Introduction to Elementary Particles (TN2811)

Theory Lecture 3

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Today's lecture

- ☑ **Fermions and bosons**: spin-statistics
- ☑ **Charged leptons** and the leptonic number
- ☑ **Neutrinos,** radioactive processes, and oscillations

Fermions and bosons

Fermions and bosons

In terms of their **spin quantum number *s*,** particles can divided into two groups:

Fermions: *half-integer spin*

$$s = \frac{\hbar}{2}, \frac{3\hbar}{2}, \ldots$$

Bosons: *integer spin*

$$s = 0, \hbar, 2\hbar, \ldots$$

Elementary particles

Composite particles

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Elementary particles	Electron, muon, tauon, neutrinos, quarks $s = \frac{\hbar}{2}$	
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Composite particles

Protons, neutrons $s = \frac{\hbar}{2}$

Some **baryons** e.g. Δ^{++} $s = \frac{3\hbar}{2}$

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Elementary particles	Electron, muon, tauon, neutrinos, quarks	$s = \frac{\hbar}{2}$	Higgs boson	$s = 0$
			Photon, gluon, *W, Z*	$s = \hbar$
			Graviton (?)	$s = 2\hbar$
Composite particles	Protons, neutrons	$s = \frac{\hbar}{2}$		
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Composite particles	Protons, neutrons	$s = \frac{\hbar}{2}$	Pions, kaons	$s = 0$
	Some **baryons** e.g. Δ^{++}	$s = \frac{3\hbar}{2}$	Some **mesons** e.g. ρ, ω	$s = 1$

Fermions and bosons

In terms of their **spin quantum number *s*,** particles can divided into two groups:

Fermions
Spin 1/2

Quarks:
up, down, strange, charm, bottom, top

Charged leptons:
electrons, muons, tauons

Neutral leptons:
electron neutrino, muon neutrino, tau neutrino

Bosons

Spin 0
Higgs boson

Spin 1
Photon (electromagnetism)
Gluon (strong force)
W,Z bosons (weak force)

Spin 2
Graviton (gravity)

Spin & statistics

Fermions and bosons do exhibit **vastly different properties** due to their different spins

Consider a quantum system composed by **two identical particles** with position $\mathbf{x_1}$ and $\mathbf{x_2}$

$$\psi_{\rm tot}(x_1, x_2) = \psi_1(x_1)\,\psi_2(x_2)$$

Now we can **exchange the position** of the two particles, and end up with:

$$\widetilde{\psi}_{\rm tot}(x_1, x_2) = \psi_1(x_2)\,\psi_2(x_1)$$

why?

How are these two total wave functions **related** among them?

Spin & statistics

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Since the particles are identical, any physical measurements carried out in the system should yield exactly **the same result**

In other words, the **probability** of finding the two particles at $\mathbf{x_1}$ and $\mathbf{x_2}$ should not change

$$|\,\psi_{\rm tot}(x_1, x_2)\,|^2 = |\,\widetilde{\psi}_{\rm tot}(x_1, x_2)\,|^2$$

What do we learn from this?

Spin & statistics

$$|\psi_{\rm tot}(x_1,x_2)|^2 = |\widetilde{\psi}_{\rm tot}(x_1,x_2)|^2$$

$$|\psi_1(x_1)\,\psi_2(x_2)|^2 = |\psi_1(x_2)\,\psi_2(x_1)|^2$$

So when we interchange the position of the two identical particles, the total wave function must be **unchanged up to a complex phase**

$$\psi_1(x_1)\,\psi_2(x_2) = e^{i\phi}\,\psi_1(x_2)\,\psi_2(x_1)$$

why? What happens if we **exchange again** the position of the particles?

What do we learn from this?

Spin & statistics

$$|\psi_{\rm tot}(x_1,x_2)|^2 = |\widetilde{\psi}_{\rm tot}(x_1,x_2)|^2$$

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$$\psi_1(x_1)\,\psi_2(x_2) = e^{i\phi}\,\psi_1(x_2)\,\psi_2(x_1)$$

What happens if we **exchange again** the position of the particles?

$$\psi_1(x_1)\,\psi_2(x_2) = e^{i\phi}\,\psi_1(x_2)\,\psi_2(x_1) = e^{i\phi}\left(e^{i\phi}\,\psi_1(x_1)\,\psi_2(x_2)\right)$$

Which implies that the complex phase can only take **two values**

$$e^{i2\phi} = 1 \;\rightarrow\; \phi = 0, \pi \;\rightarrow\; e^{i\phi} = 1\,, -1$$

Spin & statistics

Therefore the basic principles of quantum mechanics tell us that there exist **two kinds of particles** depending on how they behave under exchanging them

Bosons: if we exchange two identical bosons, the wave function is **unchanged**

$$\psi_1(x_1)\,\psi_2(x_2) = \psi_1(x_2)\,\psi_2(x_1)$$

Fermions: if we exchange two identical fermions, the wave function **changes sign**

$$\psi_1(x_1)\,\psi_2(x_2) = -\,\psi_1(x_2)\,\psi_2(x_1)$$

Why this difference is so important?

What happens if **two fermions occupy the same quantum state**?

Spin & statistics

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What happens if **two fermions occupy the same quantum state**?

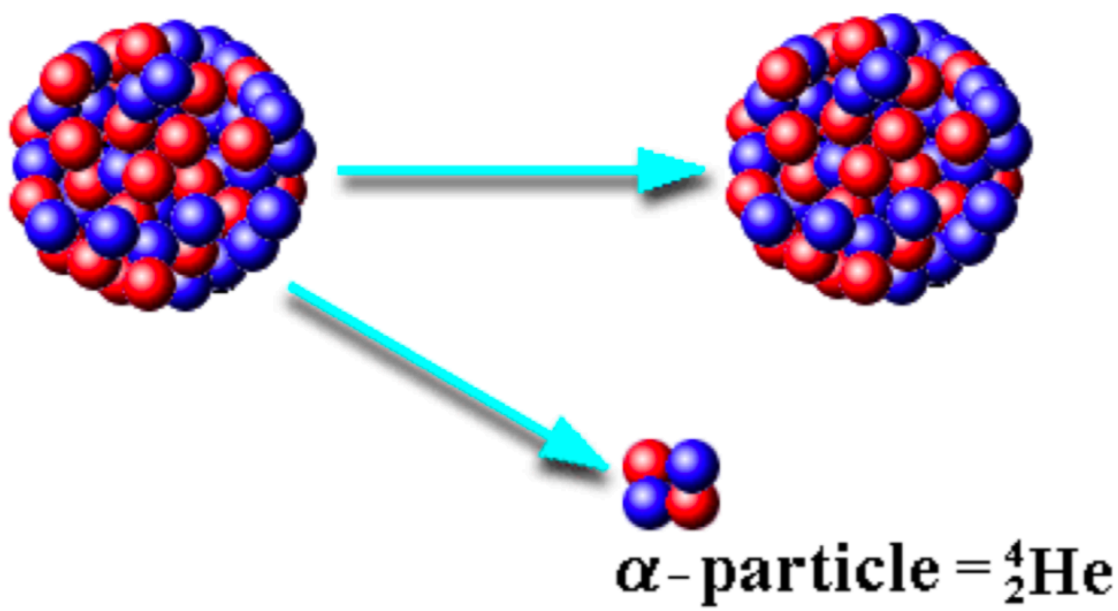
$$\psi_1(x_1)\,\psi_2(x_1) = -\,\psi_1(x_1)\,\psi_2(x_1) = 0$$

Two fermions cannot occupy the same quantum state

Radioactive decays and neutrinos

Radioactive decays

Atomic nuclei are sometimes **unstable:** they undergo radioactive decays, transforming into a **different chemical element** and emitting **energetic particles**



α-decay (strong interactions)

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}\mathrm{He}$$

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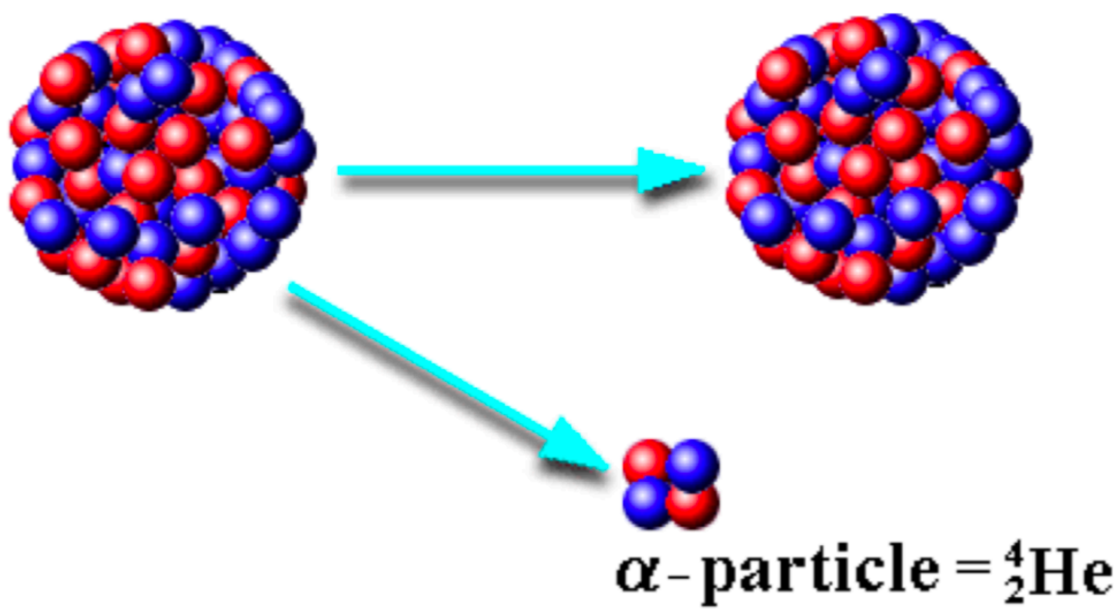

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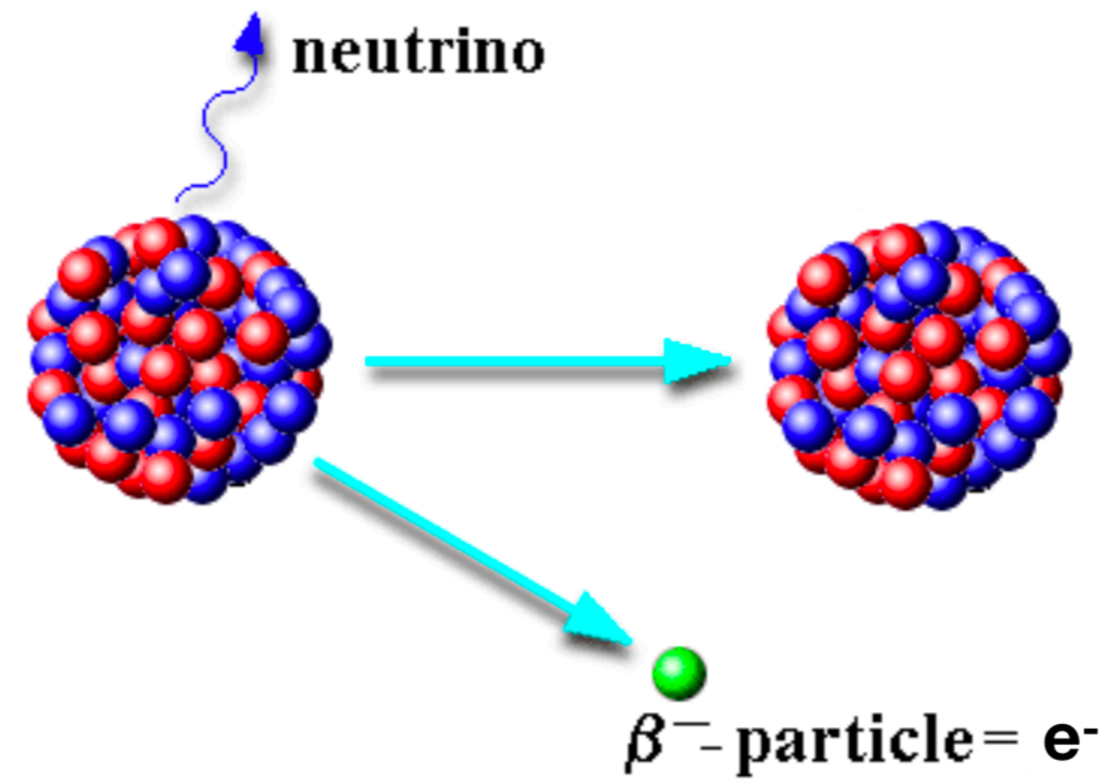

β-decay (weak interactions)

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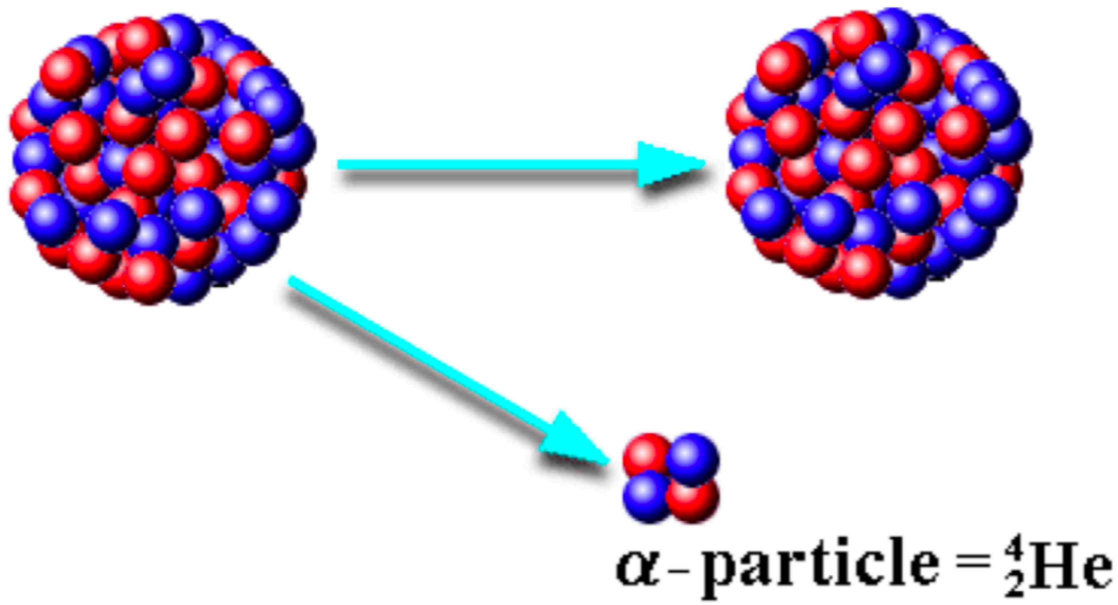

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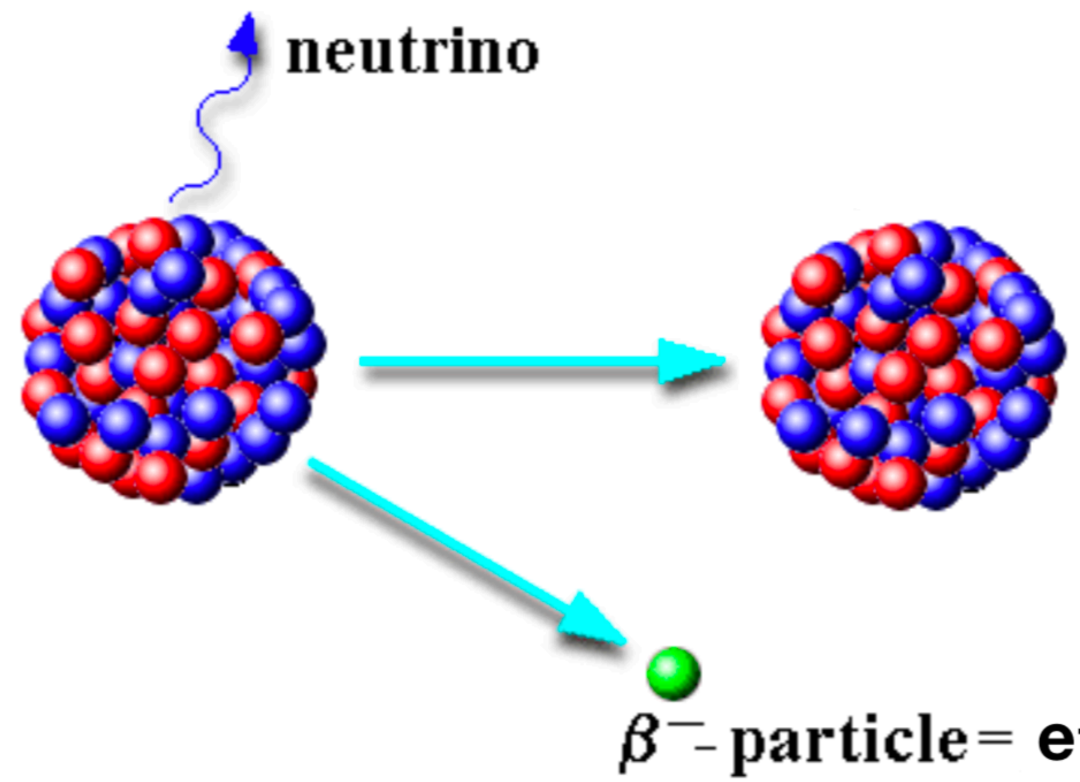

β-*decay (weak interactions)*

$$ {}^A_Z X \rightarrow {}^{A-4}_{Z-2} Y + {}^4_2 \mathrm{He} $$

$$ {}^A_Z X \rightarrow {}^{A}_{Z+1} Y + e^- + \bar{\nu}_e $$

What determines the energy of the **outgoing particles?**

exercise

Alpha decays

Let us evaluate the **energy of the alpha particle** in this type of radioactive decay

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\mathrm{He} $$

start by writing the four-momenta in the **rest frame of the decaying nucleus *X***

$$ p^{\mu}_{X} = \left(m_X c, \overrightarrow{0} \right) $$

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$$ p^{\mu}_{X} = \left(m_X c, \vec{0} \right) $$

$$ p^{\mu}_{Y} = \left(E_Y/c, \vec{p}_Y \right) $$

$$ p^{\mu}_{\alpha} = \left(E_{\alpha}/c, \vec{p}_{\alpha} \right) $$

First of all we impose **conservation of four-momentum**

$$ \vec{p}_Y = -\vec{p}_{\alpha} \equiv \vec{p} $$

$$ m_X c = E_Y/c + E_{\alpha}/c $$

We have *one equation* and *three unknowns* - what **extra info** can we use?

exercise

Alpha decays

Use the **mass-shell** condition that relates the energy, mass, and linear momentum

$$E_Y/c = \sqrt{m_Y^2 c^2 + \vec{p}^{\,2}} \qquad E_\alpha/c = \sqrt{m_\alpha^2 c^2 + \vec{p}^{\,2}}$$

Now we have *three equations* and *three unknowns* - we can solve the system!

$$E_\alpha = \frac{m_X^2 + m_\alpha^2 - m_Y^2}{2m_X} c^2$$

For a given element ***X***, the value of $\boldsymbol{E_\alpha}$ is always fixed

Recall that here ***E*** is always the relativistic energy, which includes contributions both of the **rest mass** and of the **kinetic energy**

Beta decays

Experimentally **beta decays** were observed to take the following form:

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-} $$

Applying the same reasoning as for alpha decays, we would for the electron energy

$$ E_e = \frac{m_X^2 + m_e^2 - m_Y^2}{2m_X} c^2 \simeq \frac{m_X^2 - m_Y^2}{2m_X} c^2 $$

So exactly the same electron energy in all beta decays!

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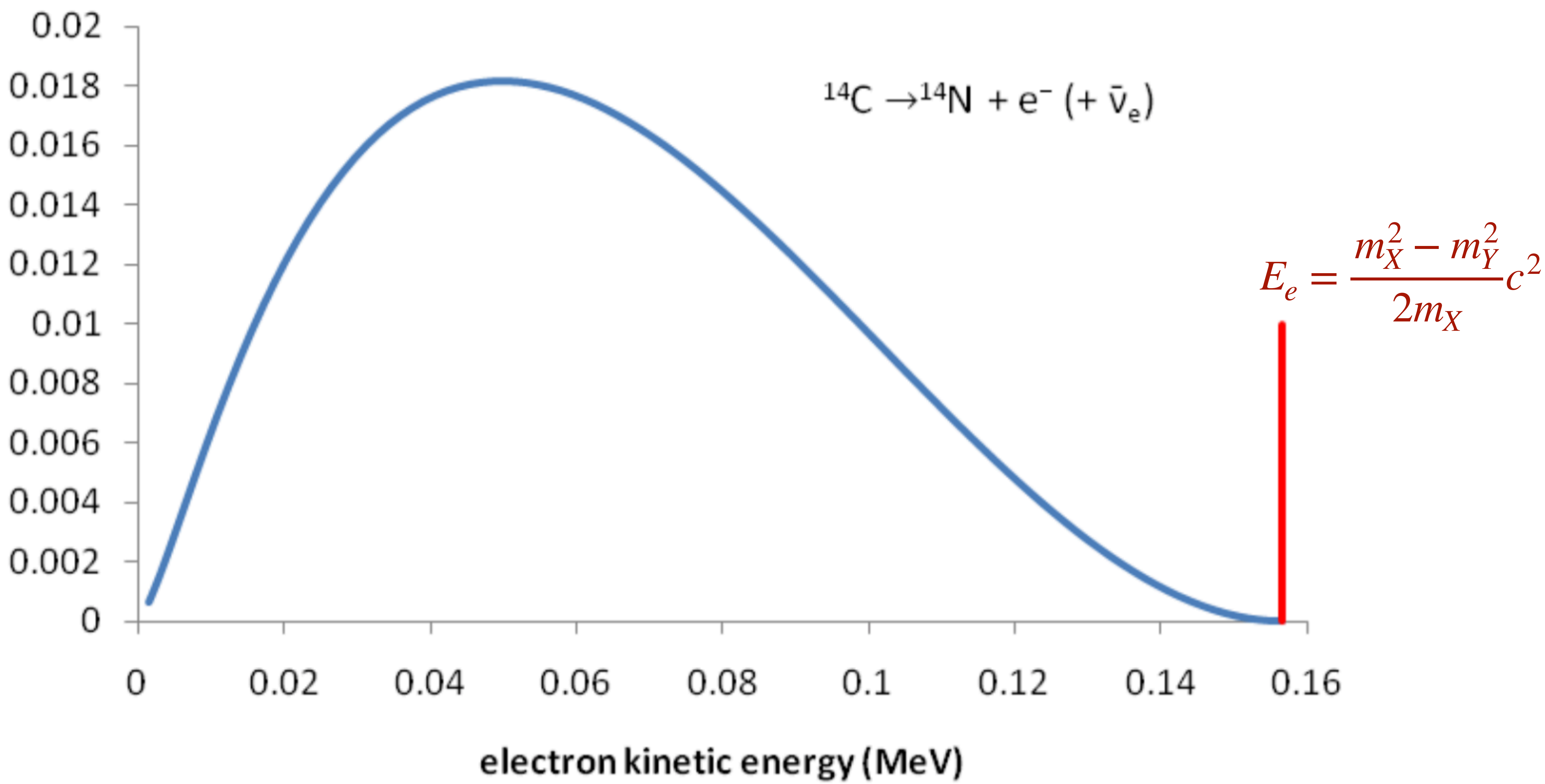
However it was found that the electron energy had a **range of possible values**



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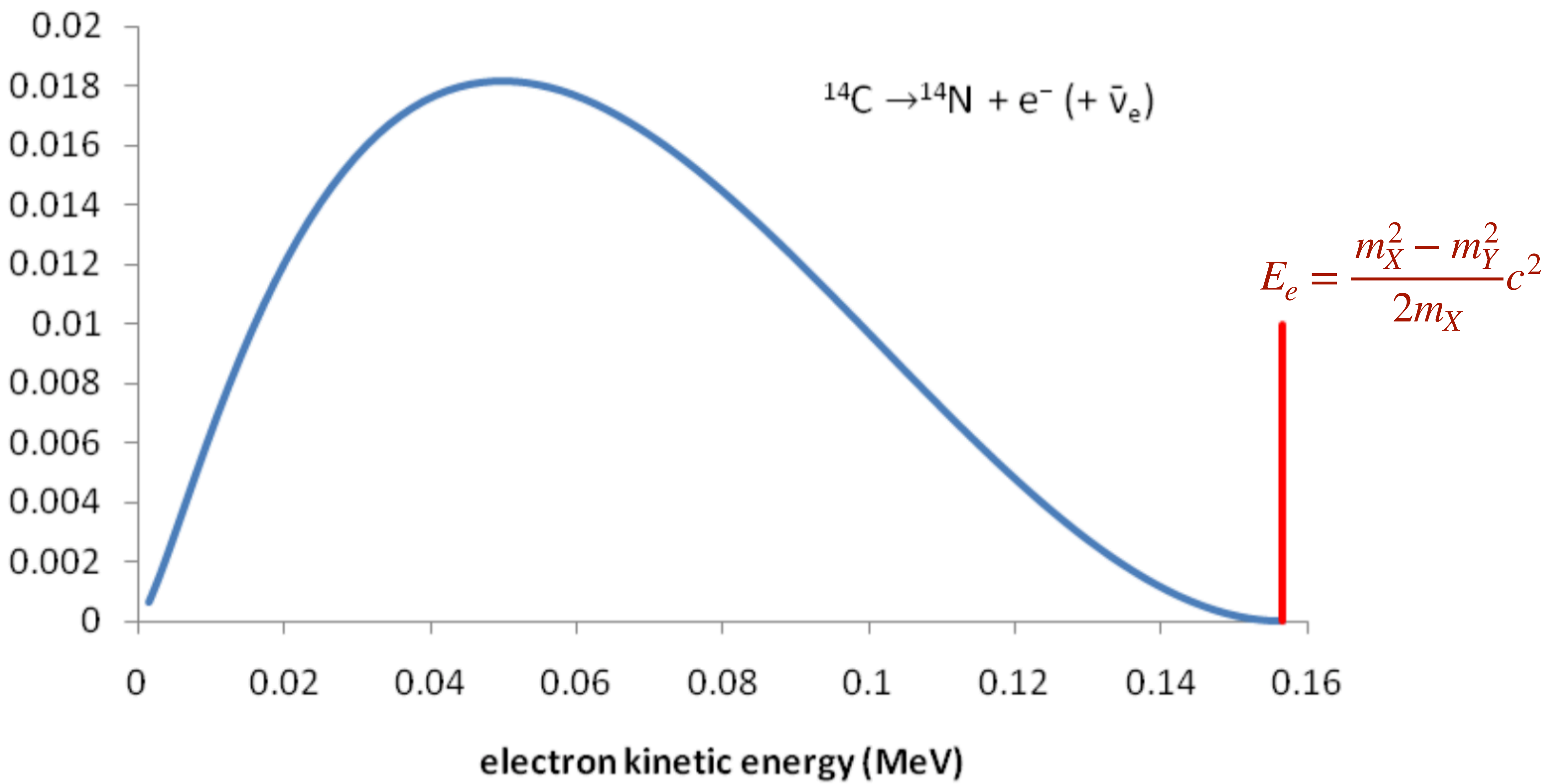
The **neutrino** was introduced to be able to describe the electron energy spectrum



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It took decades to be measured, but we knew its **properties** from the very beginning:

☑ Electric charge conservation implies that the **neutrino should be electrically neutral**

$$Q_\nu = Q_{_Z X} - Q_{_{Z+1} Y} - Q_{e^-} = Z - (Z+1) - (-1) = 0$$

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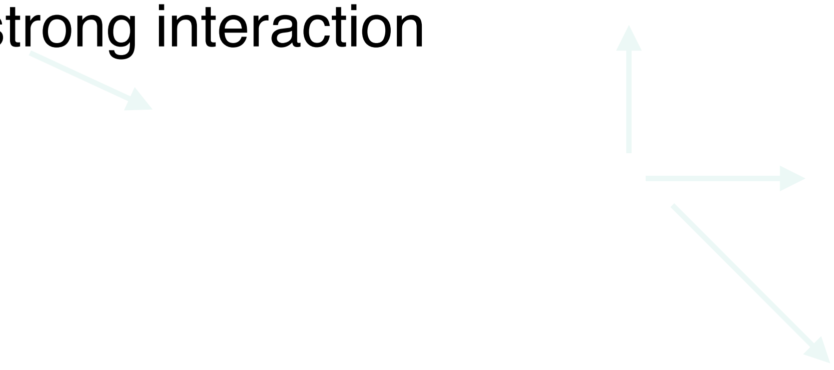

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- Normal matter is **highly transparent** to neutrinos: very weak interactions, small cross-section with matter particles

Else it should have been observed as a decay product of beta decays

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- ☑ It does not experience the strong interaction
- ☑ Normal matter is **highly transparent** to neutrinos: very weak interactions, small cross-section with matter particles
- ☑ It should either be **massless** or **have a tiny mass**, much smaller than any other particle

$$m_\nu / m_e \leq 5 \times 10^{-5}$$

from the shape of the electron energy spectrum

Charged leptons & the leptonic quantum number

Leptons

We denote as leptons all particles that **do not experience the strong force**

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with electric charge: electron, muon, tauon (+ antiparticles) → ***Charged leptons***

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$$m_e = 9.1 \times 10^{-31}\,\mathrm{kg} = 0.511\,\mathrm{MeV/c^2} \qquad Q_e = -e = -1.6 \times 10^{-19}\,C$$

$$m_\mu = 1.9 \times 10^{-28}\,\mathrm{kg} = 107.7\,\mathrm{MeV/c^2} \qquad Q_\mu = -e = -1.6 \times 10^{-19}\,C$$

$$m_\tau = 3.2 \times 10^{-27}\,\mathrm{kg} = 1.777\,\mathrm{GeV/c^2} \qquad Q_\tau = -e = -1.6 \times 10^{-19}\,C$$

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$$m_e = 9.1 \times 10^{-31}\,\mathrm{kg} = 0.511\,\mathrm{eV/c^2}$$

$$Q_e = -e = -1.6 \times 10^{-19}\,C$$

In the world of elementary particles, convenient to **measure mass** in MeV/c^2 or GeV/c^2

$$1\,\mathrm{MeV/c^2} = 1.79 \times 10^{-30}\,\mathrm{kg}$$

Leptonic number

Leptons have associated a new quantum number, called the **leptonic number *L*,** which in the Standard Model is **conserved** by all reactions involving leptons

$$\left(e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau\right) \quad \rightarrow \quad L = +1$$

As with other quantum numbers, antiparticles have the **opposite value of *L***

$$\left(e^+, \mu^+, \tau^+, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau\right) \quad \rightarrow \quad L = -1$$

What are the most important **properties** of the leptonic number?

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The leptonic number is **additive** in a system of particles

$$e^- + \mu^- \quad \rightarrow \quad L = ?$$

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$$e^- + \mu^- \quad \rightarrow \quad L = +1 + 1 = +2$$

$$e^- + e^+ + \tau^- + \tau^+ \quad \rightarrow \quad L = +1 + (-1) + 1 + (-1) = 0$$

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As with other quantum numbers, antiparticles have the **opposite value of *L***

$$\left(e^{+}, \mu^{+}, \tau^{+}, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau\right) \quad \rightarrow \quad L = -1$$

The leptonic number is **conserved** in reactions involving leptons

$$e^{-} + e^{+} \rightarrow e^{-} + e^{+} \qquad \text{allowed?}$$

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The leptonic number is **conserved** in reactions involving leptons

$$e^- + e^+ \rightarrow e^- + e^+ \qquad \text{yes}: \ \mathrm{L_{in}} = \mathrm{L_{fin}} = 0$$

$$\mu^- + \mu^+ \rightarrow \mu^- + \mu^- + \mu^+ \qquad \text{no}: \ \mathrm{L_{in}} = 0 \neq \mathrm{L_{fin}} = +1$$

Only scattering reactions that **satisfy all conservation laws** are allowed

Leptonic number

In the Standard Model, not only the **total leptonic number *L*** is conserved: also the **individual leptonic numbers** for the electron, muon, and tau are **conserved**

$$(e^-, \nu_e) \quad \rightarrow \quad L_e = +1\,, L_\mu = 0\,, L_\tau = 0$$

$$(e^+, \bar{\nu}_e) \quad \rightarrow \quad L_e = -1\,, L_\mu = 0\,, L_\tau = 0$$

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$$(e^+, \bar{\nu}_e) \quad \rightarrow \quad L_e = -1\,, L_\mu = 0\,, L_\tau = 0$$

$$\left(\mu^-, \nu_\mu\right) \quad \rightarrow \quad L_e = 0\,, L_\mu = +1\,, L_\tau = 0$$

$$\left(\mu^+, \bar{\nu}_\mu\right) \quad \rightarrow \quad L_e = 0\,, L_\mu = -1\,, L_\tau = 0$$

$$(\tau^-, \nu_\tau) \quad \rightarrow \quad L_e = 0\,, L_\mu = 0\,, L_\tau = +1$$

$$(\tau^+, \bar{\nu}_\tau) \quad \rightarrow \quad L_e = 0\,, L_\mu = 0\,, L_\tau = -1$$

Only reactions where L_e , L_μ , L_τ are **separately conserved** are allowed

Conservation laws

Enumerate **conservation laws** satisfied in scattering reactions involving **leptons**

☑ **Energy and linear momentum conservation**

$$E_{\rm in} = E_{\rm fin} \qquad \& \qquad \overrightarrow{p}_{\rm in} = \overrightarrow{p}_{\rm fin}$$

In particular energy conservation applied to **particle decays** requires that the **sum of particle masses in the final state** should be smaller than initial mass

$$m_{\rm in} \geq \sum_{i \in {\rm fin}} m_j$$

why?

Conservation laws

Enumerate **conservation laws** satisfied in scattering reactions involving **leptons**

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☑ **Individual lepton number conservation**

$$\sum_{i \in {\rm in}} L_{e,i} = \sum_{i \in {\rm fin}} L_{e,j}$$

$$\sum_{i \in {\rm in}} L_{\mu,i} = \sum_{i \in {\rm fin}} L_{\mu,j}$$

$$\sum_{i \in {\rm in}} L_{\tau,i} = \sum_{i \in {\rm fin}} L_{\tau,j}$$

exercise

Lepton scattering and decay

Determine if the following scattering reactions involving leptons are **allowed**

$$e^- \to \mu^- + \nu_e + \bar{\nu}_\mu$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

$$\mu^- \to e^+ + \nu_e$$

$$\mu^- + \mu^+ \to \tau^- + \tau^+$$

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No : $m_e < m_\mu \ (E_{\rm in} < E_{\rm fin})$

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$$\mu^- \to e^+ + \nu_e \qquad \text{No}: L_{\mu,\text{in}} = +1 \neq L_{\mu,\text{fin}} = 0$$

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No : $L_{\mu,{\rm in}} = +1 \neq L_{\mu,{\rm fin}} = 0$

$$\mu^- + \mu^+ \to \tau^- + \tau^+$$

Yes : everything conserved

$$\mu^- + \mu^+ \to e^+ + e^+ + \nu_e + \nu_e$$

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$$\mu^- + \mu^+ \to \tau^- + \tau^+$$ Yes : everything conserved

$$\mu^- + \mu^+ \to e^+ + e^+ + \nu_e + \nu_e$$ No : $Q_{\rm in} = 0 \neq Q_{\rm fin} = +2$