Introduction to Elementary Particles (TN2811)

Theory Lecture 4

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Today's lecture

- ☑ **Neutrino oscillations**: theory and practice

- ☑ **Quarks and hadrons**: strongly interacting particles

- ☑ **The quark model**: pinning down hadron spectroscopy

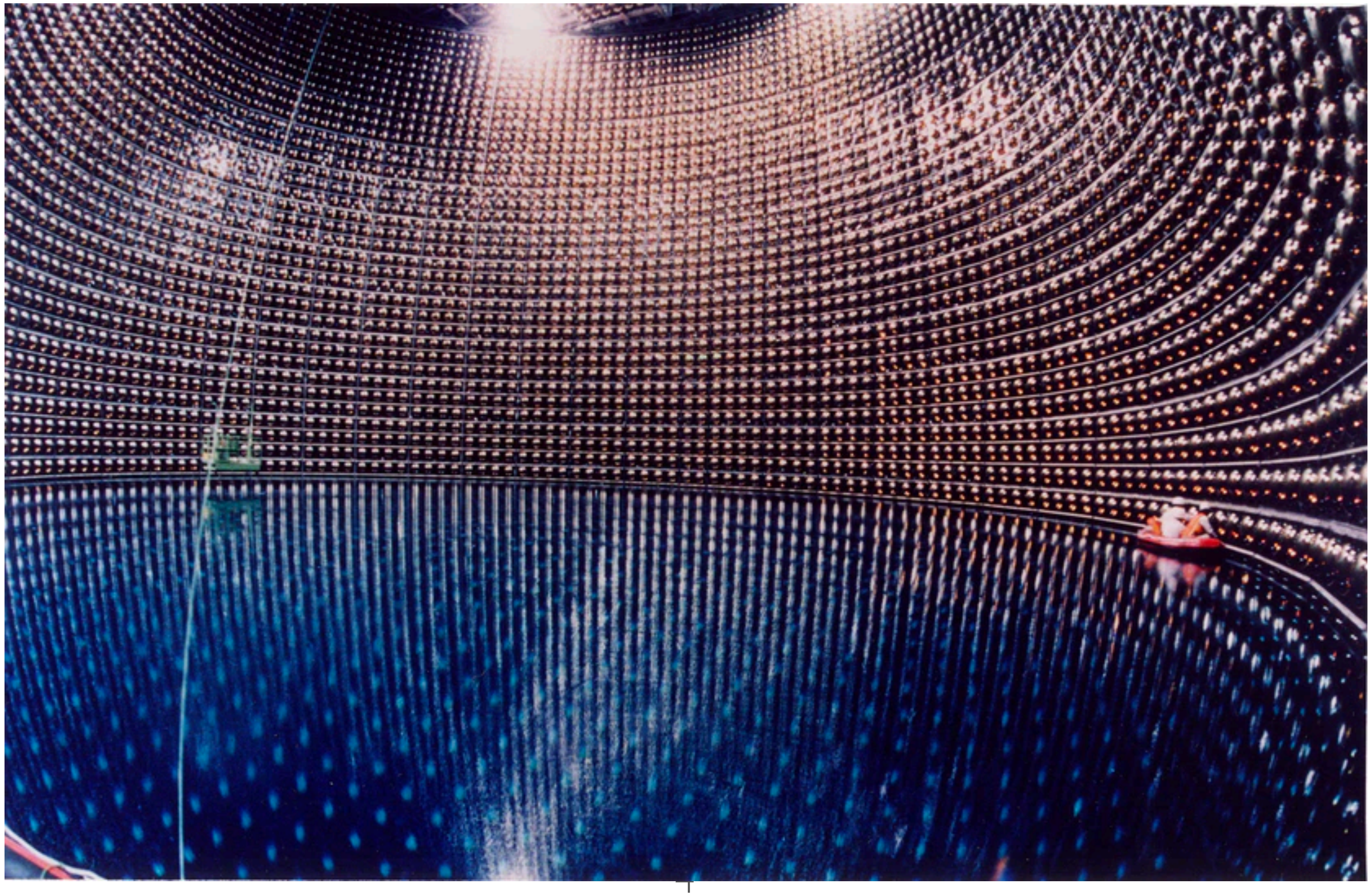
Neutrino oscillations

Leptonic number and neutrinos

- **Neutrinos** are electrically neutral, very light, and weakly interacting particles introduced by Pauli in 1930 to guarantee **energy conservation** in the beta decay process

- Due to their very low interaction rate they are extremely difficult to measure, and huge detectors are required to be able to study then

- These ghostly particles are very abundant in the Universe: every second about **100 trillion neutrinos from the Sun cross your body!**



Leptonic number and neutrinos

In the Standard Model, not only the **total leptonic number *L*** is conserved: also the **individual leptonic numbers** for the electron, muon, and tau are **conserved**

$$(e^-, \nu_e) \quad \rightarrow \quad L_e = +1, L_\mu = 0, L_\tau = 0$$

$$(e^+, \bar{\nu}_e) \quad \rightarrow \quad L_e = -1, L_\mu = 0, L_\tau = 0$$

$$\left(\mu^-, \nu_\mu\right) \quad \rightarrow \quad L_e = 0, L_\mu = +1, L_\tau = 0$$

$$\left(\mu^+, \bar{\nu}_\mu\right) \quad \rightarrow \quad L_e = 0, L_\mu = -1, L_\tau = 0$$

$$(\tau^-, \nu_\tau) \quad \rightarrow \quad L_e = 0, L_\mu = 0, L_\tau = +1$$

$$(\tau^-, \nu_\tau) \quad \rightarrow \quad L_e = 0, L_\mu = 0, L_\tau = -1$$

Only reactions where $\boldsymbol{L_e}$ **,** $\boldsymbol{L_\mu}$ **,** $\boldsymbol{L_\tau}$ are **separately conserved** are allowed

Leptonic number and neutrinos

In the Standard Model, not only the **total leptonic number *L*** is conserved: also the **individual leptonic numbers** for the electron, muon, and tau are **conserved**

In particular neutrinos cannot **spontaneously change flavour,** that is, they cannot **oscillate** between different flavours

$$\nu_\tau \to \nu_e \qquad \Delta L_e \neq 0, \Delta L_\tau \neq 0$$

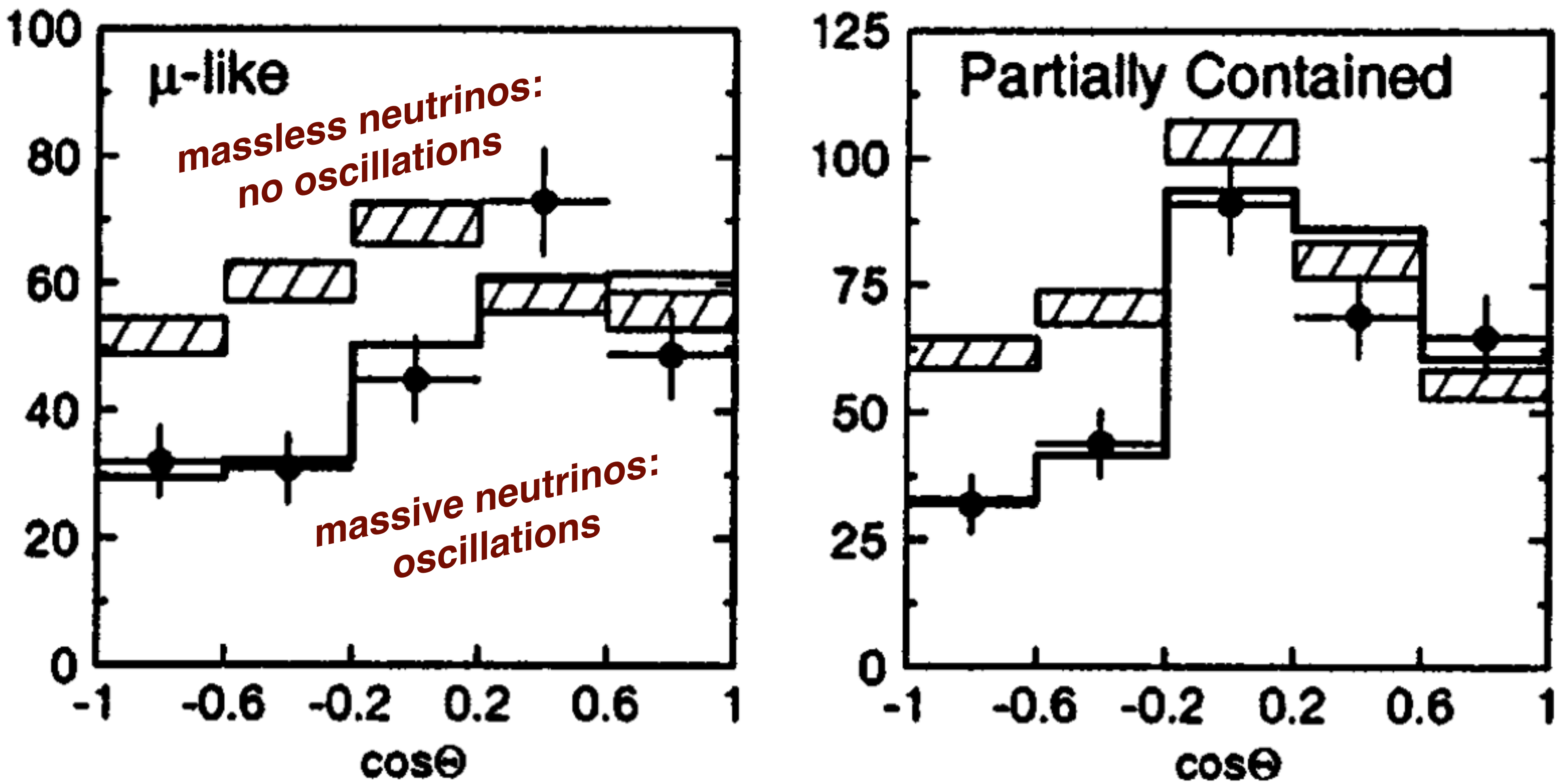
not allowed!

This is a consequence of the fact that in the SM **neutrinos are strictly massless**

Only reactions where L_e , L_μ , L_τ are **separately conserved** are allowed

Neutrino oscillations

In 1998 the discovery of **atmospheric neutrino oscillations** was announced by the Super-Kamiokande experiment: evidence for **new physics beyond the SM!**



$$\nu_\mu \rightarrow \nu_e \rightarrow \nu_\mu$$

other types of neutrino oscillations have also been observed: **neutrinos have mass**

Neutrino oscillations

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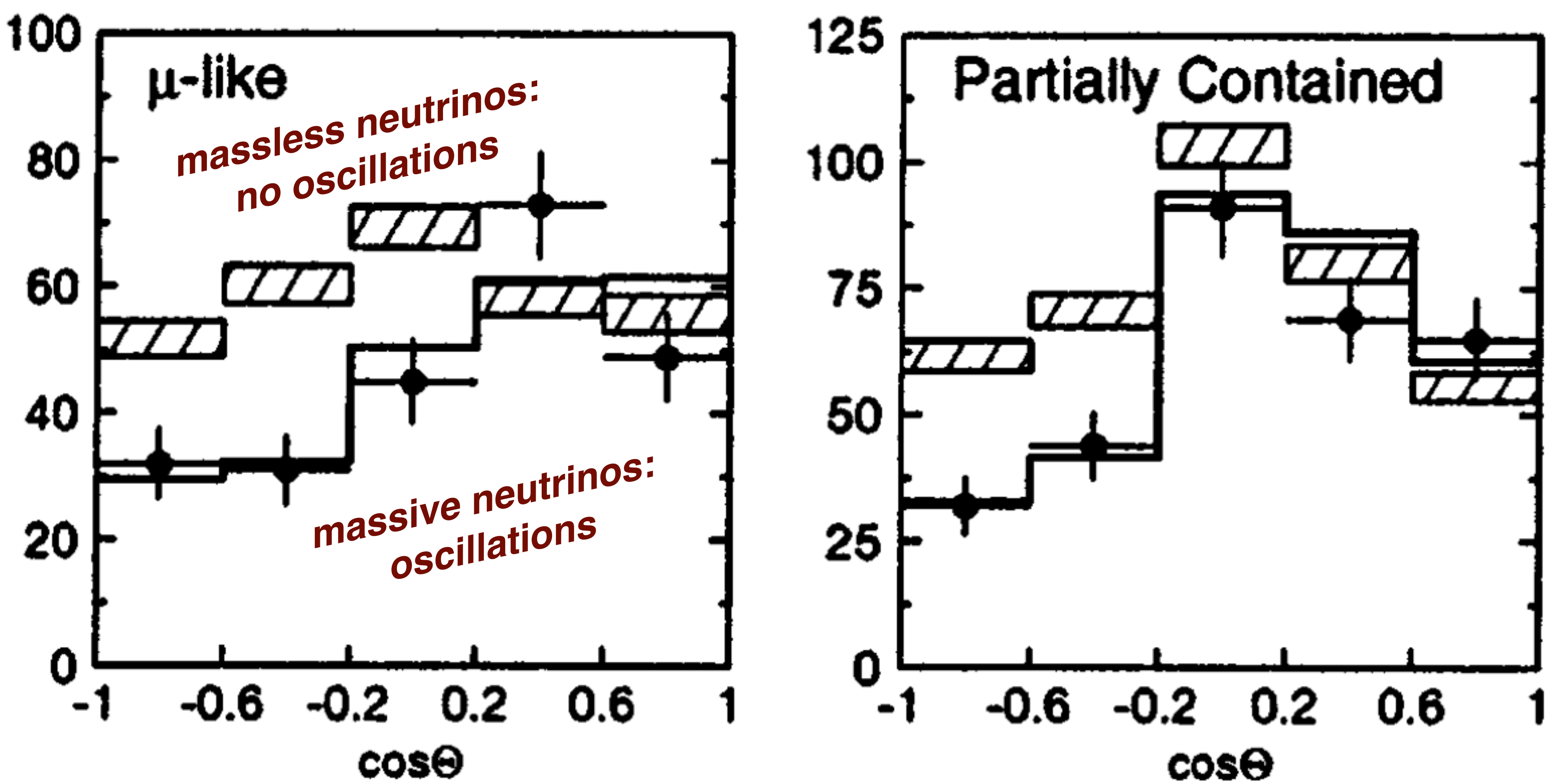

Neutrino oscillations appear only over **large distances:** in scattering reactions, the **individual leptonic numbers** are always still **conserved**

Neutrino oscillations

We can easily compute the effects of neutrino oscillations using **quantum mechanics**

Say we have a propagating neutrino with **mass *m₁***. Its **time evolution** is given by

$$\hat{H}\,|\psi_{\nu_1}(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi_{\nu_1}(t)\rangle$$

Schroedinger eq

$$|\psi_{\nu_1}(t)\rangle = e^{-i\hat{H}t/\hbar}\,|\psi_{\nu_1}(t=0)\rangle$$

We can simplify this by using that the **neutrino mass is tiny** and so they move at **ultra-relativistic velocities** close to c

$$\hat{H} = \sqrt{\vec{p}^{\,2}c^2 + m_{\nu_1}^2 c^4} \simeq |\vec{p}\,|\,c\left(1 + \frac{m_{\nu_1}^2 c^2}{2|\vec{p}\,|^2}\right) \simeq E + \frac{m_{\nu_1}^2 c^4}{2E}$$

So the quantum-mechanical time-evolution depends (a little) on the **neutrino mass**

Neutrino oscillations

Neutrinos with a well-defined mass are **quantum mechanical superpositions** of neutrinos with well-defined flavour. Assume for simplicity **only two neutrino flavours**

$$|\psi_{\nu_1}\rangle = \cos\theta\,|\psi_{\nu_e}\rangle + \sin\theta\,|\psi_{\nu_\mu}\rangle$$

mixing angle θ quantifies degree of superposition between flavour and mass eigenstates

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$$\theta = 0 \quad \rightarrow \quad |\psi_{\nu_1}\rangle = |\psi_{\nu_e}\rangle$$ ***no mixing***

$$\theta = \frac{\pi}{4} \quad \rightarrow \quad |\psi_{\nu_1}\rangle = \frac{1}{\sqrt{2}}|\psi_{\nu_e}\rangle + \frac{1}{\sqrt{2}}|\psi_{\nu_\mu}\rangle$$ ***maximal mixing?***

Neutrino mass implies mixing between mass and flavour eigenstates, and in turn this leads to the phenomenon of **neutrino oscillations**

Neutrino oscillations

What is the probability that an **electron neutrino** produced at ***t=0*** changes flavour and becomes a **muon neutrino** at time ***t*** ?

First of all express electron and muon neutrinos in terms of **mass eigenstates**

$$|\psi_{\nu_e}\rangle = \cos\theta|\psi_{\nu_1}\rangle + \sin\theta|\psi_{\nu_2}\rangle$$

$$|\psi_{\nu_\mu}\rangle = -\sin\theta|\psi_{\nu_1}\rangle + \cos\theta|\psi_{\nu_2}\rangle$$

You can check that as expected electron and muon neutrinos are **orthonormal**

$$\langle\psi_{\nu_e}|\psi_{\nu_e}\rangle = \cos^2\theta + \sin^2\theta = 1$$

$$\langle\psi_{\nu_e}|\psi_{\nu_\mu}\rangle = -\cos\theta\sin\theta + \cos\theta\sin\theta = 0$$

So how can an electro neutrino turn into an electron neutrino?

Via quantum-mechanical **time-evolution effects!**

Neutrino oscillations

Compute the time evolution of an electron neutrino

$$|\psi_{\nu_e}(t=0)\rangle = \cos\theta\,|\psi_{\nu_1}\rangle + \sin\theta\,|\psi_{\nu_2}\rangle$$

$$|\psi_{\nu_e}(t)\rangle = \cos\theta \exp\left(-it/\hbar(E + m_{\nu_1}^2/2E)\right)|\psi_{\nu_1}\rangle$$
$$+\sin\theta \exp\left(-it/\hbar(E + m_{\nu_2}^2/2E)\right)|\psi_{\nu_2}\rangle$$

Now compute the **probability** that at the time ***t*** the original electron neutrino has turned into a muon neutrino

$$\langle\psi_{\nu_\mu}|\psi_{\nu_e}(t)\rangle = \left(-\sin\theta\langle\psi_{\nu_1}| + \cos\theta\langle\psi_{\nu_2}|\right)\cos\theta \exp\left(-it/\hbar(E + m_{\nu_1}^2/2E)\right)|\psi_{\nu_1}\rangle$$
$$+\left(-\sin\theta\langle\psi_{\nu_1}| + \cos\theta\langle\psi_{\nu_2}|\right)\sin\theta \exp\left(-it/\hbar(E + m_{\nu_2}^2/2E)\right)|\psi_{\nu_2}\rangle$$

We can simply this matrix element by recalling that **mass eigenstates are orthonormal**

Neutrino oscillations

$$\langle\psi_{\nu_\mu}|\psi_{\nu_e}(t)\rangle = \left(-\sin\theta\langle\psi_{\nu_1}| + \cos\theta\langle\psi_{\nu_2}|\right)\cos\theta\exp\left(-it/\hbar(E + m^2_{\nu_1}/2E)\right)|\psi_{\nu_1}\rangle$$

$$+\left(-\sin\theta\langle\psi_{\nu_1}| + \cos\theta\langle\psi_{\nu_2}|\right)\sin\theta\exp\left(-it/\hbar(E + m^2_{\nu_2}/2E)\right)|\psi_{\nu_2}\rangle$$

$$\langle\psi_{\nu_\mu}|\psi_{\nu_e}(t)\rangle = (-\sin\theta\cos\theta)\exp\left(-it/\hbar(E + m^2_{\nu_1}/2E)\right)$$

$$+(-\sin\theta\cos\theta)\exp\left(-it/\hbar(E + m^2_{\nu_2}/2E)\right)$$

Now we square to compute the **sought-for probability,** and after some algebra

$$P_{\nu_e\to\nu_\mu}(t) = |\langle\psi_{\nu_\mu}|\psi_{\nu_e}(t)\rangle|^2 = \sin^2(2\theta)\sin^2\left(\frac{tc^4}{4\hbar E}\Delta m^2_{12}\right)$$

In terms of the **neutrino mixing angle** and their **mass-square difference**

$$\Delta m^2_{12} \equiv m^2_{\nu_1} - m^2_{\nu_2}$$

Neutrino oscillations

Finally since neutrino travel at almost the speed of light ***L = c t*** and the final result is:

$$P_{\nu_e \to \nu_\mu}(L) = |\langle \psi_{\nu_\mu} | \psi_{\nu_e}(t) \rangle|^2 = \sin^2(2\theta)\sin^2\left(\frac{Lc^3}{4\hbar E}\Delta m_{12}^2\right)$$

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If neutrinos **have the same mass** then they do not oscillate

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If neutrinos **do not mix (θ = 0)** then they do not oscillate

If neutrinos **have the same mass** then they do not oscillate

- More energetic neutrinos **oscillate more frequently** that low-energy neutrinos

- To measure neutrino oscillations, the ***baseline lenght L*** needs to account for both the neutrino energy and the mass difference

Neutrino oscillations

Finally since neutrino travel at almost the speed of light ***L = c t*** and the final result is:

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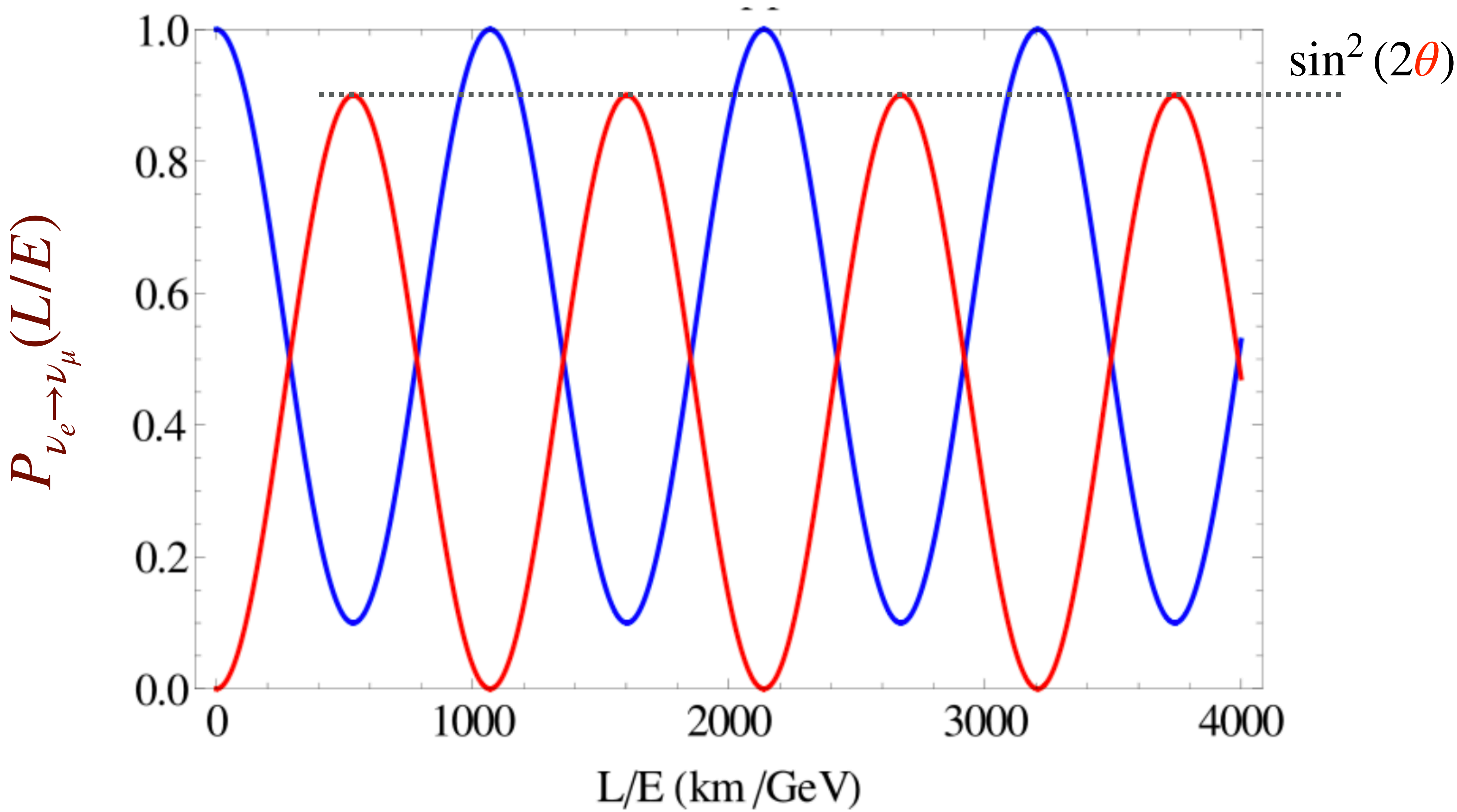

Neutrino oscillations summary

https://www.youtube.com/watch?v=7fgKBJDMO54

Application to neutrino detectors

exercise

$$P_{\nu_e \to \nu_\mu}(L) = |\langle \psi_{\nu_\mu} | \psi_{\nu_e}(t) \rangle|^2 = \sin^2(2\theta)\sin^2\left(\frac{Lc^3}{4\hbar E}\Delta m_{12}^2\right)$$

Assume we have a **muon neutrino beam**, and determine the smallest value of ***L*** for which the **probability of conversion to electron neutrinos** is bigger than ***P = 0.75***

$$\theta_{12} = 33.62^\circ \qquad \Delta m_{12}^2 = 7.4 \times 10^{-5}\,\mathrm{eV}^2 \qquad E = 1\,\mathrm{GeV}$$

$$1\ \mathrm{GeV} = 5.1 \times 10^{15}\,\mathrm{m}^{-1}$$

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$$1\ \mathrm{GeV} = 5.1 \times 10^{15}\,\mathrm{m}^{-1}$$

$$0.75 = 0.85\sin\left(\frac{Lc^3}{4\hbar E}\Delta m_{12}^2\right) \qquad L = 1.22\left(\frac{4\hbar E}{c^3\Delta m_{12}^2}\right)$$

using **natural units** to simplify the calculation

$$L = 1.22\left(\frac{4E}{\Delta m_{12}^2}\right) = 6.5 \times 10^{22}\,\mathrm{GeV}^{-1} = 1.3 \times 10^4\,\mathrm{km}$$

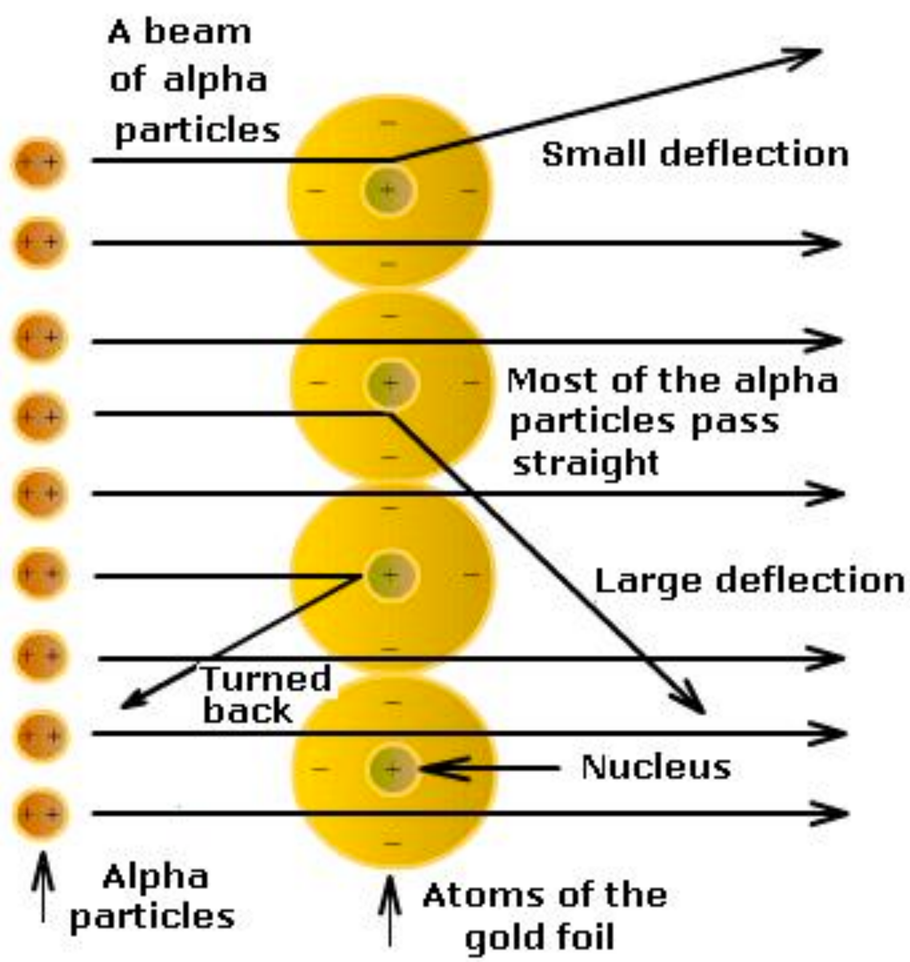
Quarks and hadrons

Quarks and protons

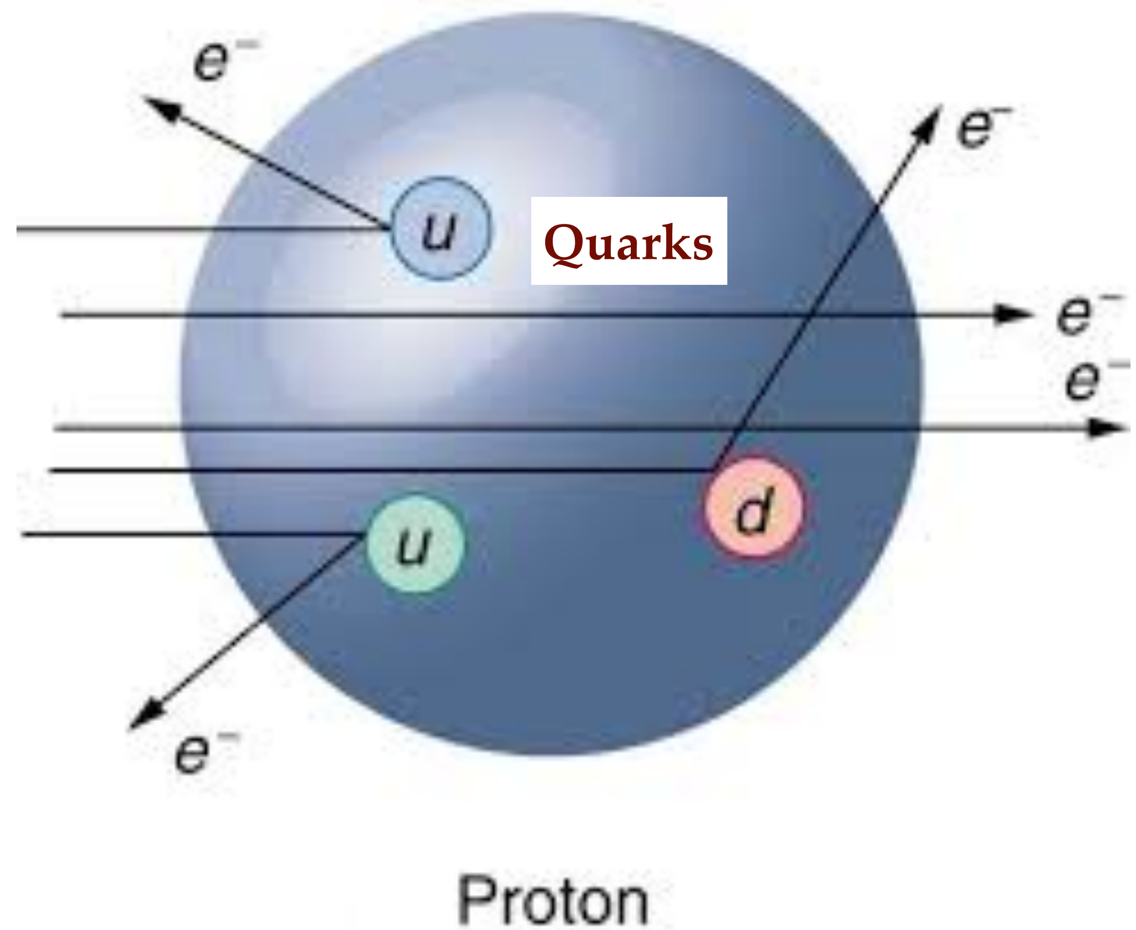
- Scattering of **α particles (He nuclei) off atoms** lead in 1911 Rutherford to **discovery of internal structure of atoms**: a **point-like nucleus** and layers of electrons

- 70 years later, the **scattering of energetic electrons off protons** lead to equally surprising result: the **internal structure of protons**, composed by point-like **quarks**

Rutherford experiment:
Atoms have internal structure!



Electron-proton collisions at Stanford Linear Accelerator:
Protons have internal structure!



The hadron zoo

name	symbol	m (MeV/c^2)	s ($\hbar$)
Pion	π^+	139.6	0
	π^0	135.0	0
Kaon	K^+	493.7	0
	K^0	497.7	0
Phi	Φ	1019.5	1
D-meson	D^+	1869.4	0
	D^0	1864.5	0
	D_s^+	1968	0
J/psi	J/ψ	3097	1
B-meson	B^+	5279	0
	B^0	5279	0
	B_s^0	5366	0
	B_c^+	6277	0
Upsilon	Υ	9460	1
Proton	p	938.3	1/2
Neutron	n	939.6	1/2
Delta	Δ^+	1232	3/2
	Δ^{++}	1232	3/2
Lambda	Λ^0	1116	1/2
Sigma	Σ^+	1189	1/2

There exists a very large number of particles that experience the **strong interaction**

We denote as **hadrons** all particles that are affected by the strong force

What determines their properties, such as **mass and spin**?

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Why the **proton** and the **neutron** have almost identical masses?

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Why some hadrons are **much heavier** than others?

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Why some hadrons have a **higher spin** than others?

The quark model of hadrons

Hadrons are **not** elementary particles: they are composed by **quarks**

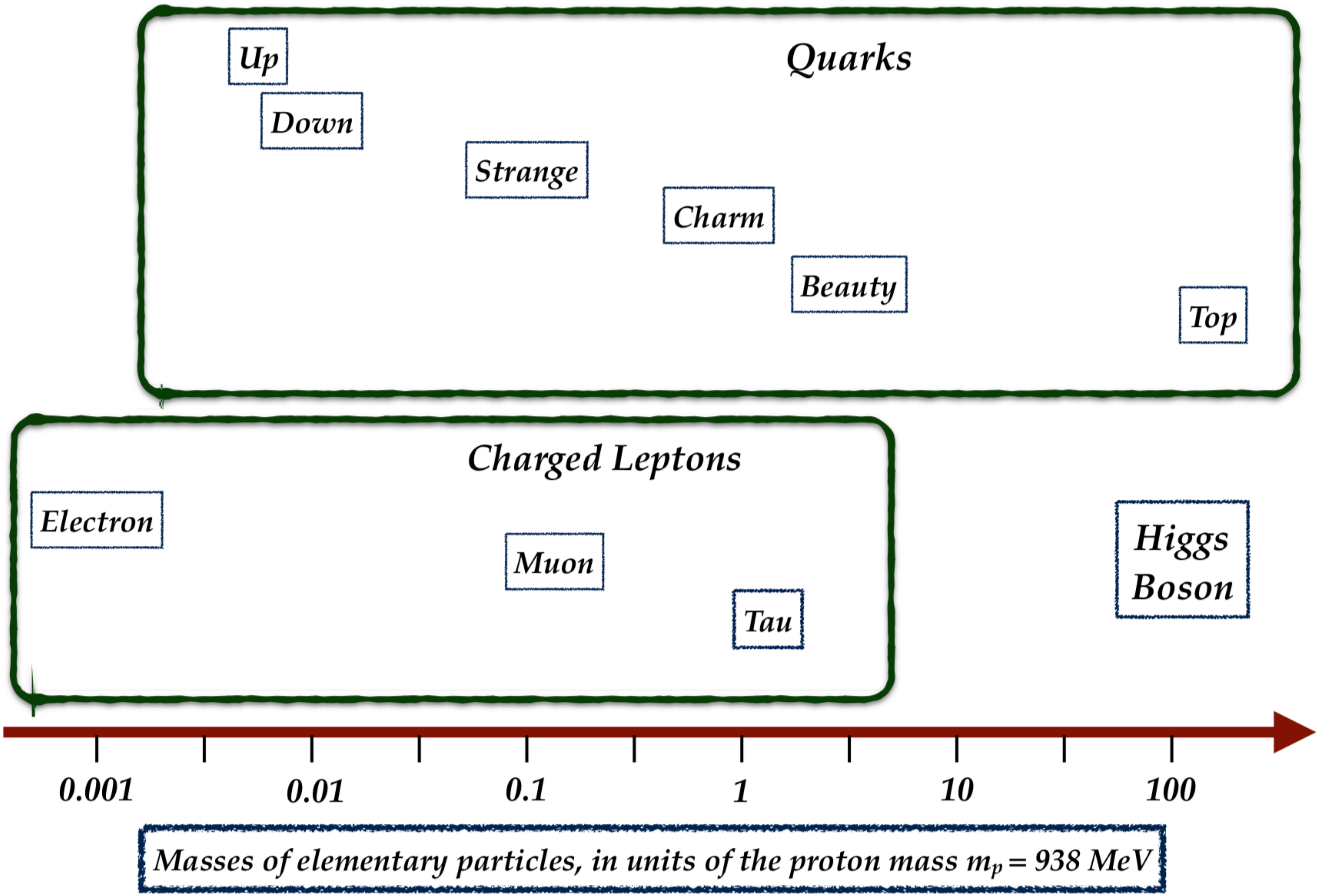
The **properties of hadrons** become transparent once we identify their quark substructure

To describe hundreds of hadrons, we need to know only the **properties of 5 quarks**

	Flavour	Mass m	Electric charge Q
First family	up	$\sim$ 5 MeV	$+2/3$
	down	$\sim$ 10 MeV	$-1/3$
Second family	strange	$\sim$ 100 MeV	-1/3
	charm	1.3 GeV	+2/3
Third family	~~top~~	~~175 GeV~~	~~$+2/3$~~
	bottom/beauty	4.5 GeV	$-1/3$

Moreover all quarks have the **same spin:** $s = \hbar/2$

Quarks vs leptons



Mesons and baryons

Depending on the number of quarks they contain, we can **divide hadrons** as follows:

Mesons: composed by a **quark-antiquark pair** (can be different flavours)

Mesons have integer spin (0 or 1) and are thus **bosons**

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e.g.: the positively charged pion is composed by an **up quark** and a **down antiquark**

$$\pi^{+} = \left(u\, \bar{d} \right)$$

Note how the constituent quark **electric charges add up** to that of the hadron

$$Q_{\pi^+} = +1 = Q_u + Q_{\bar{d}} = +\frac{2}{3} + \frac{1}{3}$$

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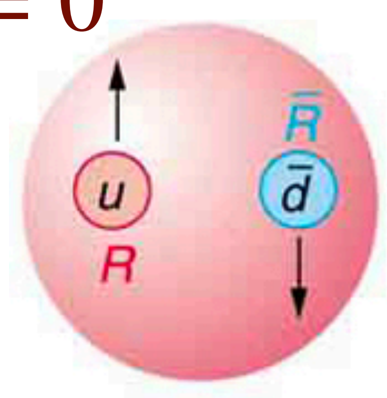
$$s_{\pi^+} = 0$$



The pion has **zero spin** since the two constituent quarks are aligned along opposite directions

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Baryons: composed by **three quarks** or **three antiquarks** (can be different flavours). Baryons have half-integer spin (1/2 or 3/2) and are thus **fermions**

Mesons and baryons

Depending on the number of quarks they contain, we can **divide hadrons** as follows:

e.g.: the proton is composed by **two up quarks** and **one down quark**

$$p = \left(u\,u\,d\right)$$

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Note how the constituent quark **electric charges add up** to that of the hadron

$$Q_p = +1 = 2Q_u + Q_d = +2 \times \frac{2}{3} - \frac{1}{3}$$

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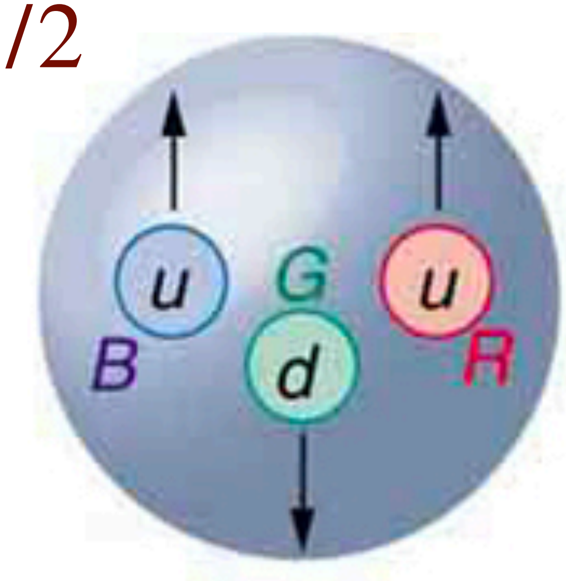
$$s_p = 1/2$$



The proton has **spin one-half** because two constituent quarks are aligned along the opposite directions than the other quark

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Tetraquarks / pentaquarks: composed by **four / five quarks** or **antiquarks**

The baryonic number

Strongly interacting particles carry a new quantum number: the **baryonic number**

As for the leptonic number, this baryonic quantum number ***B*** is **conserved** in all reactions involving hadrons and the strong interaction

For quarks we have that $B_q = +1/3$

exercise

Work out the values of ***B*** for the **proton** and the **pion**

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exercise

Work out the values of ***B*** for the **proton** and the **pion**

$$\pi^+ = (u\,\bar{d}) \qquad B_{\pi^+} = B_u + B_{\bar{d}} = +\frac{1}{3} + \left(-\frac{1}{3}\right) = 0$$

As for other **quantum charges, *B*** for antiquarks is the opposite that for quarks

Same pattern for all other **mesons: *B* = 0**