Higgs boson (the interaction cross-section σ) but also the total number of proton-proton collisions that will take place, called the collider *integrated luminosity* $\mathcal{L}$. The relation is

$$N_{\rm higgs} \, = \, \mathcal{L}_{pp} \times \sigma_{pp \to h} \,. \tag{4.25}$$

For example, the LHC has so far accumulated at the center-of-mass energy of $\sqrt{s} = 13$ TeV an integrated luminosity of around $\mathcal{L} \simeq 100$ fb^{-1}. Therefore, in this period the number of Higgs bosons produced is

$$N_{\rm higgs} = 100 \times 10^3 \text{ pb}^{-1} \times 40 \text{ pb} = 4 \times 10^6 \,, \tag{4.26}$$

so around four millions. Unfortunately we are only able to detect a small fraction of them, since the Higgs bosons decay very fast into different final states (such as $h \to b\bar{b}$ or $h \to \gamma\gamma$) more or less difficult to detect.

In order to be able to detect the Higgs boson, we need to ensure not only that the *signal* (number of Higgs bosons produced) is large enough, but also that the *background* (processes that *look like* coming from Higgs production but actually correspond to other processes) is reasonably small. For example, the decay channel where the Higgs boson goes into a $b\bar{b}$ leads to the larger *signal cross-sections*, but in the strong interaction there are many ways to produce $b\bar{b}$ pairs unrelated to the Higgs boson. Therefore the *background* for this process is very large, making difficult the observation of the $h \to b\bar{b}$ decay.

4.6 The Standard Model of particle physics

It is now time to step back and summarise all that we have learned up to now. The various elementary particles, together with the Higgs boson, and the three fundamental interactions that admit a quantum description, constitute the *Standard Model (SM) of particle physics*. This is arguably one of the most powerful scientific theories even constructed by humankind. Based on the combination of quantum theory, special relativity, and electromagnetism, the SM allows a high precision description of a wide variety of different phenomena. So in order to present a summary of what we have learned so far about the Standard Model of particles physics, we could highlight the following points:

- The electric and magnetic forces can be described within the framework of quantum theory: *Quantum Electrodynamics*.
- The strong nuclear force can also be described within the quantum theory framework: *Quantum Chromodynamics*.
- The *weak nuclear force* seems inconsistent with quantum theory, unless we introduce the Higgs boson to give mass to the W and Z bosons, as well as to all other elementary particles.
- All subatomic phenomena can thus be described by a unified theory of elementary particles and their interactions: *the Standard Model*.
- The only of the four fundamental forces that is not described (yet) by quantum theory is *gravity*.

Despite all of its successes, the Standard Model cannot be considered a *fully satisfactory theory of elementary particles*, since it leaves unexplained a number of important aspects of the microscopic world. In the next and final part of the course, we explain what are these limitations of the Standard Model and briefly present some of the proposed avenues towards an even better fundamental theory of Nature.

5 Beyond the Standard Model of particle physics

In this last part of the course we will discuss a number of open questions and shortcoming within the Standard Model, such as the nature of dark matter and dark energy, or the asymmetry between matter and antimatter. Naturally, this will lead us to discuss the interplay between elementary particles and cosmology, that is, between the smallest and the largest length scales in the Universe. We will also briefly present some of the possible avenues that are currently explored to build a new theory of the fundamental particles and interactions of Nature beyond the Standard Model.

Learning goals of the lecture:

(a) To understand that the Standard Model is not a complete theory, and that it has important shortcomings that also connected with open problems in cosmology.

(b) To become familiar with some of the proposed theories that aim to improve these shortcomings and build an improved theory of elementary particles beyond the Standard Model.

(c) To be able to identify which measurements or experiments are more promising to find develop this beyond the Standard Model theory.

5.1 The shortcomings of the Standard Model

Up to this point we have presented the Standard Model as if it was a fully complete theory of the fundamental interactions between elementary particles. And though certainly the Standard Model is extremely successful in describing a wide array of different phenomena with astonishing precision (one could argue that the SM is the most successful physical theory ever constructed) there are a number of important questions that are left unanswered. In this lecture we discuss the drawbacks and limitations of the Standard Model, emphasizing that we still don't know how to address these limitations with an even better (and deeper) theory.

What are some of this open questions?

- Why do we have three copies (*families/generations*) of quark and leptons, identical in all their properties except to their masses?
- Is there an underlying principle that determines the mass patterns of quarks and leptons? Why the difference between the lightest quark (up) and heaviest (top) quark is as large as a factor of $m_t/m_u \simeq 10^6$?
- The Universe contains five much times *dark matter* (matter that does not interact electromagnetically) than normal visible matter. The Standard Model does not contain any suitable candidate to act as this dark matter.
- The Universe is undergoing a period of accelerated expansion driven by some form of dark energy. Again, in the Standard Model there is no mechanism that explains this dark energy.

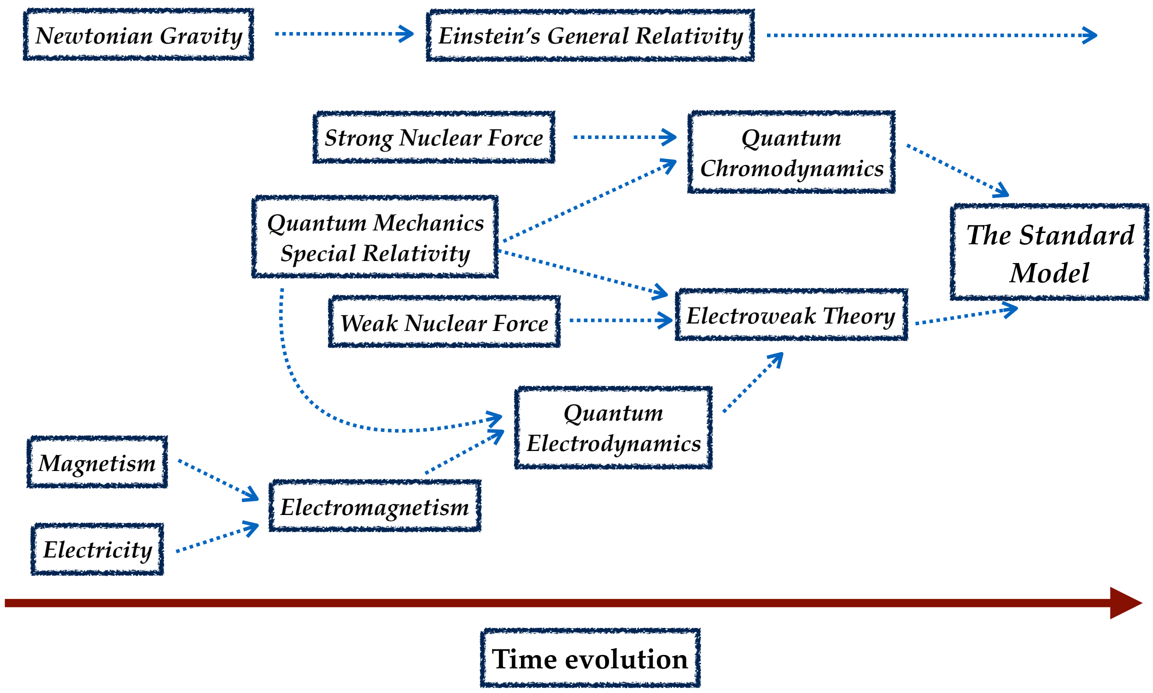

Figure 5.1. Schematic representation of the time evolution of our understanding of the various fundamental interactions and their subsequent unification into the Standard Model of elementary particle physics.

- Neutrinos are massless in the Standard Model, but recent measurements have demonstrated that they have a small yet finite mass, $m_\nu \neq 0$. This opens a number of interesting questions, such as if the Higgs mechanism also gives mass to neutrinos, or there is some altogether different mechanism.

In the following we discuss these open questions in turn, and highlight briefly some of the possible avenues to overcome this limitations and develop and improved theory of the fundamental forces and interactions.

5.2 The unification of all fundamental forces

In Sect. 4 we have discussed the properties of the three fundamental interactions that are relevant to describe the world of elementary particles: *electromagnetism*, the *weak* and the *strong* nuclear forces. An obvious question is, why *these three forces*, and not say two or ten? Perhaps there is a unique underlying interaction, of which these three forces (and perhaps even also gravity) are different manifestations? The idea has certain appeal if we recall that the history of physics has many instances of such *unification of seemingly unrelated phenomena*, such as celestial and earth mechanics with Newton and electricity and magnetism with Maxwell.

Actually, we already know that electromagnetism and the weak interactions are actually two aspects of the same underlying phenomenon, the *electroweak interaction*. The electromagnetic and weak interactions appear very differently at low energies because the masses of their corresponding force carries (the photon and the W, Z bosons) are very different, and thus the *range* of these interactions is also very different (very long and very short range interactions, respectively). But if we go to very high energies, in the sense that $E \gg m_Z$, then we observe the electromagnetic and weak interactions *unify* into a single electroweak interaction. So actually the Standard Model contains only *two* fundamental interactions: the strong and the electroweak forces.

In Fig. 5.1 we provide an schematic representation of the time evolution of our understanding of the various fundamental interactions and their subsequent unification into the Standard Model of elementary particle physics. Gravity, electricity, and magnetism were known since ancient times, but only around the 17th century a mathematical framework to describe these interactions that was able to describe phenomena was developed. Electricity and magnetism where unified into the theory of electromagnetism at the end of the 19th century following the work of Maxwell and several others.

In the beginning of the 20th century, the strong and weak nuclear forces were discovered. The combination of quantum mechanics and special relativity lead to the formulation of Quantum Chromodynamics and of the Electroweak Theory for the strong, weak, and electromagnetic interactions respectively. The strong and electroweak interactions constitute the Standard Model of particle physics, which is as of today, as discussed in the previous section, our best theory for the world of elementary particles.

This trend towards unification might suggest that by going to even higher energies the strong and electroweak interactions could unify into a single fundamental force. This seems difficult, since the *relative strengths* of these two interactions are rather different, so how can they arise from the same underlying interaction? Here a crucial consequence of quantum mechanics is that the strength of a specific interaction is not fixed, but actually depends on the *characteristic energy scale* of the interaction. For instance, in the specific case of the strong interaction, its coupling constant *decreases* as we increase the energy, while for the electroweak force, their couplings *increase* at higher energies.

Let us quantify a bit more this point. In both QED and QCD, the value of the coupling constant $\alpha(Q)$ depends on the energy Q of the process as follows:

$$\alpha(Q) = \frac{1}{b_0 \log Q^2/\Lambda^2} \,, \tag{5.1}$$

where Λ is some reference scale which is introduced in order to define the coupling constant in terms of experimental measurements. The *running* of the coupling constants then is then determined by the values of b_0 and Λ. For QCD one has

$$b_0 = \frac{33 - 2n_f}{12\pi} \,, \quad \Lambda \simeq 300 \text{ MeV} \,. \tag{5.2}$$

while for QED instead one has

$$b_0^{\rm QED} = -\frac{4n_f}{12\pi} \,, \quad \Lambda \simeq 10^{90} \text{ GeV} \,, \tag{5.3}$$

with n_f being the number of quarks with mass $2m_q \geq Q$. Since b_0 has a different sign, the sign of the running of the two coupling constants will be different. For example, in the case of QCD we have that at $Q = 1$ GeV, $\alpha(Q) \sim 0.3$, but at $Q = 1$ TeV, $\alpha(Q) \sim 0.1$ instead. Note that when $Q \to \Lambda$ the coupling constant becomes infinite: this is called the *Landau pole* and indicates where the perturbative description of the theory ceases to be valid.

To further illustrate this behaviour, in Fig. 5.2 we show the inverse of the characteristic strength of the electromagnetic, weak and strong forces as a function of either the energy (in GeV) or the spatial resolution

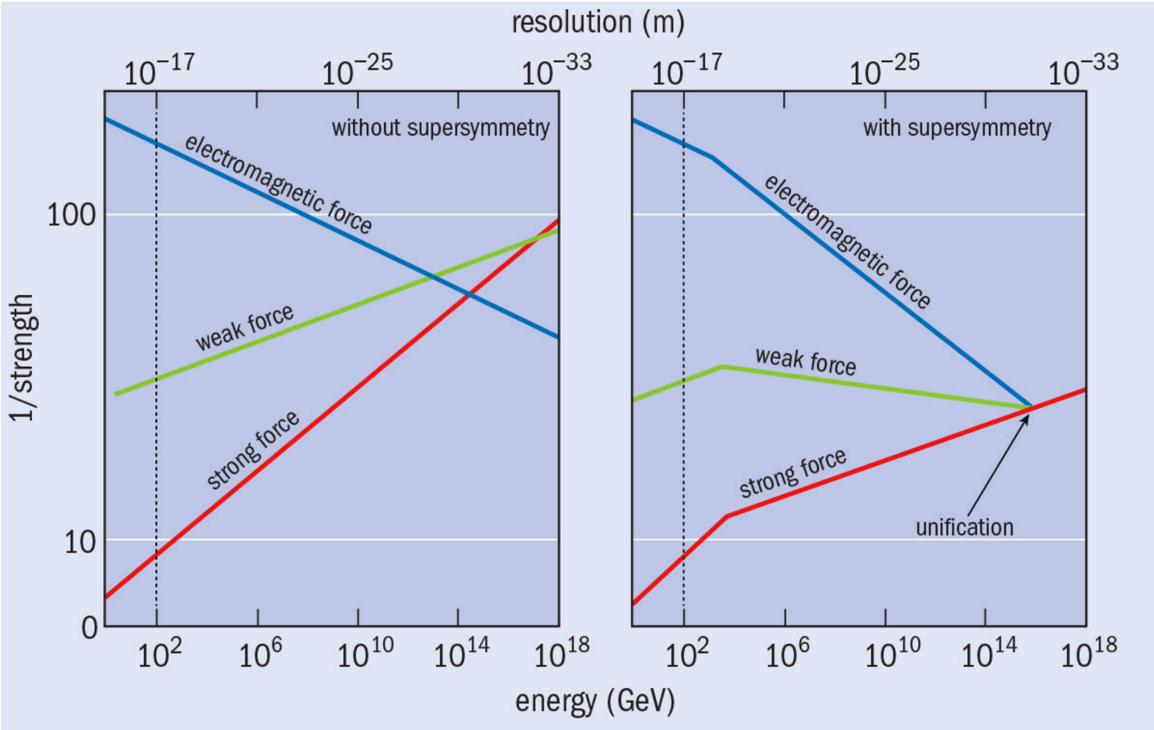

Figure 5.2. The inverse of the characteristic strength of the electromagnetic, weak and strong forces as a function of either the energy (in GeV) or the spatial resolution (in m) of the process. We show the results corresponding both to the Standard Model (left plot) and to the Standard Model extended to include low-scale supersymmetry (right plot).

(in meters) of the process. We show the results corresponding both to the Standard Model (left plot) and to the Standard Model extended to include low-scale supersymmetry (right plot), which will be discussed below. In the Standard Model, we find that the three couplings become similar (but not identical) at energies of around $E \simeq 10^{14}$ GeV, corresponding to distances of the order of $r \simeq 10^{-29}$ m. In the case of the SM extended with supersymmetry, the unification of the coupling constants of the fundamental interactions is much better, and takes place at an energy scale of $E \simeq 10^{15}$ GeV, which is the so-called *Grand Unification Scale*.

So in principle it is possible that under specific conditions, as shown in Fig. 5.2, the electromagnetic, weak, and strong interactions unify into a unique force, with a common value of the coupling constant. Note that however in this picture gravity is still excluded. On the other hand, so far we do not have experimental evidence of this unification, which might require probing energies much higher that those that are currently within our reach. All tests performed so far looking for a *Grand Unified Theory* have failed, such as the searches for the *decays of the proton*, which is predicted by such theories.

5.3 Dark matter

From a variety of astronomical and cosmological measurements we know that the *visible mass* of the universe (for example, the mass of visible stars) is only a *small fraction* of the total mass. We call *Dark Matter* this additional source of matter, which does not interact via electromagnetism (hence it is *dark*) and thus that we cannot see directly, but can only infer indirectly via its gravitational effects

As an example of this, in Fig. 5.3 we show a schematic representation of the *galactic rotation curves*, namely the dependence of the velocity $v(R)$ of its stars as a function of the distance R with respect to the galactic center. The dotted curve is the expectation from the visible mass of the galaxy, while the solid line is the best fit to the experimental data. Clearly the observed velocity profile is very different as compared to the expectations based on visible matter, suggesting the presence of additional, *dark*, sources of matter in the galaxy, which increase the rotational velocity $v(R)$ for large R.

Could this mysterious *Dark Matter* be composed by one or more of the Standard Model particles? Let us see if in the SM we have any particle that satisfies the properties of a suitable DM candidate:

- DM is *electrically neutral*, since it does *not emit light*. Therefore, a candidate for DM should have $Q = 0$. This eliminates all the quarks as well as the charged leptons, $e^{\pm}$, $\mu^{\pm}$ and $\tau^{\pm}$, and the charged weak bosons $W^{\pm}$.

- DM candidates should have a *non-zero mass* $m_{\rm DM} \neq 0$. This further eliminates the photon γ and the gluon g, both of which are massless.

- A suitable DM candidate should be *stable on cosmological scales* in order to seed galactic structure. This eliminates the Z^0 and h bosons, both of which have very short lifetimes.

The only SM particles that satisfies these three conditions (being electrically neutral, massive, and stable on cosmological time-scales) are the neutrinos ν. However, due to a number of reasons which we don't have the time to discuss here, the neutrinos cannot be the explanation for Dark Matter (at least for the greatest part of it). So the nature of DM is still basically unknown to us.

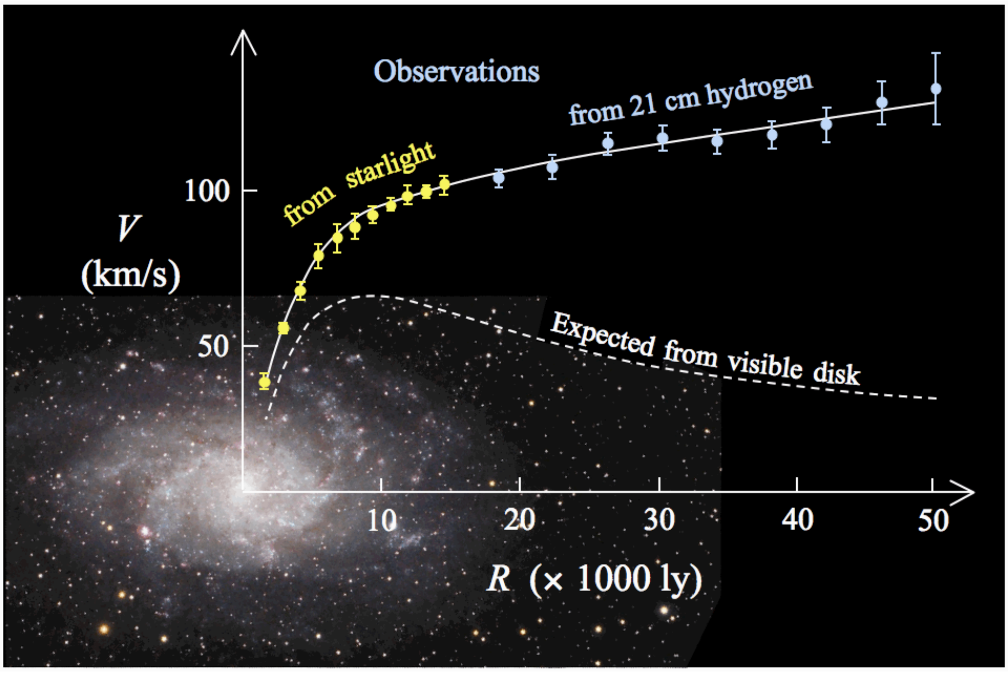

Figure 5.3. Schematic representation of the *galactic rotation curves*, namely the dependence of the velocity v of its stars as a function of the distance R with respect to the galactic center. The dotted curve is the expectation from the visible mass of the galaxy, while the solid line is the best fit to the experimental data.

It is illustrative to remind ourselves how the velocity profile $v(R)$ in Fig. 5.3 provides information on the galaxy's matter content. To begin with, the integral form for Gauss's law for gravity gives

$$\int_{\delta V} \mathbf{g} \cdot d\mathbf{A} = -4\pi G M(V)\,, \tag{5.4}$$

where $\mathbf{g}$ is the gravitational field, $M(V)$ is the mass enclosed in the volume V and δV is its surface. For circular motion, the acceleration of a star will then be given by

$$a = \frac{v^2}{R} = G\frac{M(R)}{R^2} \tag{5.5}$$

so that the velocity profiles of a star at a distance R of the galactic center is

$$v(R) = \sqrt{\frac{GM(R)}{R}}\,, \tag{5.6}$$

For a galaxy approximated by a cylinder of height h, we have that the enclosed mass $M(R)$ is

$$M(R) = \int_0^R dr\,\rho(r) 2\pi r h\,, \tag{5.7}$$

with $\rho(r)$ being the *galactic mass density*. Therefore, if $\rho(r) \sim 1/r^n$, we see that

$$M(R) \sim \int_0^R dr\, r^{-n+1} \simeq R^{-n+2}\,, \tag{5.8}$$

and therefore the velocity profile will be proportional to

$$v(R) \sim \sqrt{\frac{M(R)}{R}} \sim R^{(-n+1)/2}\,, \tag{5.9}$$

so a measurement of $v(R)$ provides direct information on n, the exponent of the galactic mass density. For example, if $n = -1$ $v(R) \sim R^0$ would be independent of the distance to the galactic center R.

In Fig. 5.4 we show an schematic representation of our current understanding of the matter/energy content of the Universe. The known particles of the Standard Model add up to meager *5% of this matter/energy content*. There are around 5 times more *dark matter* than normal matter: in other words, for each galaxy that we see in the Universe, there exists the equivalent of five galaxies composed of Dark Matter. The nature and properties of this mysterious dark matter, which certainly cannot be explained by the Standard Model of elementary particles, is now being actively searched for. Dark Matter for example could be produced at high-energy colliders such as the LHC. Direct searches for DM particles are being pursued in big underground experiences, shielded from all contamination from normal matter. Finally, hints for DM annihilation are also searched for in astrophysical measurements.

From Fig. 5.4 we also see that the so-called *Dark Energy* constitutes the majority of the matter/energy content of the universe. The nature of this Dark Energy is also unknown: we can only infer its presence via its indirect effects, first and foremost driving the present phase of cosmic acceleration of the Universe. Information about dark matter and dark energy can be obtained from astronomical measurements such as

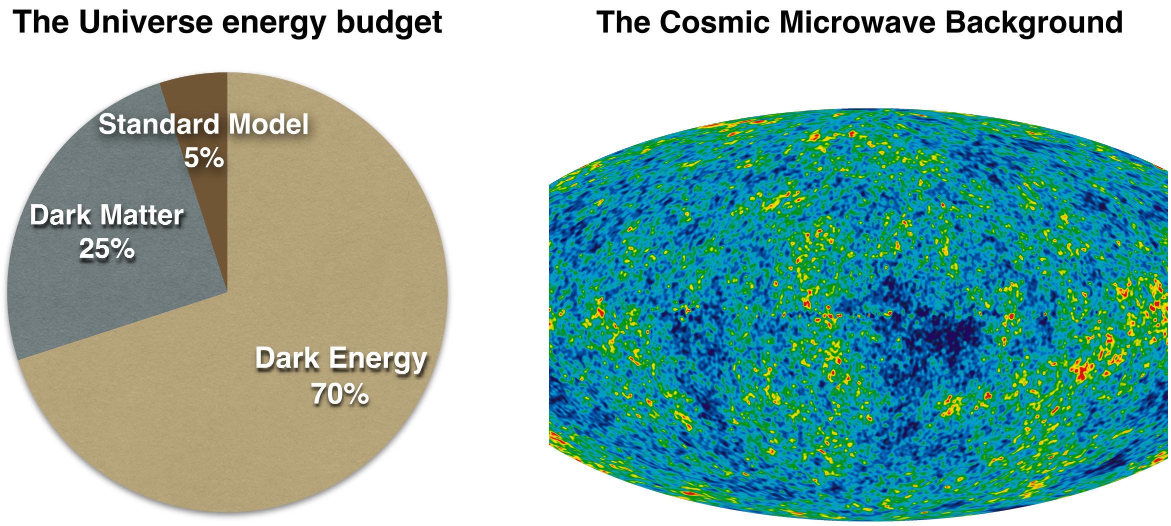

Figure 5.4. Left plot: schematic representation of our current understanding of the matter/energy content of the Universe. The known particles of the Standard Model add up to meager 5% of this matter/energy content. There are around 5 times more dark matter than normal matter. Then dark energy, driving cosmic acceleration, makes up most of the energy content of the universe. Right plot: information about dark matter and dark energy can be obtained from astronomical measurements such as those of the Cosmic Microwave Background, a kind of after glow of the Big Bang, that provides a snapshot of the very early Universe.

those of the Cosmic Microwave Background, a kind of after glow of the Big Bang, that provides a snapshot of the very early Universe. A recent measurement of this Cosmic Microwave Background is shown in Fig. 5.4.

5.4 Quantum gravity

As shown in Fig. 5.1, while we now have a unified description of all known elementary particles and of their interactions by means of the strong, weak, and electromagnetic forces within the highly successful framework of the Standard Model, we still don't really know how to fit gravity in the picture. The main limitation is that we still have not managed to construct a quantum theory of gravity, or to be more precise, a quantum version of Einstein's theory of *General Relativity*.

As we discussed in Sect. 4.1, the effects of the gravitational interaction are really tiny in the world of elementary particles. This has one important corollary: testing a theory of quantum gravity is difficult since experimental measurements sensitive to quantum effects of the gravitational force are, for the time being, very far from our reach. This statement can be quantified as follows. The energy scale at which the quantum effects of gravity will become important is given by *Planck's energy*, a fundamental constant of Nature that is determined by Planck's constant h, Newton's gravitational constant c and the speed of light c as follows:

$$E_P = \sqrt{\frac{\hbar c^5}{G}} \simeq 10^{19}\ \text{GeV}\,. \tag{5.10}$$

If we recall that the highest energies that we can achieve at man-made particle colliders is $E_{\rm lhc} \simeq 10^4$ GeV, we see that we are still *15 orders of magnitude* in energy below Planck's energy. Therefore, we are still a big way behind before we will be able to experimentally study quantum effects of gravity.[16]

[16] Actually, there are a number of theoretical frameworks in which the quantum effects of gravity can be accessed with

How large should a particle collider be if we want to be able to perform particle collisions at the Planck's energy E_p, Eq. (5.10)? To answer this question, we note that in circular colliders such as the LHC, the maximum energy of an ultra-relativistic particle is determined by the radius of the ring R and by the average magnetic field B of bending magnets as follows

$$E_{\rm max}\,[{\rm GeV}] = 0.3 \times B\,[{\rm T}] \times R\,[{\rm m}] \ . \tag{5.11}$$

For instance, the LHC has $E_{\rm max} \simeq 7$ TeV (that is, the maximum energy of its proton beam) and $R \simeq 4.3$ km (corresponding to a circumference of 27 km), implying that its magnets can reach fields as high as roughly $B \simeq 6$ Tesla. If we now assume that we can accelerate particles up to Planck's energy, Eq. (5.10), in order to probe the quantum effects of gravity, and optimistically assuming that we can produce magnets 100 times more powerful than those used now at the LHC, we find that we need to build a collider with circumference L as large as

$$L = 2\pi R = 2\pi \times \frac{E_P}{0.3 \times 100 \times B_{\rm LHC}} \simeq 10^{17} \ {\rm m}\,. \tag{5.12}$$

which corresponds to around 10^6 Astronomical Units (1 AU = average distance from the Earth to the Sun), roughly the *distance between the Sun and its nearest start*, Proxima Centauri. Therefore, unless a *galactic collider* is built, or we develop brand new methods to accelerate particles at the highest energies (such as Plasma Wakefield Acceleration) it will be very challenging to experimentally test quantum gravity, at least with colliders.

Planck's energy E_P has also distance and mass counterparts, which again reflect the typical distances and mass scales for which the effects of quantum gravity becomes relevant. In the case of distance scales, Planck's length L_P is given by

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \simeq 1.61 \times 10^{-35}\,{\rm m}\,, \tag{5.13}$$

again much smaller than any distance that we can probe now or in the foreseeable future. Recall that the proton radius us about $R_p \simeq 10^{-15}$ m, and therefore Planck's length is $\sim 10^{-20}$ times smaller than the proton radius. With present and near-future particle colliders, we can at most probe distances as small as $\sim 10^{-20}$ meters, much larger than Planck's length Eq. (5.13), again preventing us from testing for the time being the quantum effects of gravity.

Despite the challenges is experimentally testing quantum gravity, this has not prevented theorists to developing new frameworks to unify the Standard Model with a quantum theory of gravity. There are a numbers of proposals that are being actively researched, such as string theory, loop quantum gravity, asymptotically free gravity, causal sets dynamical triangulation, and may others. In string theory for instance, elementary particles are replaced by replaced by finite-size one-dimensional strings, which avoids some of the problems affecting the quantum version of gravity. The different properties of each type of string then determine particle properties such as its mass and its spin. In Fig. 5.5 we show how a high-energy collision between two elementary particles is represented in string theory: two closed strings fuse into a single string which then decays into two closed strings again. Beyond its potential to represent a quantum theory of

present-day technology. Unfortunately for the time being these gravitational quantum effects have proven to be very elusive.

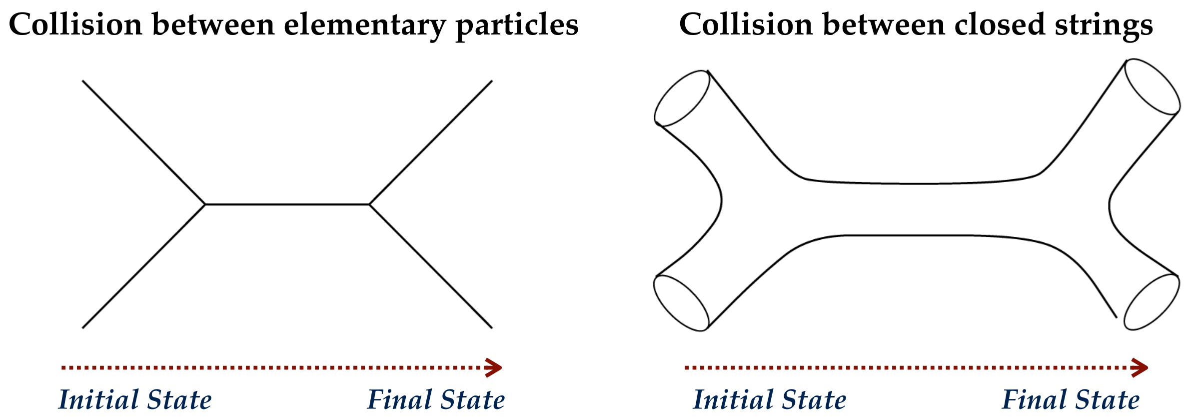

Figure 5.5. In string theory, elementary particles are replaced by finite-size one-dimensional strings. Here we show how a high-energy collision between two elementary particles is represented in string theory: two closed strings fuse into a single string which then decays into two closed strings again.

gravity, string theory has found applications to many other fields from pure mathematics to condensed matter systems and heavy ion collisions.

5.5 What is next?

At this point we have reached the end of our journey through the world of elementary particles. Let us know step back and recap what we have learned during these four weeks traveling around the particle zoo:

- A wide variety of extremely different phenomena can be explained in a rather economical way by just a handful of elementary particles and interactions: the *Standard Model of particle physics*.
- In addition to the well-known gravitational and electromagnetic forces, we have seen that two other interactions are present in the world of elementary particles: the *strong nuclear force* and the *weak nuclear force*.

  Their effects are visible in our everyday life: for instance, the Sun emits light thanks to nuclear reactions mediated by the weak force in its interior, and the fact that the protons inside atomic nuclei do not repeal themselves and that matter is thus stable is a direct consequence of the properties of the strong force.
- Even without entering into the details of how each of the three fundamental interactions work, we have seen that there exists a number of *symmetries* and *conservation principles* that determine very tightly whether or not a given particle reaction is physically allowed.
- The world of elementary particles is much richer than what normal matter, composed by only protons, neutrons, and electrons, would seem to imply. These additional particles have crucial implications even if their existence is fleeting.

  For instance, the Higgs boson is responsible to give mass to all elementary particles, and the fact that there are three generations of quark and leptons might be crucial to explain the asymmetry between matter and antimatter in the Universe, and so on.

Despite all the successes of the Standard Model, we have also seen in this last part of the course that this theory is ultimately incomplete, since it leaves open a number of crucial questions such as the nature of dark matter and dark energy, the origin of its flavour structure, the possible unification of the three fundamental forces among them and with gravity, and the explanation for the observed origin of the matter-antimatter asymmetry in the Universe. With this motivation, particle physicists are currently working very hard, both from the theoretical and experimental points of view, to unveil the properties of a better, more fundamental, theory of elementary particles beyond the Standard Model.

To conclude, let me emphasize that the field of elementary particles is a extremely alive and active area of research. There have been a number of breakthrough discoveries in the last 25 years, and only the future can tell which further discoveries lie ahead. Specifically, in the last 25 years the high-energy physics community has:

- Discovered the *top quark*, the heaviest of all known elementary particles, with a mass as large as in Ytterbium ${}^{173}_{70}$Yb atom.
- Discovered the *Higgs boson*, responsible for giving mass to all elementary particles and to the weak force carries W, Z, which completes the Standard Model.
- Found evidence for neutrino oscillations, indicating that *neutrinos have mass*, that is, that neutrinos can change its flavour. Neutrino masses are one possible gateway to new physics beyond the SM, and imply that the *individual leptonic quantum numbers* are not conserved at large distances.
- Mapped the properties of *Dark Matter* to an unprecedented level of precision from a variety of experiments, and excluded a wide range of models for DM candidates.
- Identified the *accelerated expansion of the universe*, driven by the mysterious Dark Energy for which we do not have currently any explanation.
- Discovered *gravitational waves*, opening a new window to the universe, mapping extreme astrophysical events such as black hole and neutron star mergers.

As a last comment, let me mention that in this course we have only been able to covered a very reduced snapshot of the richness and fascination of the world of elementary particles. The interested student is encouraged to take a look at the suggested further reading to learn more about the topics we have discussed here.

5.6 Summary

Let us summarize what we have learned in this final part of the course about the limitations and possible forward avenues with the Standard Model:

- The Standard Model leaves open many important questions about the world of elementary particles such as the origin of the neutrino masses, the nature of dark matter, or the reason for the observed asymmetry between matter and antimatter in the Universe.
- Some of this problems have a direct connection with astronomy and cosmology, for example, all experimental information that we have dark matter and dark energy comes from looking at the universe rather than from investigating the microcosm.
- While there are several proposed theories to go beyond the Standard Model, we are still far from establishing the next fundamental theory of elementary particles.
- Some of these theories that improve the SM can be tested at present and near-future colliders, such as the LHC, as well as in other experiments. Other bSM candidates theory, such as String Theory, are still very far from any possibility of experimental verification.

A Physical constants and conversion factors

In this appendix we collect a list of useful physical constants and conversion factors that might be useful to follow the derivations in the lecture notes as well as to carry out the homework exercises. Additional values of conversion factors and physical constants relevant for the study of elementary particles can be found *i.e.* in the Particle Data Group (PDG) webpage: http://pdg.lbl.gov/2017/reviews/rpp2017-rev-phys-constants.pdf.

A.1 Conversion factors

To begin with, we provide here a series of conversion factors to relate physical magnitudes in different units.

- From Joules to eV: 1 J $= 6.242 \times 10^{18}$ eV.
- From MeV to Joules: 1 MeV $= 1.609 \times 10^{-13}$ J.
- From the atomic mass unit u to kg: 1 u1.661×10^{27} kg.
- From kilograms to GeV (in natural units): 1 GeV $= 1.783 \times 10^{27}$ kg.
- From MeV/c^2 to kilograms: $1\ MeV/c^2 = 1.79 \times 10^{-30}$ kg.
- From GeV to meters (in natural units): 1 GeV $= 5.06 \times 10^{15}$ m^{-1}.

A.2 Physical constants

Next we provide a list of useful physical constants that are used through the lecture notes and that are sometimes needed to carry out some of the exercises in the problem sets.

- The speed of light in vacuum: $c = 2.99792458 \times 10^8$ m/s.
- Electron mass: $m_e = 9.11 \times 10^{-31}\,\mathrm{kg} = 0.511$ MeV.
- Electron charge: $Q_e = 1.6 \times 10^{-19}$ C.
- Proton mass: $m_p = 1.67 \times 10^{-27}\,\mathrm{kg} = 938.27$ MeV.
- Coulomb's constant: $k = 9 \times 10^9$ Nm^{2}C^{-2}.
- Newton's gravitational constant: $G = 6.673 \times 10^{-11}$ Nm^{2}kg^{-2}.
- Planck's constant: $h = 6.626 \times 10^{-34}$ m^{2}kg/s $= 4.136 \times 10^{15}$ eV.
- Reduced Planck's constant $\hbar = 1.0545 \times 10^{34} = 6.582 \times 10^{16}$.
- Planck's length: $L_p = 1.616 \times 10^{35}$ m.

B Homework exercises I

This set of exercises should be completed and submitted via **Brightspace** by **Wednesday 17th of January 2018** before 17h.

1) We have a sample of 1.49 μg of pure ${}^{13}_{7}$N, characterized by a half-life of $\tau = 600$ s.
Answer the following questions:
a) How many atoms are there in this sample of pure nitrogen?
b) How large is the activity at $t = 0$?
c) How large is the activity after one hour?
d) How much ${}^{13}_{7}$N do we still have after one hour?
e) Assume that ${}^{13}_{7}$N undergoes a beta decay of the form ${}^{13}_{7}\mathrm{N} \to \mathrm{X} + e^+ + \nu$. What is X here? Is this reaction actually possible?

2) Consider the radiative decay of polonium-210: ${}^{210}_{84}\mathrm{Po} \to \mathrm{X} + \alpha$.
a) What is X in this reaction, and why?
b) Determine the energy Q released in this decay (in MeV), using that $m_{\mathrm{Po}} = 209.93676$ u, $m_{\mathrm{X}} = 205.92945$ u, and $m_\alpha = 4.0012$ u.
c) Compute the kinetic energy that the α particle has in this decay process. Make use of the conservation of energy and momentum, and assume that the Po nuclei was at rest before decaying. Assume also Newtonian (non-relativistic) mechanics.
d) Compute the velocity of the radiated α particle. Should we worry about relativistic effects? Why?

3) What is the energy that we need to supply to a proton in order to accelerate it from $\beta = 0$ to $\beta = 0.9$? And from $\beta = 0.9$ to $\beta = 0.99$? And from $\beta = 0.99$ to $\beta = 0.999$? Is is possible to accelerate then the proton up to $\beta = 1$? Why?

4) An electron in a cathodic rays tube, starting from rest, experiences a potential difference of $\Delta V = 5 \times 10^4$ V between an initial position and a screen that can measure its position upon impact. With which speed does the electron reach the screen? Perform the calculation using both classical mechanics and relativistic mechanics. How do the results change if now the potential difference is increased up to $\Delta V = 2 \times 10^5$ V? And to $\Delta V = 10^6$ V? For which values of the potential difference is the classical approximation sensible and for which others one needs the relativistic expressions?

5) An unstable particle at rest, denoted by X, decays into two lighter particles A and B. We know that particle A has a mass of 6.67×10^{-27} kg and a speed of $\beta = 0.6$. We also know that particle B has a mass of 1.67×10^{-27} kg. Determine the speed of particle B. Compute also the mass of the initial particle X.

6) Consider an anti-proton, traveling with momentum 1 TeV/c in the z direction.
a) Assume that this anti-proton collides head-on with a proton carrying the same momentum but traveling in the opposite direction. Assume that the result of this collision is a new particle. Using energy-momentum conservation, compute the four-momentum of this new particle. What is its mass?
b) Same situation as above, but now with the proton standing at rest.
c) Discuss what are the advantages and advantages of the two configurations described above.

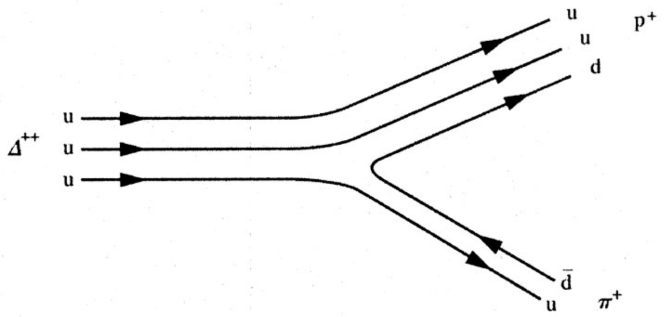

Figure C.1. The decay $\Delta^{++} \to p + \pi^+$ represented within the constituent quark model framework.

C Homework exercises II

This set of exercises should be completed and submitted via **Brightspace** by **Wednesday 24th of January 2018** before 17h.

1) Consider that we have detected photons and neutrinos, both with an energy of 1 MeV, coming from a supernova explosion which is at a distance of 10^5 light years from us. We measure that the arrival time on Earth of these photons and neutrinos differ by less than 10 seconds. Given that photons are massless, derive from this observation an upper bound on the neutrino mass.

2) Construct the four-momentum vector $(E/c, \vec{p})$ for the following particles. Verify explicitly in all cases that the mass-shell condition holds.
a) A proton at rest.
b) A neutron with a total energy of 1500 MeV, moving the z direction.
c) An electron with kinetic energy of 500 keV, moving in the z direction.
d) A proton moving in the z direction with speed $\beta = 0.9$.
e) A photon with wavelength $\lambda = 10^{-15}$ m, moving in the z direction.

3) Consider the Φ meson, see Table 3.2. In the framework of the quark model, can you give a possible quark content of this meson? Is this choice unique? This Φ meson has a width (a fundamental uncertainty in its rest mass) of around 4.3 MeV/c^2. What can we say about its expected lifetime?

4) Consider the following decay modes of the tau leptons, where X represents one or more unknown particles. Indicate which of these decay modes are actually possible, and if they are not possible why is so. For the allowed decays, indicate what particle(s) X stands for.

a) $\tau^- \to e^- + X$
b) $\tau^- \to \mu^- + X$
c) $\tau^+ \to \pi^+ + X$
d) $\tau^+ \to p + X$
e) $\tau^+ \to \nu_\mu + X$

5) The decay $\Delta^{++} \to p + \pi^+$ can be interpreted in terms of the constituent quark model as indicated in Fig. C.1.

Based on the example of Fig. C.1, draw the corresponding diagrams, taking into account the quark composition of the initial and final-state hadrons, for the following processes:

a) $\pi^0 + p \to n + \pi^+$
b) $\Phi \to K^+ + K^-$
c) $\pi^- + p \to \Lambda^0 + K^0$
d) $\Lambda^0 \to p + \pi^-$

In all cases describe any assumption that you might have used. Note that the first three processes are mediated by the strong interaction, and therefore the only allowed lines are those connecting the same quark in the initial and in the final state, and those connecting a quark and an antiquark of the same flavor in either the initial or the final state. Discuss why the last reaction is different and cannot be mediated by the strong interaction.

D Homework exercises III

This set of exercises should be completed and submitted via **Brightspace** by **Wednesday 31st of January 2018** before 17h.

1) In all known interactions between elementary particles, the conservation of electric charge, individual leptonic number, and baryon number applies. Taking these considerations into account, indicate which of the following processes are possible and which are impossible. In each case, provide a motivation for your choice. For those processes which are not possible, indicate the conservation rules that are not satisfied. For the physically allowed processes, discuss which of the three fundamental interactions (strong, weak, and/or electromagnetic) might mediate each process.

a) $\mu^+ \to e^+ + \nu_e$
b) $n \to p + e^- + \bar{\nu}_e$
c) $\pi^- + p \to n + \pi^0$
d) $p \to e^+ + \pi^0 + \nu_e$
e) $\Delta^0 \to p + \pi^-$
f) $p + \mu^- \to n + \bar{\nu}_\mu$
g) $e^+ + e^- \to \nu_\mu + \bar{\nu}_\mu$
h) $\nu_\tau + n \to p + \tau^-$

2) A Σ^- baryon at rest decays into a neutron and a π^-, that is, $\Sigma^- \to n + \pi^-$. Describe this decay process at the quark level, including drawing the corresponding diagram, and discuss which interaction might be responsible for it.

Calculate the energy that is released in the $\Sigma^- \to n + \pi^-$ decay process. Discuss what happens with this energy. Who receives more energy, the neutron or the pion? Compute the speed of the outgoing pion, explaining any assumptions that you might have used.

3) Draw the Feynman diagrams, including the particles that mediate the fundamental interactions, for the following processes:

a) $J/\psi \to e^+ + e^-$
b) $\pi^+ \to \mu^+ + \nu_\mu$
c) $e^+ + e^- \to b + \bar{b}$
d) $b \to c + \tau^- + \bar{\nu}_\tau$

Take into account how quantum numbers change between the initial and final state in order to determine which interaction is mediating the process.

4) Consider a very high energy muon neutrino ν_μ (with $E_\nu \gg 1$ GeV) that is traveling in water. The neutrino interacts with either the proton or the neutron of a water molecule by means of the following reaction, $\nu_\mu + X \to \mu^- + Y$, where X stands by either the proton or the neutron. Write the corresponding Feynman diagrams for these reactions (including the particles that mediate the interactions), and determine what is Y both when X is a proton and when it is a neutron. Which interactions has a higher likelihood to happen, $\nu_\mu + p$ or $\nu_\mu + n$?