Introduction to Elementary Particles (TN2811)

Theory Lecture 2

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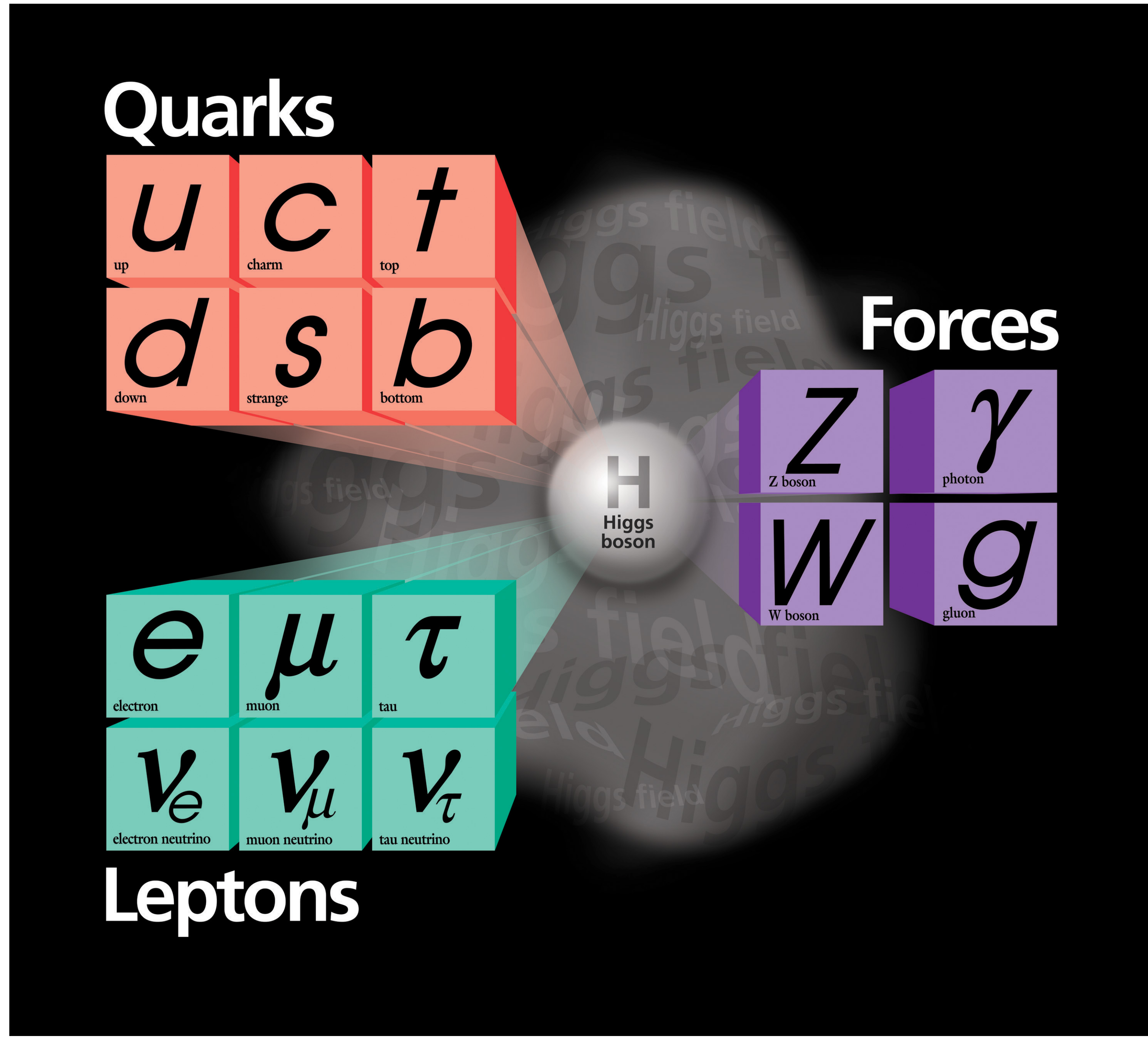
Recap: the Standard Model

- The **Standard Model (SM) of particle physics** explains a wide variety of microscopic phenomena in a unified framework: **Quantum Field Theory**

- **Matter content** composed by **six quarks** and **six leptons,** organised in **three families**

- Interactions **between matter particles** are governed by **gauge bosons**: **photons** (electromagnetism), **W and Z bosons** (weak force), and **gluons** (strong interaction)

- The last ingredient is the **Higgs Boson,** provides mechanism by which **particles acquire mass**



Today's lecture

- Spin and statistics: fermions and bosons

- More about neutrinos: **how large a neutrino detector** needs to be?

- The gravitational interaction, black holes, and **gravitational wave detection**

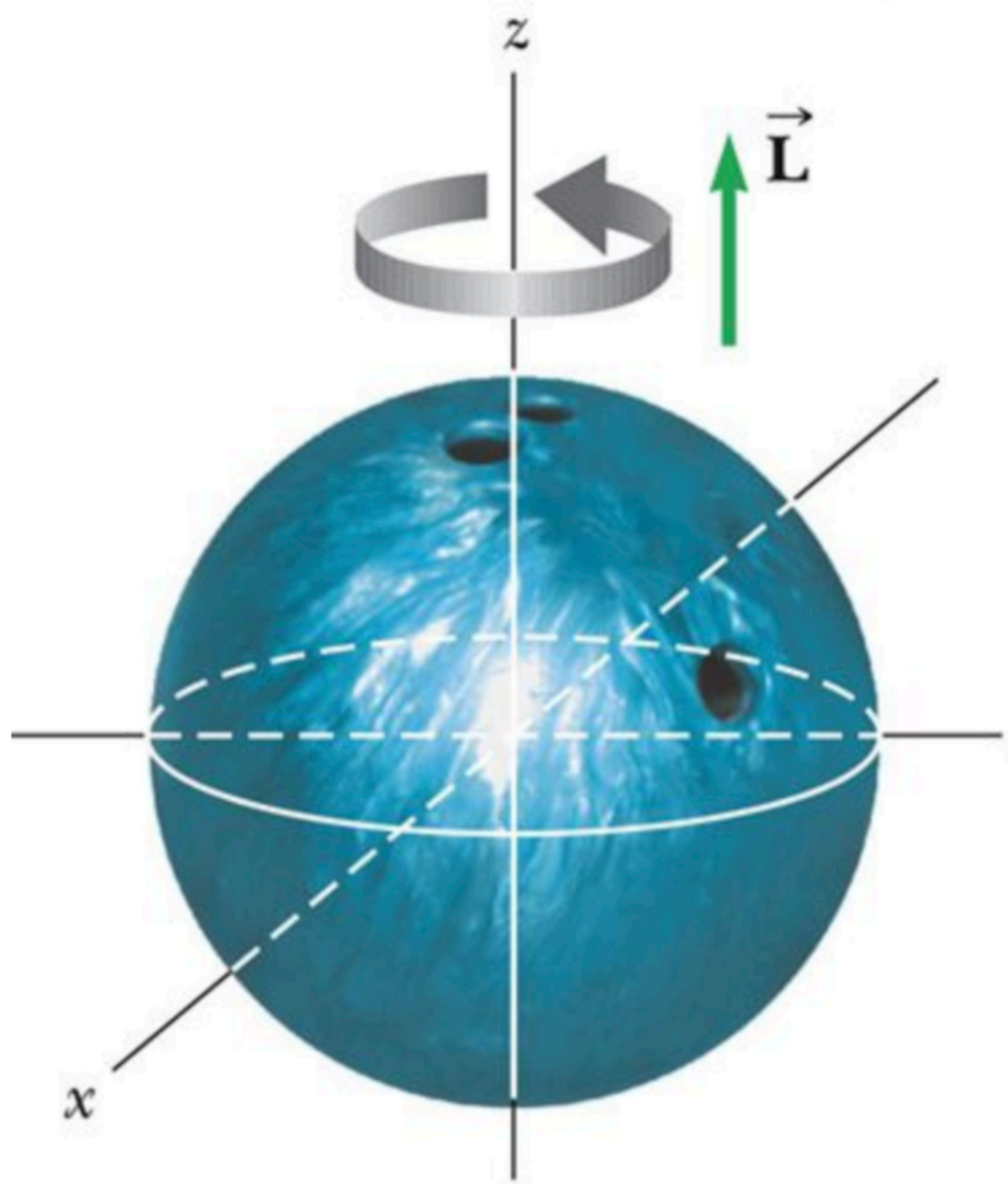
- Elements for **Special Relativity** relevant for Elementary Particle Physics

Spin in quantum theory

Intrinsic angular momentum

- In classical mechanics, a solid body rotating around one axis has associated **angular momentum,** which is **conserved** in the absence of external forces

Bowling ball (classical mechanics)



$$\left|\vec{L}\right| = I \times \omega = \left(\frac{2}{5}MR^2\right)\omega$$

moment of inertia (I)

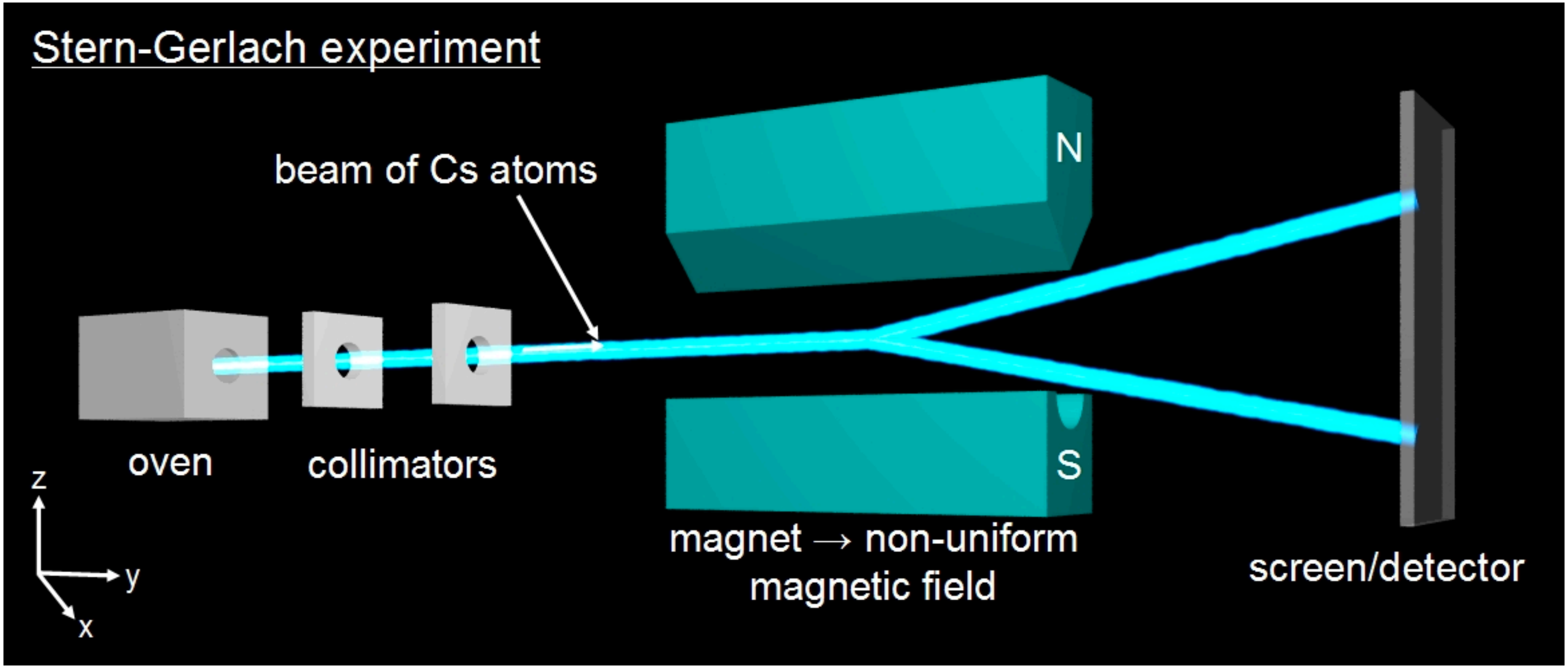
angular velocity (ω)

Intrinsic angular momentum

- In classical mechanics, a solid body rotating around one axis has associated **angular momentum,** which is **conserved** in the absence of external forces

Around 1920, it was found that the electron appeared to have some **intrinsic angular momentum,** with only **two orientations** possible

$$\left|\vec{L}_e\right| = \frac{\hbar}{2} \quad L_{e,z} = \pm\frac{\hbar}{2}$$



exercise Intrinsic angular momentum

Can we understand intrinsic angular momentum of electron using **classical mechanics?**

classical mechanics

$$\left|\overrightarrow{L}_e\right| = I \times \omega = \left(\frac{2}{5} m_e R_e^2\right) \omega$$

experiment

$$\left|\overrightarrow{L}_e\right| = \frac{\hbar}{2}$$

If we assume the electron is a solid (classical) body, **at which speed** it needs to rotate?

$$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$$

$$r_e \sim 10^{-18}\,\mathrm{m}$$

$$\hbar = 1.05 \times 10^{-34}\,\mathrm{J\,s}$$

exercise Intrinsic angular momentum

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$$r_e \sim 10^{-18}\,\mathrm{m}$$

$$\hbar = 1.05 \times 10^{-34}\,\mathrm{J\,s}$$

$$\left|\vec{L}_e\right| = \left(\frac{2}{5} m_e R_e^2\right) \frac{v}{R_e} = \frac{\hbar}{2} \quad \rightarrow \quad v = \frac{5\hbar}{4 m_e r_e} \simeq 1.5 \times 10^{14}\,\mathrm{m/s}$$

If we think of the intrinsic angular momentum of the electron in classical terms, its surface should move **faster than the speed of light**, which is impossible!

Spin

In quantum theory, we call **spin** the **intrinsic angular momentum** of elementary particles, which as we have just seen does not admit a classical interpretation

Spin is an example of a **quantum number:** intrinsic properties of elementary particles that might (electric charge) or not (color charge, spin) have classical counterparts

Spin

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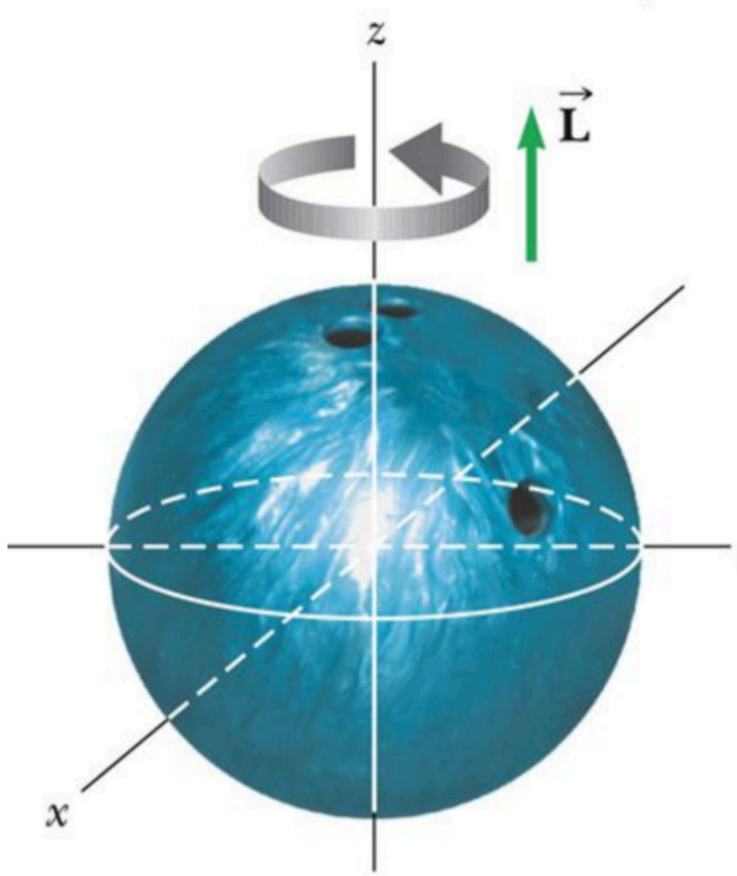
Spin is quantised: it cannot take arbitrary values, but only a finite subset of them

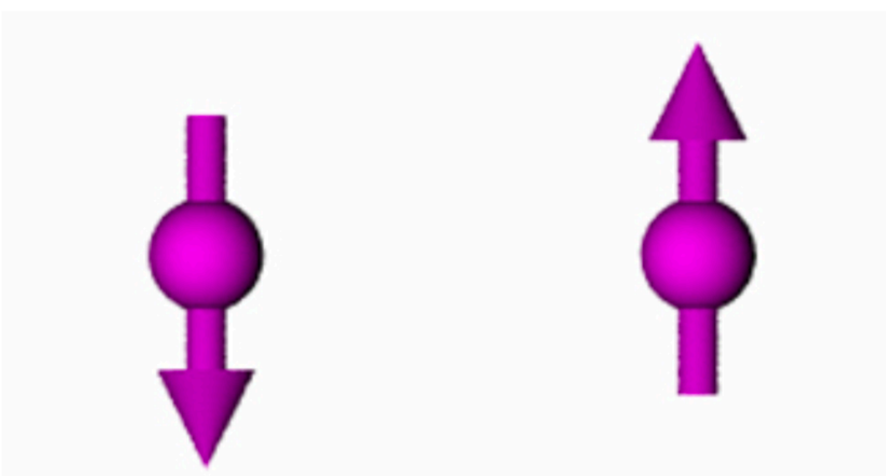
Bowling ball (classical mechanics)



$$L = I\omega = \left(\frac{2}{5}MR\right)\omega$$

Electrons (quantum mechanics)

$|-\rangle$ $|+\rangle$



$$L = \frac{\hbar}{2}, L_z = -\frac{\hbar}{2} \qquad L = \frac{\hbar}{2}, L_z = +\frac{\hbar}{2}$$

Spin

In quantum theory, we call **spin** the **intrinsic angular momentum** of elementary particles, which as we have just seen does not admit a classical interpretation

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Electrons (quantum mechanics)

Electron with spin up

$$|\varphi\rangle_{\rm up} = |+\rangle = |\uparrow\rangle$$

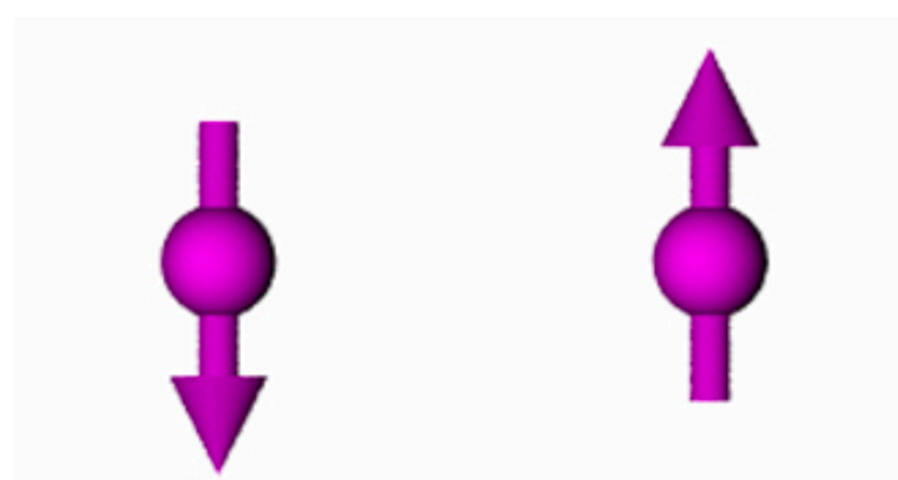
Electron with spin down

$$|\varphi\rangle_{\rm down} = |-\rangle = |\downarrow\rangle$$

Electron with spin superposition

$$|\varphi\rangle_{\rm mixed} = |+\rangle + |-\rangle = |\uparrow\rangle + |\downarrow\rangle$$

$|-\rangle$ $|+\rangle$



$L = \frac{\hbar}{2}, L_z = -\frac{\hbar}{2}$ $L = \frac{\hbar}{2}, L_z = +\frac{\hbar}{2}$

Fermions and bosons

Fermions and bosons

In terms of their **spin quantum number *s*,** particles can divided into two groups:

Fermions: *half-integer spin*

$$s = \frac{\hbar}{2}, \frac{3\hbar}{2}, \ldots$$

Bosons: *integer spin*

$$s = 0, \hbar, 2\hbar, \ldots$$

Elementary particles

Composite particles

Fermions and bosons

In terms of their **spin quantum number *s*,** particles can divided into two groups:

	Fermions: *half-integer spin*	***Bosons:*** *integer spin*
	$s = \frac{\hbar}{2}, \frac{3\hbar}{2}, \ldots$	$s = 0, \hbar, 2\hbar, \ldots$
Elementary particles	Electron, muon, tauon, neutrinos, quarks $s = \frac{\hbar}{2}$	
Composite particles		

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Elementary particles

Electron, muon, tauon, neutrinos, quarks $s = \frac{\hbar}{2}$

Composite particles

Protons, neutrons $s = \frac{\hbar}{2}$

Some **baryons** e.g. Δ^{++} $s = \frac{3\hbar}{2}$

Bosons: *integer spin*

$$s = 0, \hbar, 2\hbar, \ldots$$

Fermions and bosons

In terms of their **spin quantum number *s*,** particles can divided into two groups:

	Fermions: *half-integer spin*		***Bosons:*** *integer spin*	
	$s = \frac{\hbar}{2}, \frac{3\hbar}{2}, \ldots$		$s = 0, \hbar, 2\hbar, \ldots$	
Elementary particles	Electron, muon, tauon, neutrinos, quarks	$s = \frac{\hbar}{2}$	Higgs boson	$s = 0$
			Photon, gluon, *W, Z*	$s = \hbar$
			Graviton (?)	$s = 2\hbar$
Composite particles	Protons, neutrons	$s = \frac{\hbar}{2}$		
	Some **baryons** e.g. Δ^{++}	$s = \frac{3\hbar}{2}$		

Fermions and bosons

In terms of their **spin quantum number *s*,** particles can divided into two groups:

	Fermions: *half-integer spin*		***Bosons:*** *integer spin*	
	$s = \frac{\hbar}{2}, \frac{3\hbar}{2}, \ldots$		$s = 0, \hbar, 2\hbar, \ldots$	
Elementary particles	Electron, muon, tauon, neutrinos, quarks	$s = \frac{\hbar}{2}$	Higgs boson	$s = 0$
			Photon, gluon, *W, Z*	$s = \hbar$
			Graviton (?)	$s = 2\hbar$
Composite particles	Protons, neutrons	$s = \frac{\hbar}{2}$	Pions, kaons	$s = 0$
	Some **baryons** e.g. Δ^{++}	$s = \frac{3\hbar}{2}$	Some **mesons** e.g. ρ, ω	$s = 1$

Fermions and bosons

In terms of their **spin quantum number *s*,** particles can divided into two groups:

Fermions
Spin 1/2

Quarks:
up, down, strange,
charm, bottom, top

Charged leptons:
electrons, muons, tauons

Neutral leptons:
electron neutrino,
muon neutrino,
tau neutrino

Bosons

Spin 0
Higgs boson

Spin 1
Photon (electromagnetism)
Gluon (strong force)
W,Z bosons (weak force)

Spin 2
Graviton (gravity)

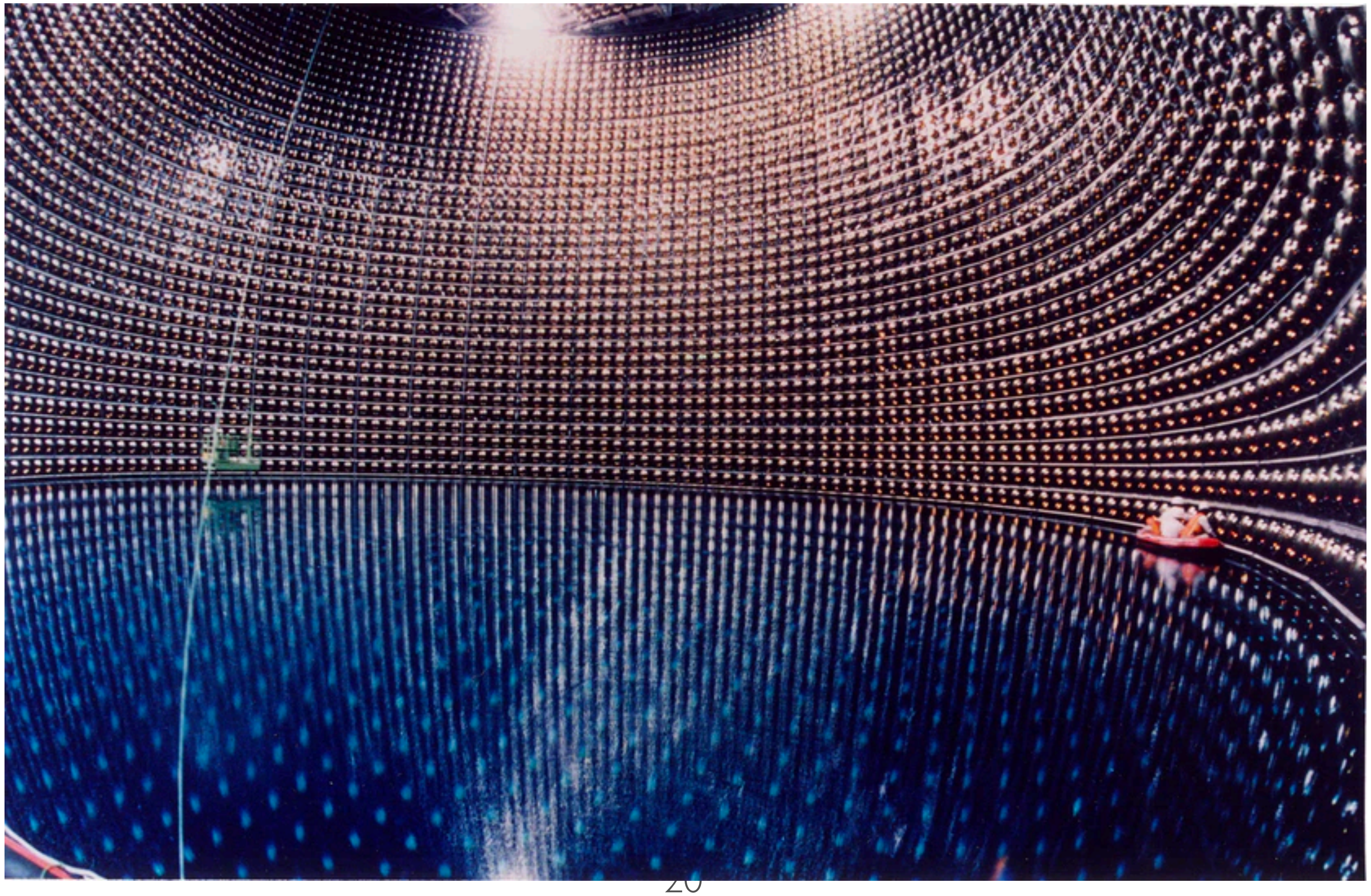
How large a neutrino detector needs to be?

Neutrinos: the ghost particles

- **Neutrinos** are electrically neutral, very light, and weakly interacting particles introduced by Pauli in 1930 to guarantee **energy conservation** in the beta decay process

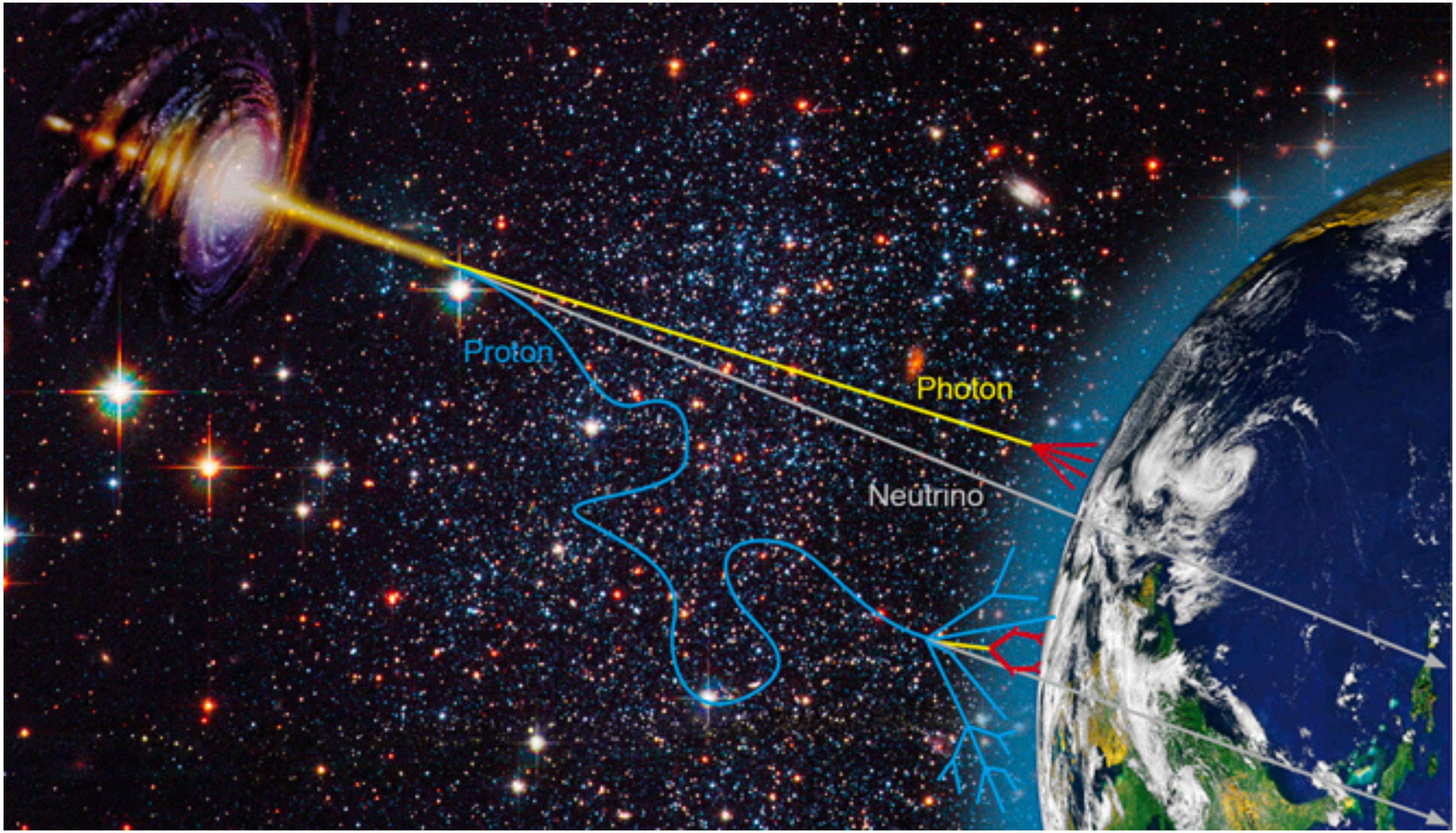
- Due to their very low interaction rate they are extremely difficult to measure, and huge detectors are required to be able to study then

- These ghostly particles are very abundant in the Universe: every second about **100 trillion neutrinos from the Sun cross your body!**



The neutrino universe

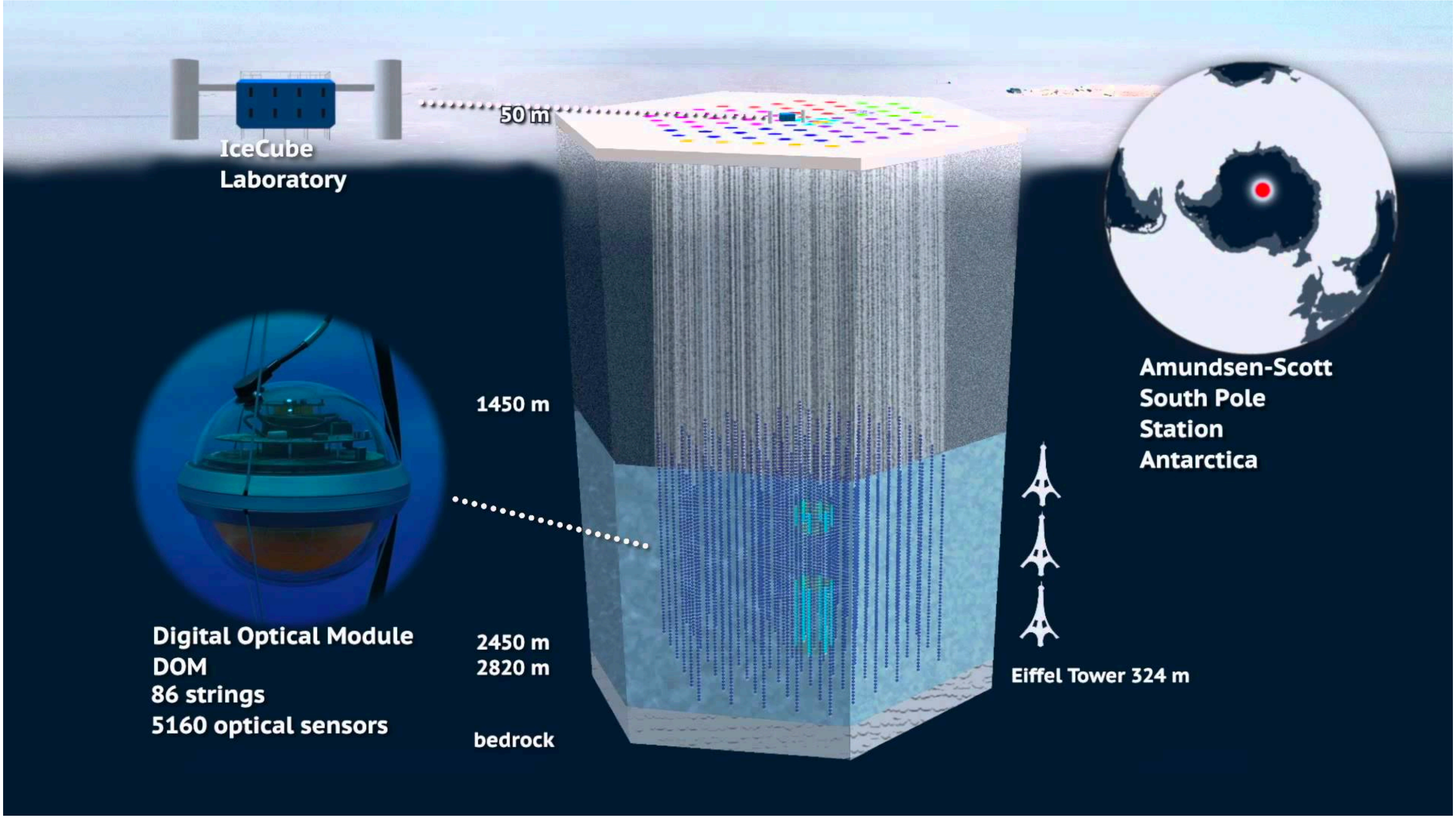
*Neutrinos are not **deflected or attenuated**: unique probes of extreme astrophysical events*



Neutrinos interact very weakly: need to **instrument huge volumes,** around **1 km^3**, to have significant event rates

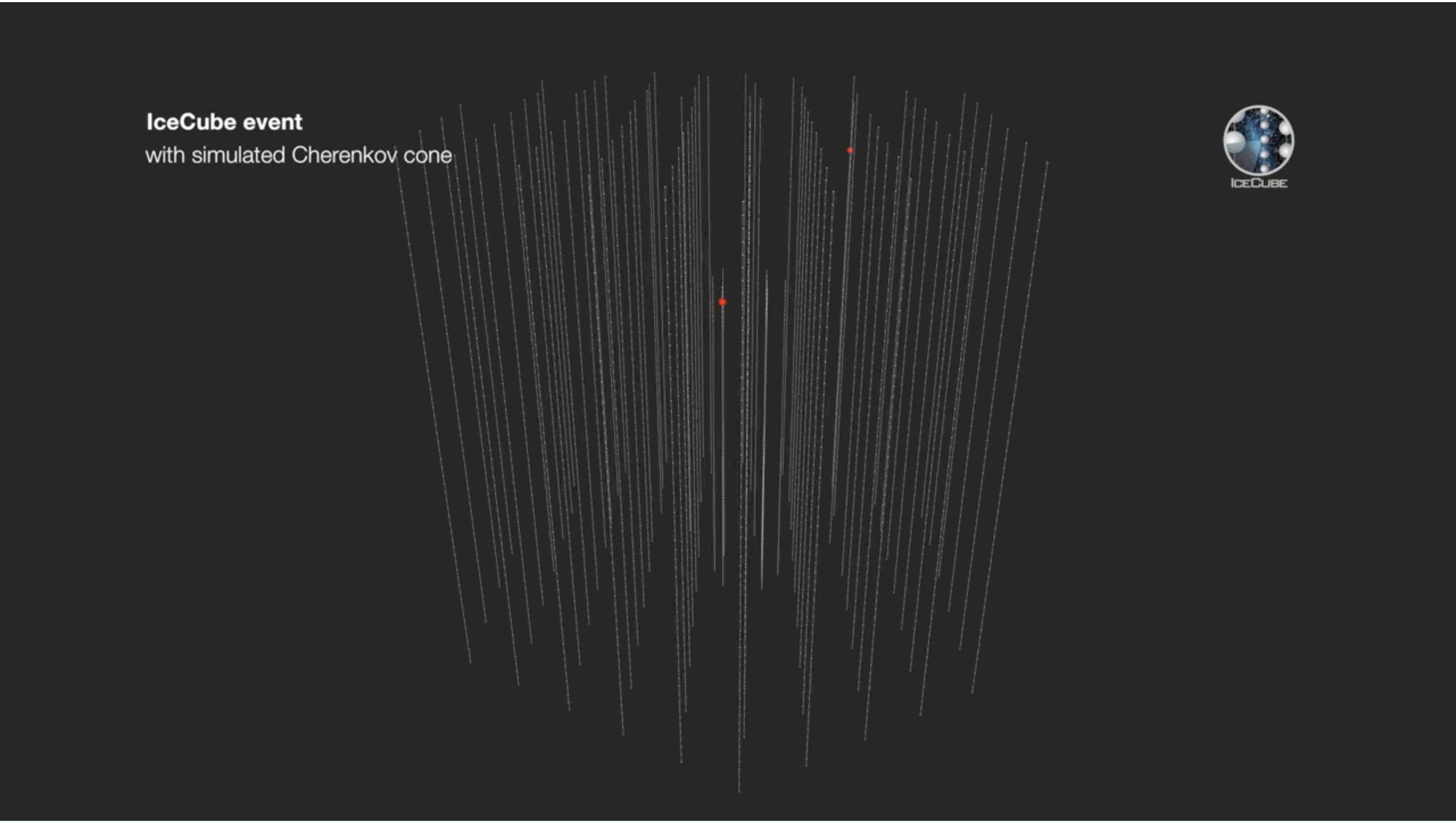
Neutrino telescopes

Ultra-high energy (UHE) neutrinos: novel window to the **extreme Universe**



Neutrino telescopes

Neutrinos interact very weakly: need to **instrument huge volumes** (1 km^3) to have significant event rates



Neutrino telescopes

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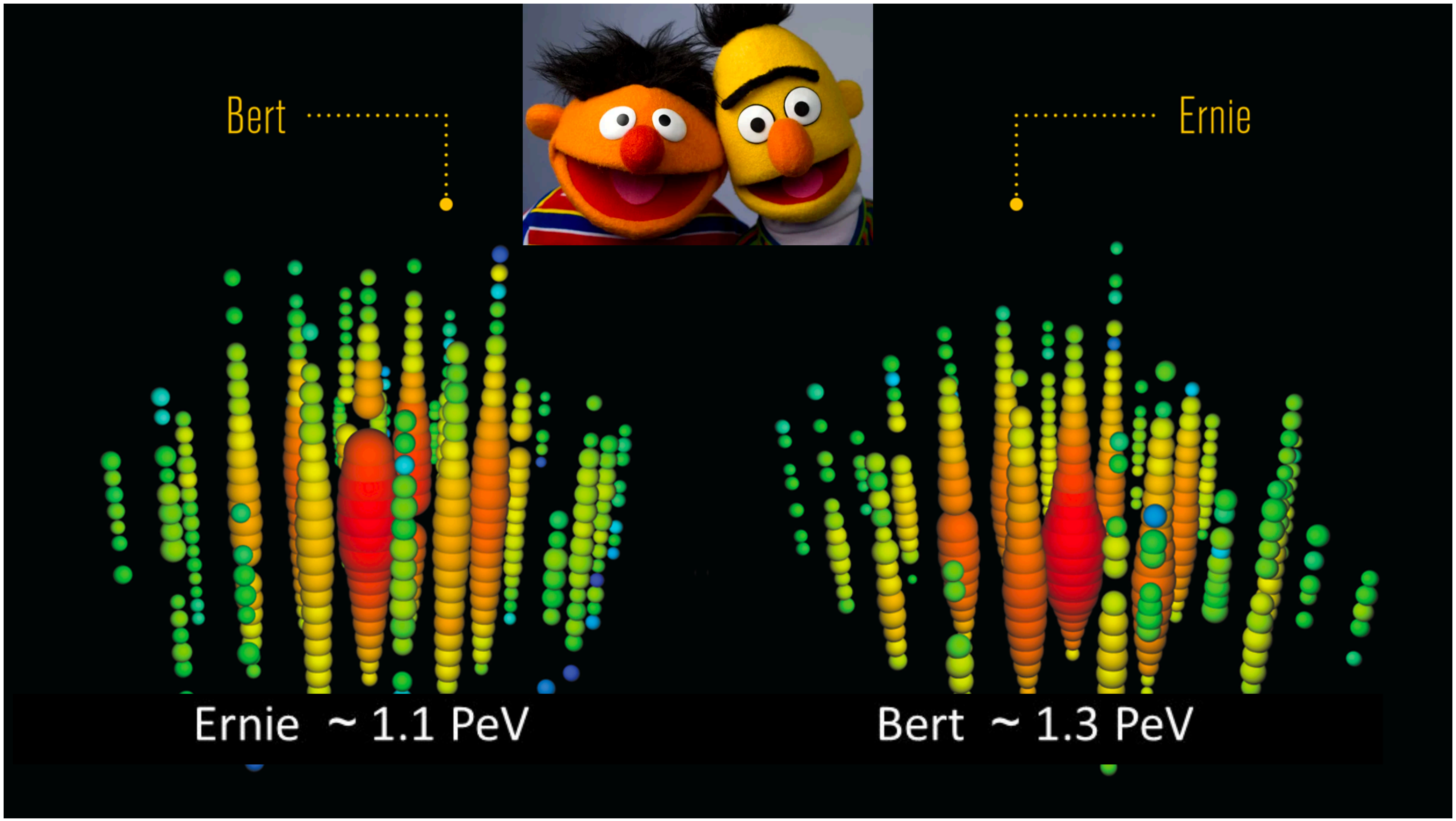
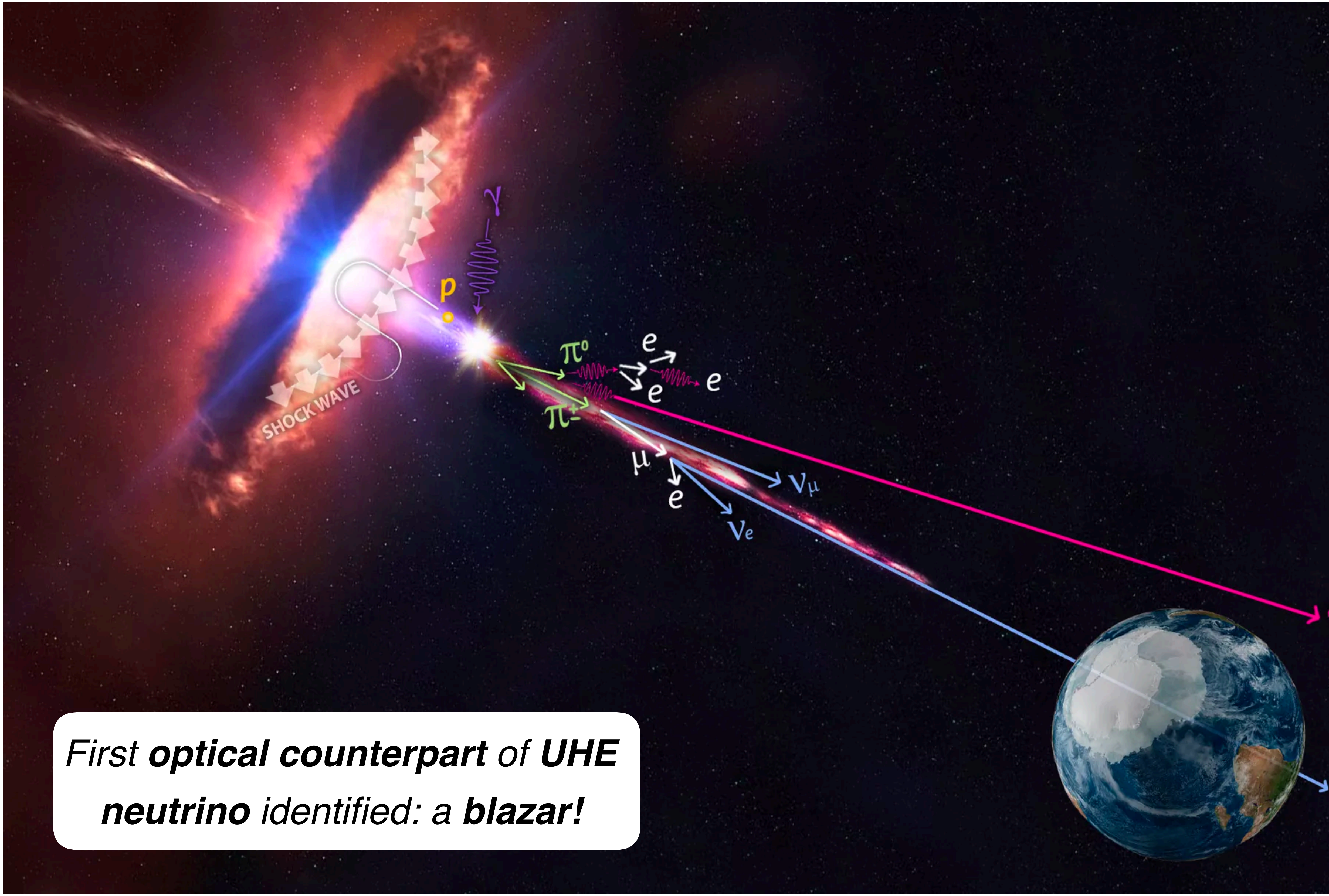


Same or higher centre-of-mass energy than in ***LHC collisions***

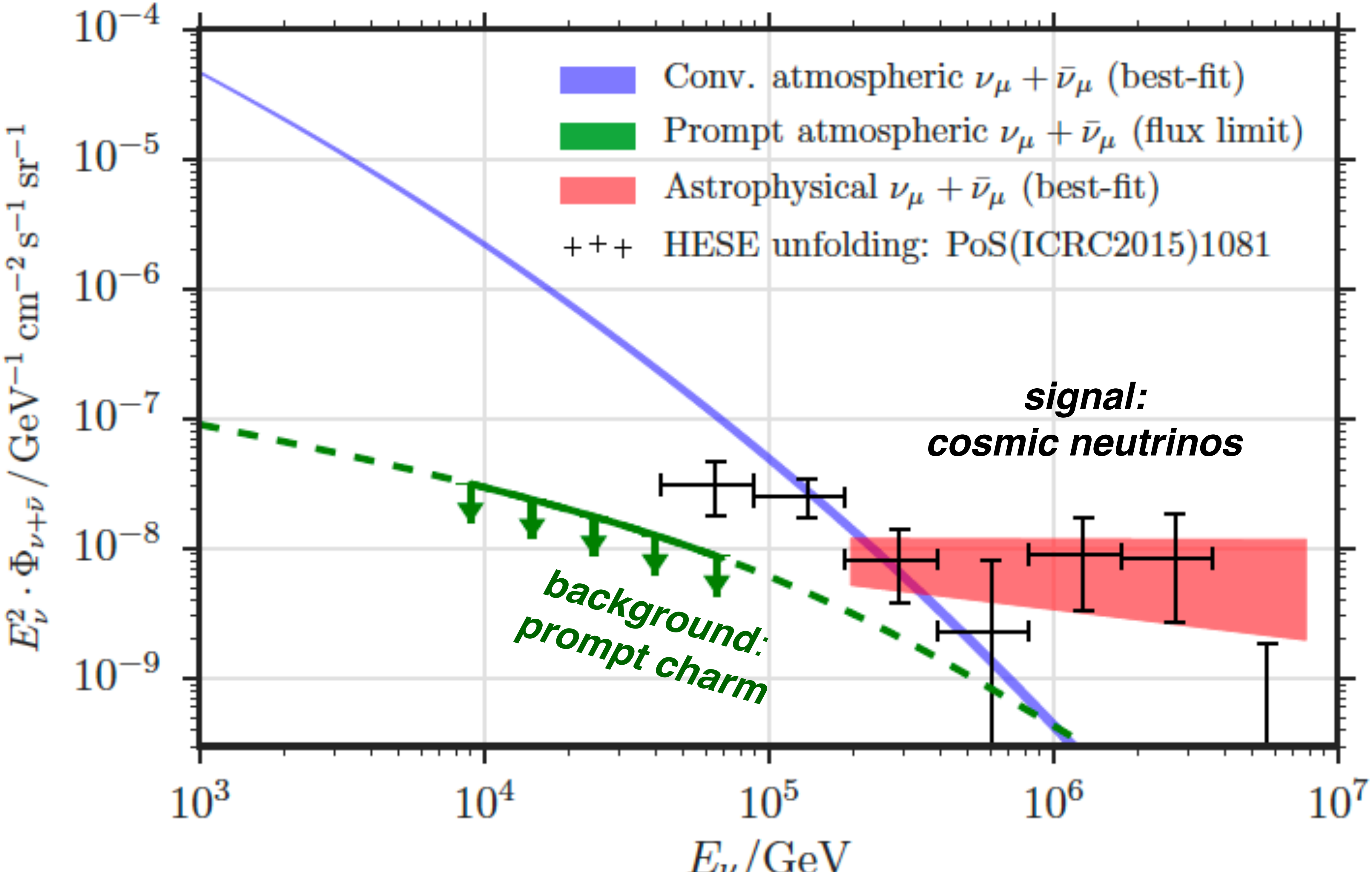
Unveiling cosmic neutrino origin



Neutrino telescopes as particle microscopes

***signal**: cosmic neutrino - nucleus scattering*

***background**: prompt charm production*



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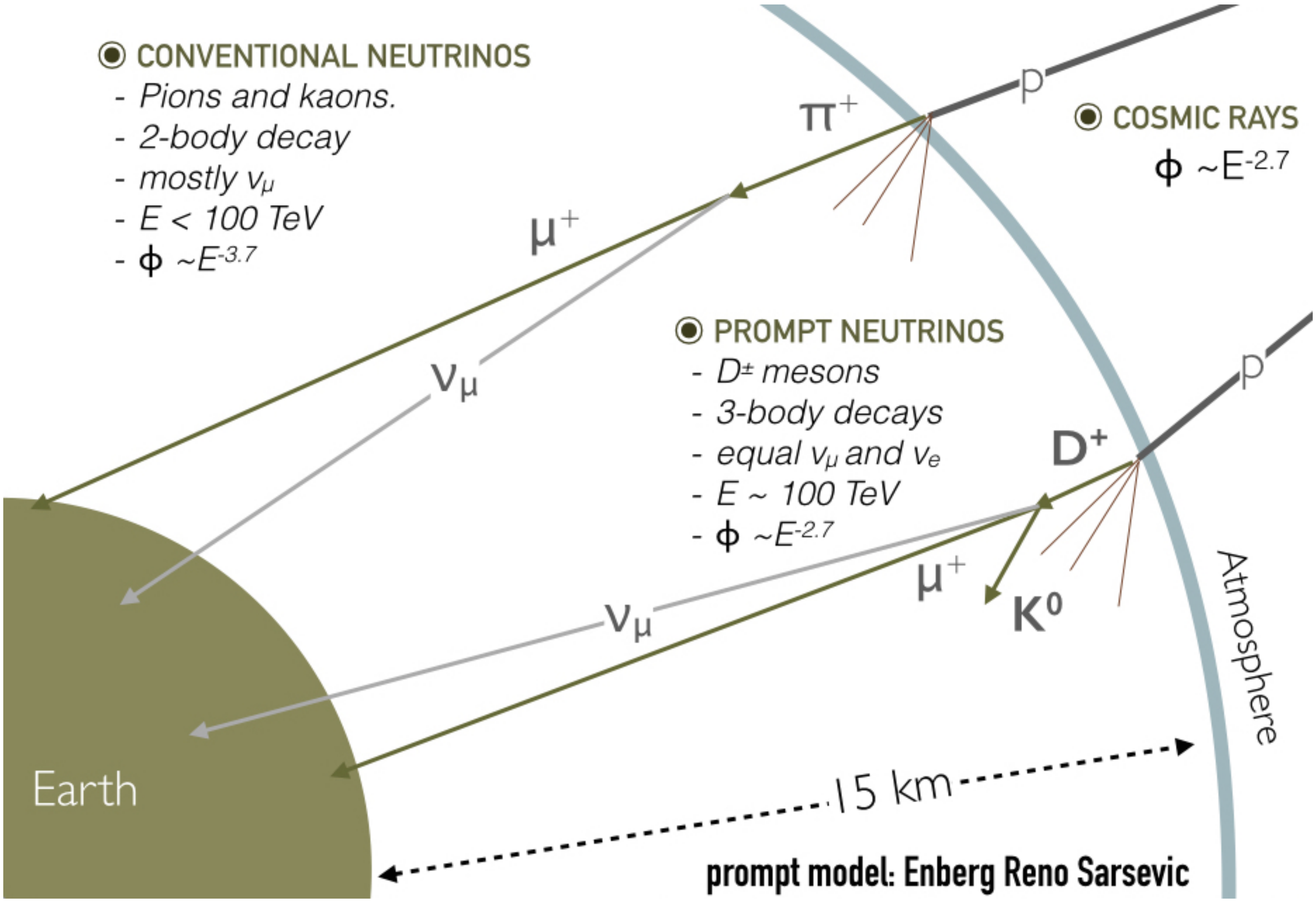

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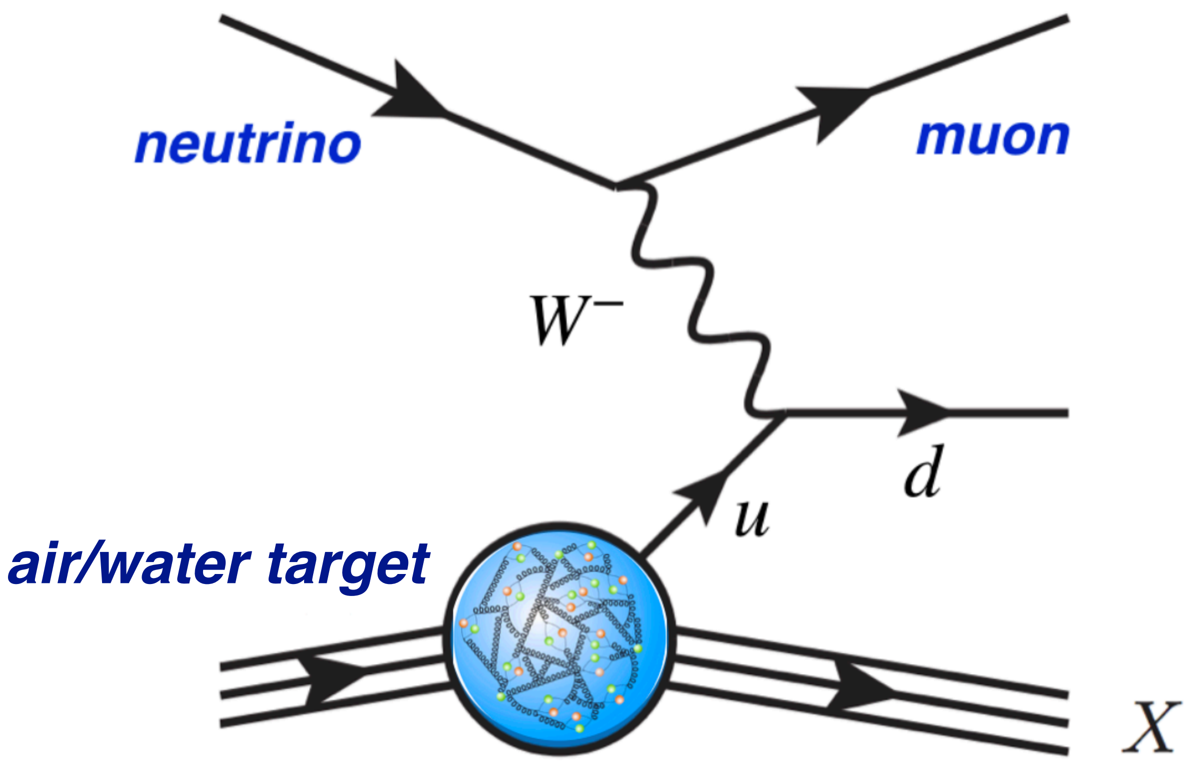
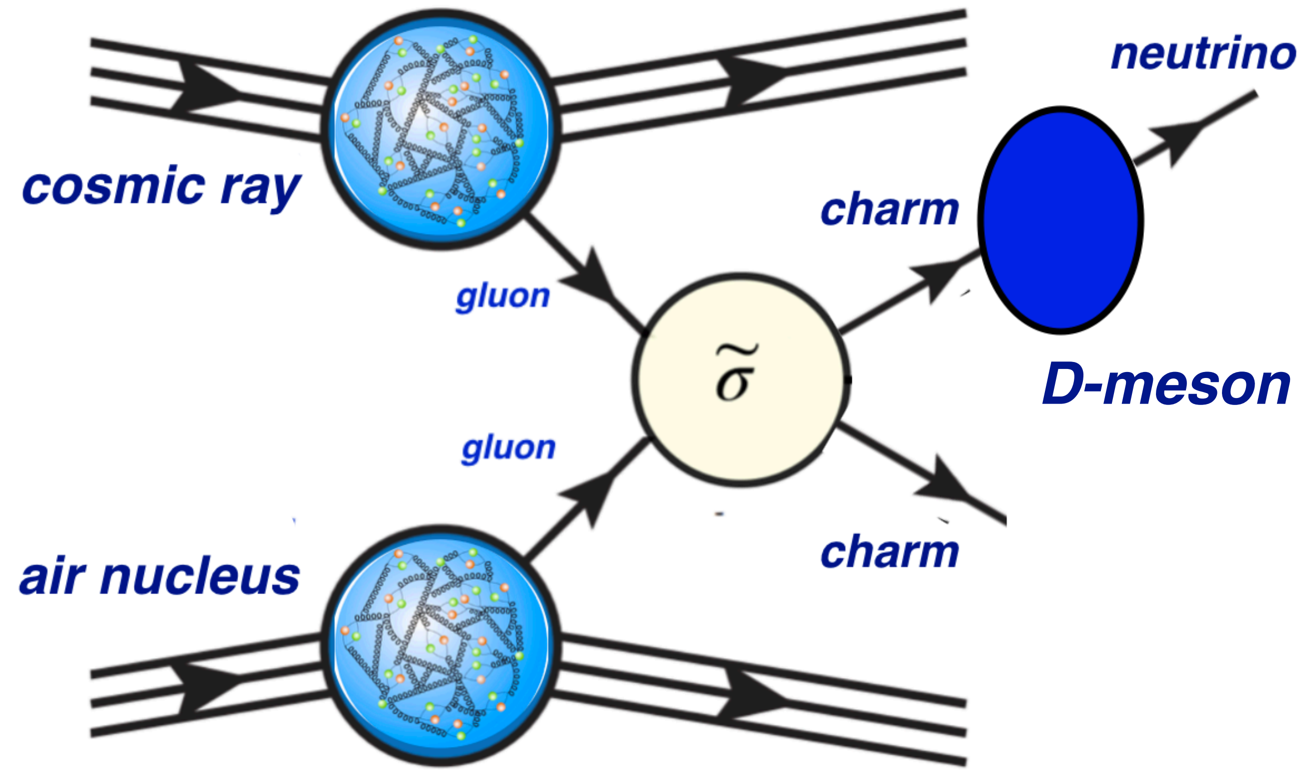




***High-energy particle astrophysics**: unique connection between elementary particle physics, astronomy, and cosmology*

exercise

Why 1 km³ for UHE neutrinos?

The **interaction cross-section σ** measures how likely a given scattering reaction is to take place. It is a kind of **effective collision area** and the units are cm^{-2}

In general, the number of scattering events is given by

$$N_{\rm coll} = \mathscr{L} \times \sigma \times T$$

where ***T*** is the exposure time, and the instantaneous **luminosity L** is given by

$$\mathscr{L} = J \times n_t \times \Delta x$$

J → Flux of incoming particles

n_t → Target number density

Δx → Target thickness

Compute the **thickness of an ice target** if we want to detect at least **10 neutrinos per year** with energy of **10⁵ GeV,** using the following data

$$\sigma(\nu N, E_\nu = 10^5\,{\rm GeV}) = 2 \times 10^{-34}\ {\rm cm}^2$$

$$n_t = 5.5 \times 10^{24}\,{\rm cm}^{-3} \qquad J(E_\nu = 10^5\,{\rm GeV}) = 3 \times 10^{-3}\,{\rm s}^{-1}$$

exercise

Why 1 km^3 for UHE neutrinos?

First of all, compute the **instantaneous luminosity** as a function of the thickness

$$\mathscr{L} = 3 \times 10^{-3}\,\mathrm{s}^{-1} \times 5.5 \times 10^{24}\,\mathrm{cm}^{-3} \times \Delta x = (1.6 \times 10^{22} \times \Delta x)\,\mathrm{cm}^{-3}\mathrm{s}^{-1}$$

Then evaluate the **number of scattering collisions** expected during one year

$$N_{\mathrm{coll}} = \mathscr{L} \times \sigma \times T = (1.6 \times 10^{22} \times \Delta x)\,\mathrm{cm}^{-3}\mathrm{s}^{-1} \times 2 \times 10^{-34}\,\mathrm{cm}^2 \times 3 \times 10^7\,s$$

Finally demand this number to be greater than 10 collisions per year

$$N_{\mathrm{coll}} \sim 10^{-4}\,\mathrm{cm}^{-1} \times \Delta x > 10$$

$$\Delta x > 10^5\,\mathrm{cm} \sim 1\,\mathrm{km}$$

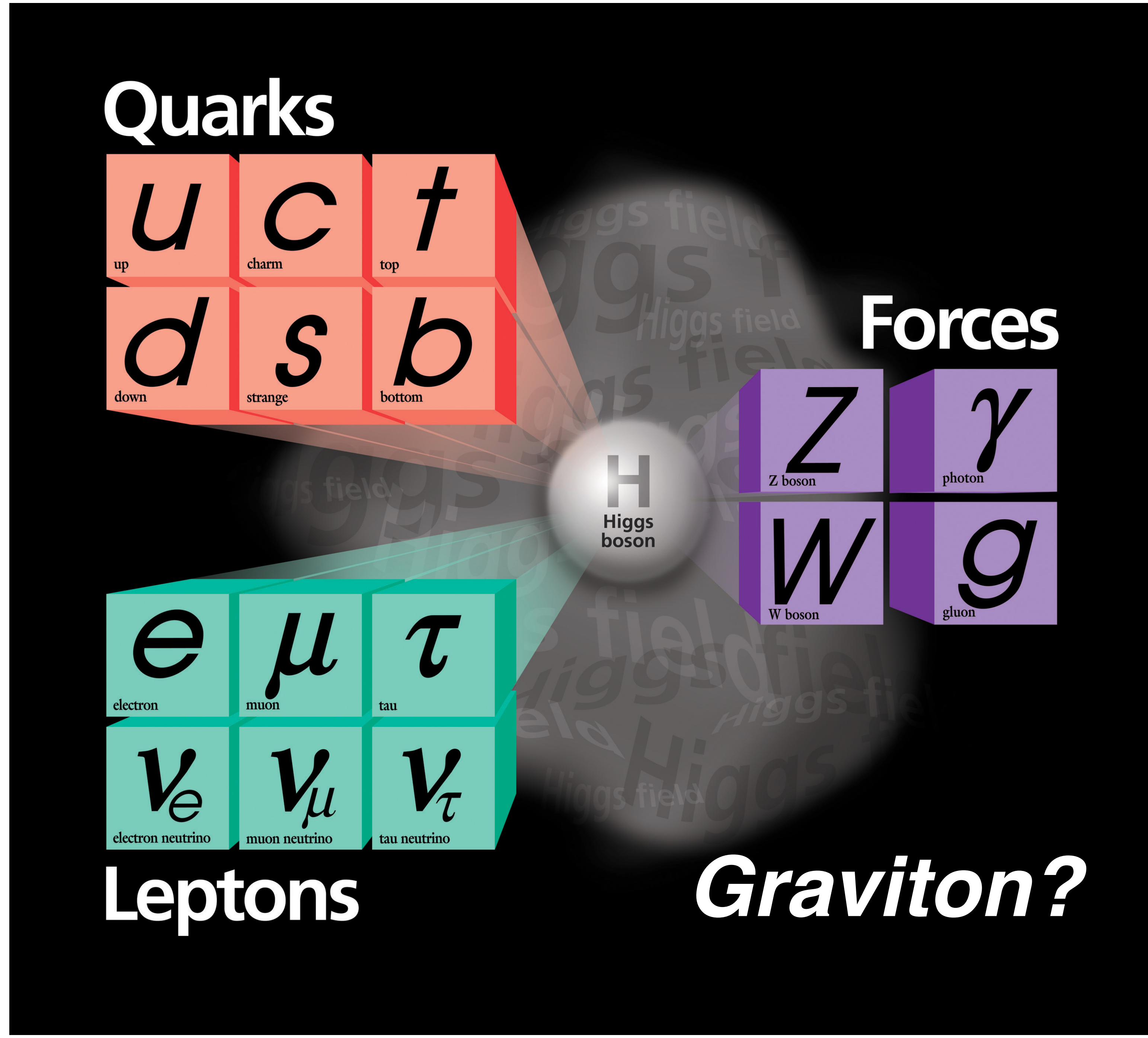
Therefore we need an **ice target of 1 km** to be able to record enough UHE events

This is size instrumented by **IceCube** (south pole) and **KM3NET** (Mediterranean sea)

Gravity and gravitational waves

The gravitational interaction

- The Standard Model does not include the **gravitational interaction**
- **Gravity** is described by Einstein's theory of **General Relativity**
- So far we have not been able to construct a **quantum theory of gravity ….**



exercise

Is gravity strong or weak?

- Of the four fundamental interactions, **gravity's presence** is the one most familiar in everyday life
- Since we notice gravity more than other forces, does it mean that **gravity is much stronger**?

Compare the **gravitational** and **electric forces** between a **proton** and an **electron** within an hydrogen atom: which interaction is stronger?

$$F_g = G\frac{m_p m_e}{r^2} \qquad F_e = k\frac{q_p q_e}{r^2}$$

using the following data:

$$G = 6.7 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}} \qquad k = 9 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}}$$

$$m_e = 9.11 \times 10^{-31}\,\mathrm{kg} \qquad q_e = -q_p = -1.6 \times 10^{-19}\,\mathrm{C}$$

$$m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$$

exercise

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$$F_g = G\frac{m_p m_e}{r^2} \qquad F_e = k\frac{q_p q_e}{r^2}$$

$$\frac{F_g}{F_e} = 10^{-39}$$

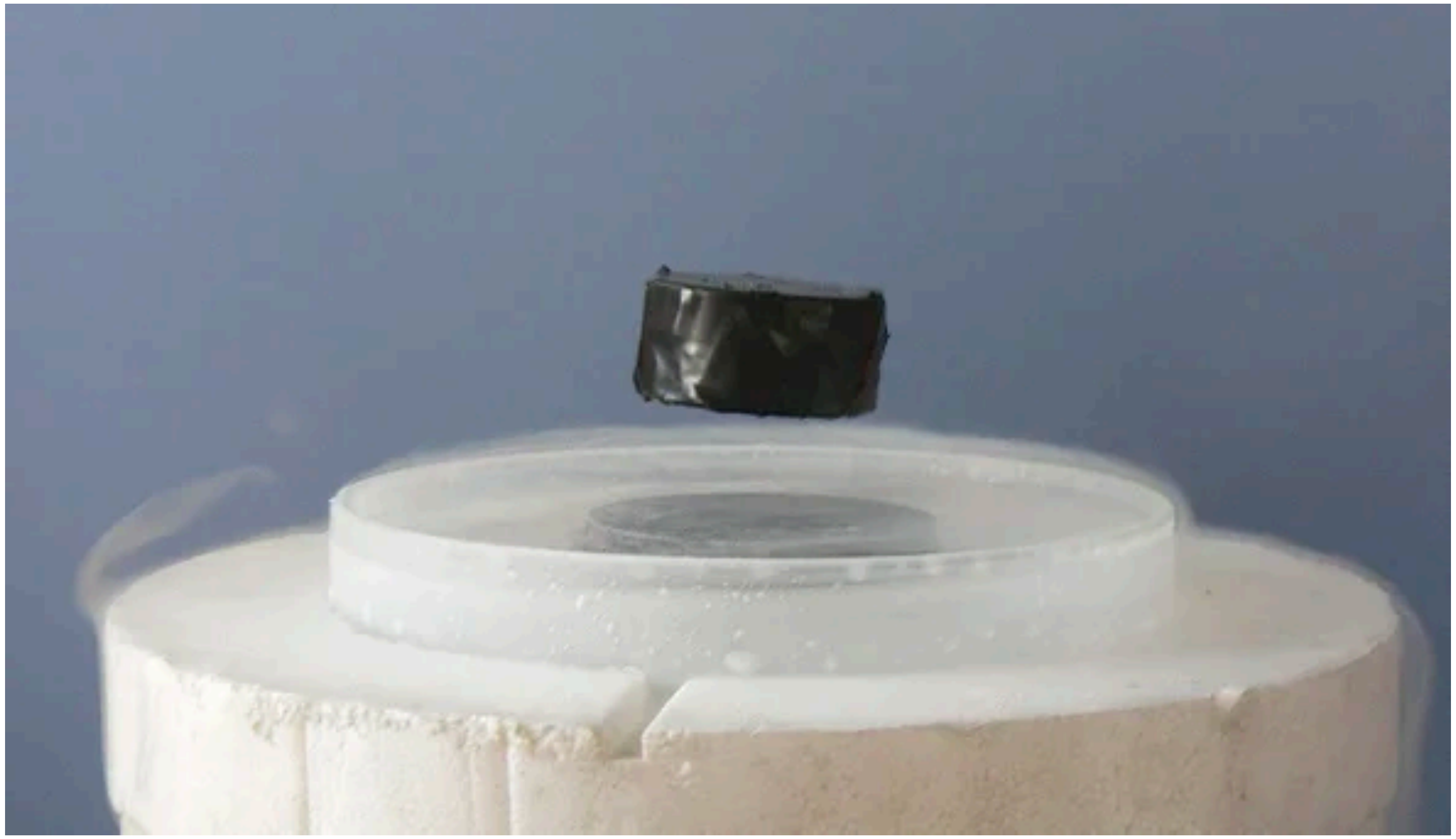
Gravity is **immensely weaker** than electromagnetism!

exercise

Is gravity strong or weak?

A small magnetic field can lift a magnet, **overcoming the gravitational pull** of the whole planet Earth

Why then **appears** that gravity is much stronger than the electric force?



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exercise

Is gravity strong or weak?

A small magnetic field can lift a magnet, **overcoming the gravitational pull** of the whole planet Earth

Why then **appears** that gravity is much stronger than the electric force?

Gravity

All masses are *positive:* gravitational interaction **always attractive**

Electromagnetism

Electric charges can be positive and negative, thus electric interaction either **attractive or repulsive:** large bodies electrically neutral

Gravity is **immensely weaker** than electromagnetism!

Ripples in space-time

Ripples in space-time

Einstein's theory of General Relativity describes **gravity** as **deformations of space-time**

Massive enough bodies, such as black holes, deform space-time and **slow the local time**

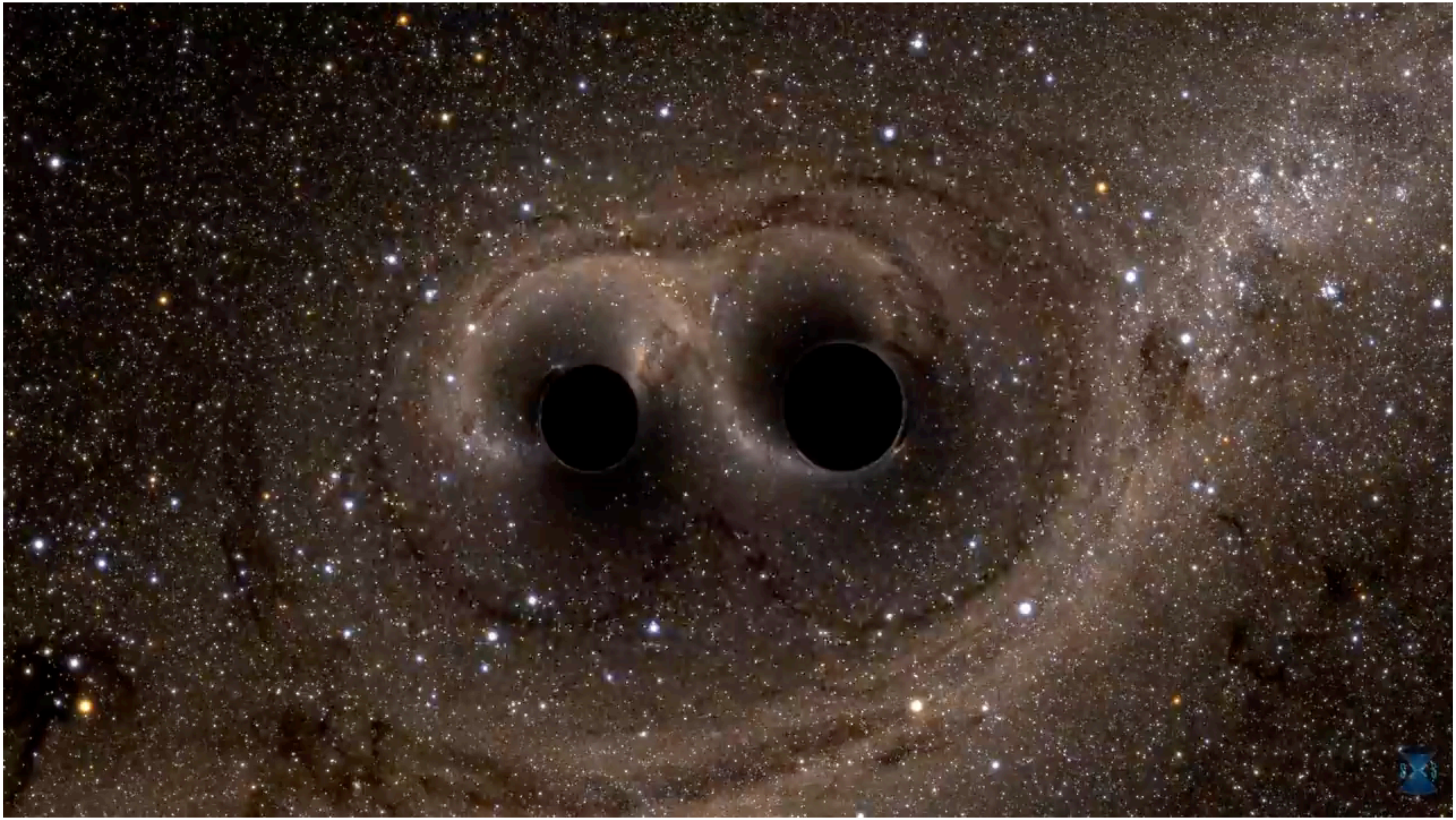
Black holes

Black holes are one of the most fascinating objects in the Universe: **nothing can escape from their attraction**, not even light!

When worlds collide

Black holes can be found in **pairs**, remnants from binary star systems

Black hole mergers are some of the most cataclysmic events that take place in the Universe



If a black hole is, well, black, then how is all this energy released?

When worlds collide

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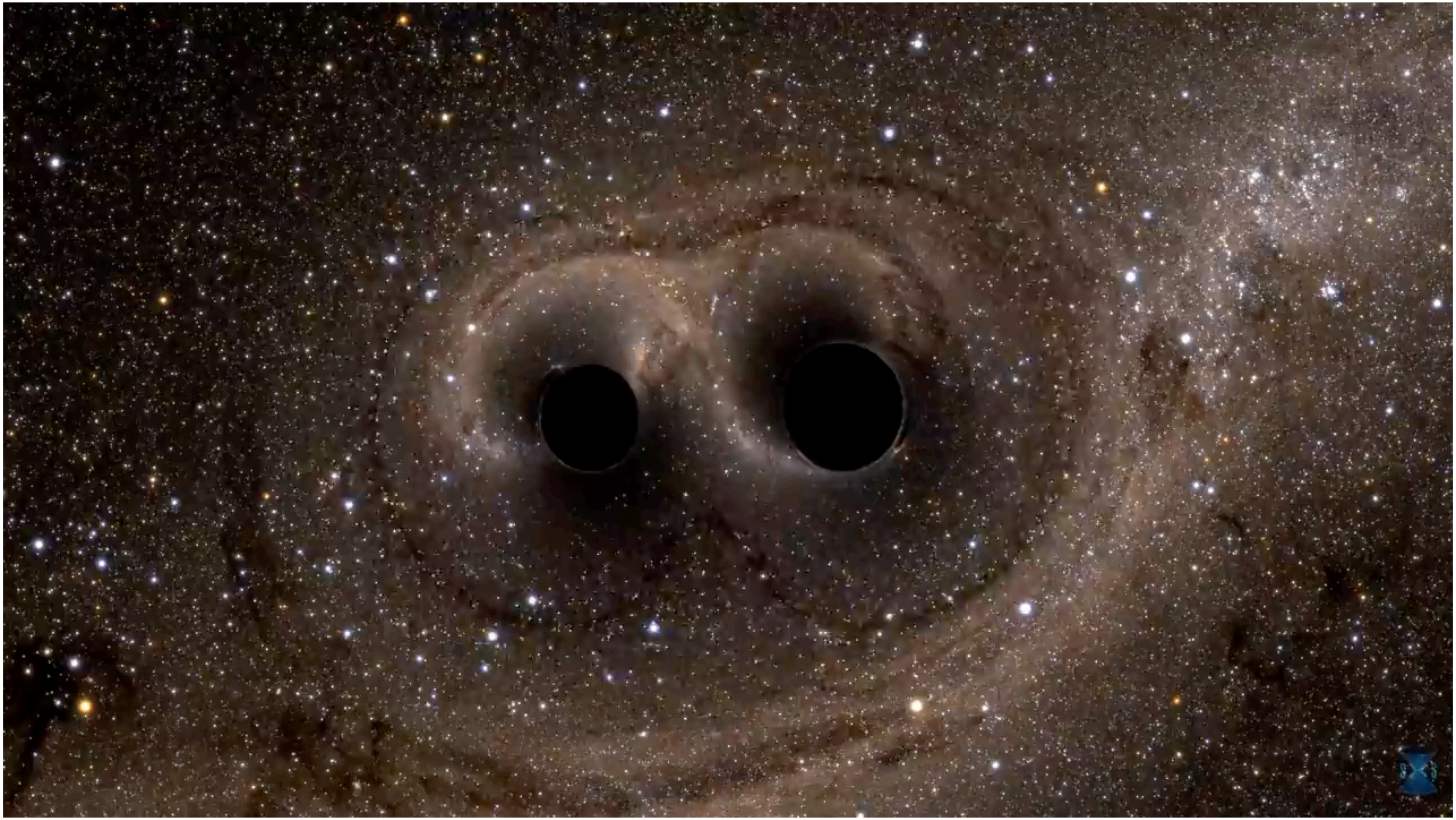


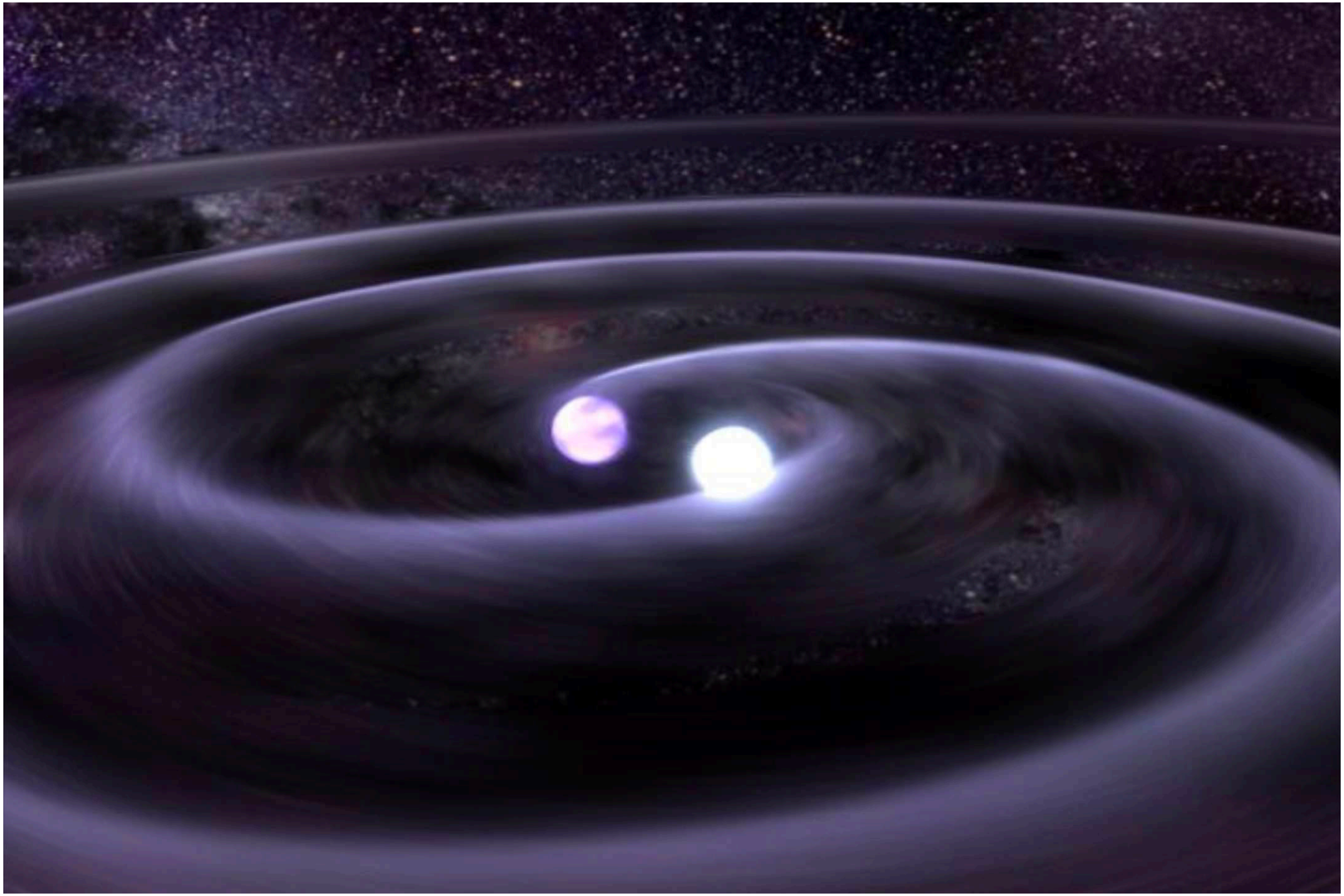
If a black hole is, well, black, then how is all this energy released?

In terms of gravitational waves, ripples of space-time itself!

Ripples in space time

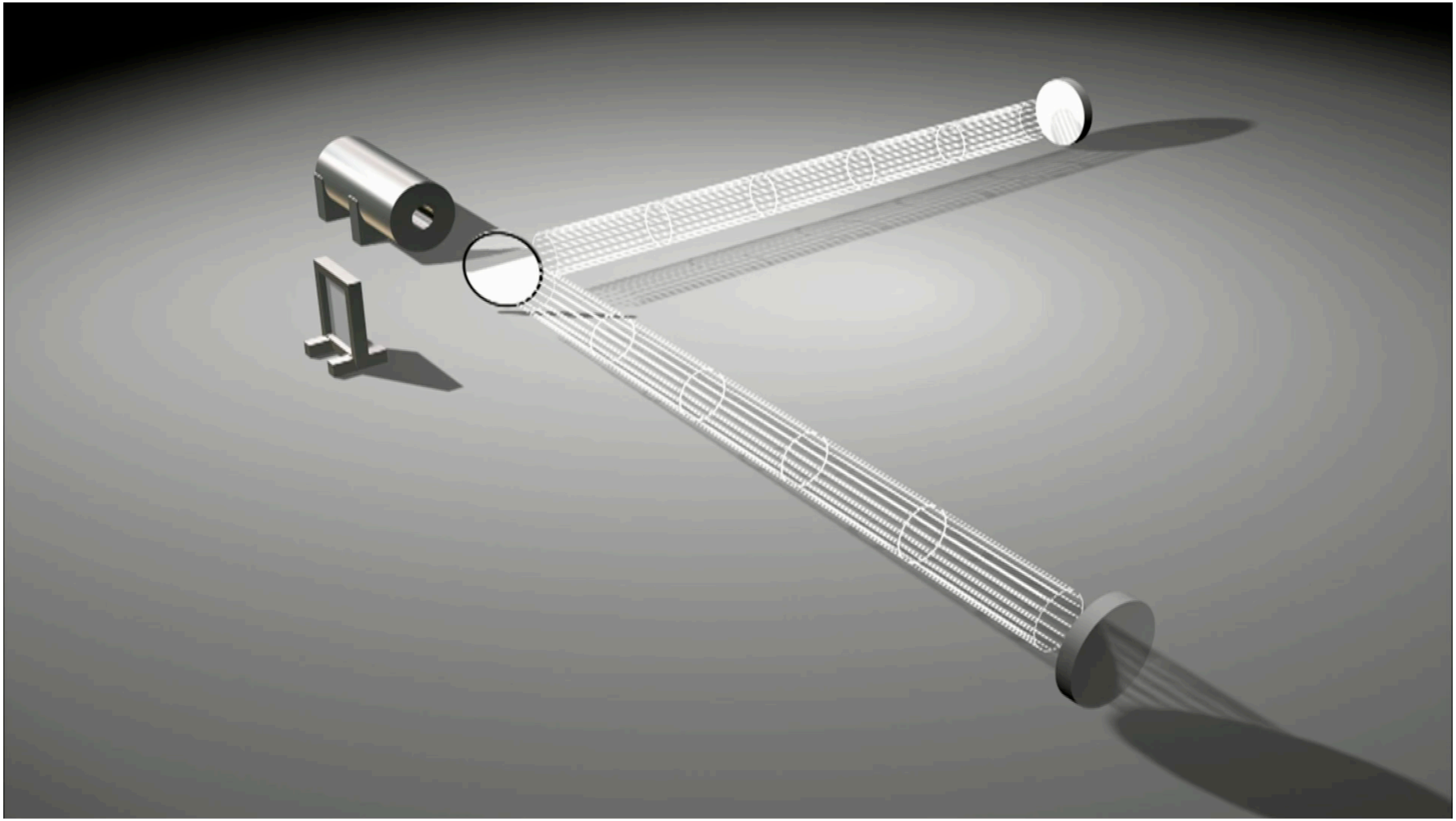
Gravitational waves, unlike matter waves, do not propagate on top of something

space-time itself oscillates*, propagating energy across the universe*



Detecting gravitational waves

Gravitational waves can be detected with **ultra-precise laser interferometers**



Need to measure **length variations of less that 1/1000** of a proton size in the interferometer arms of 4 km each!

Detecting gravitational waves

Nobel Prize in Physics 2017!

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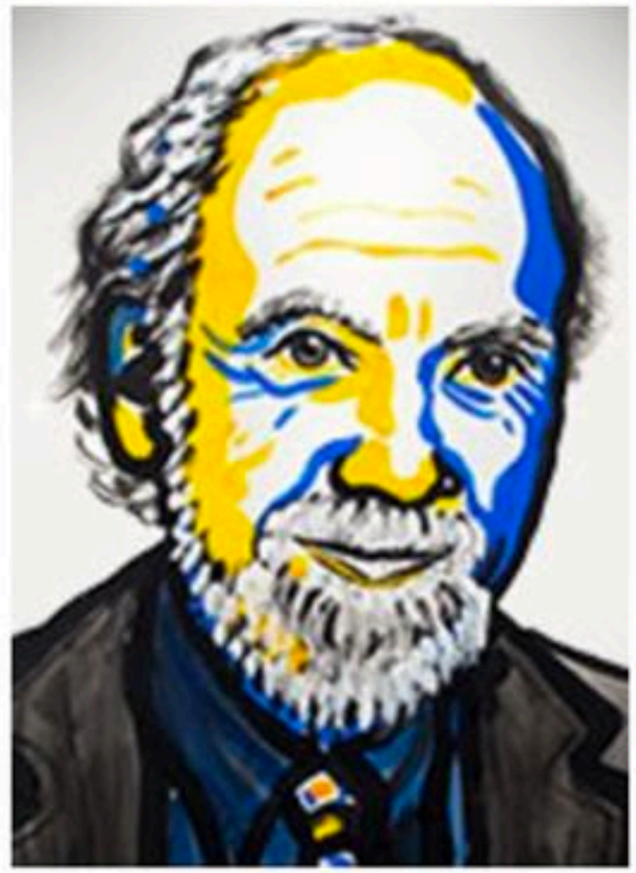
Rainer Weiss

Prize share: 1/2



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Barry C. Barish

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Kip S. Thorne

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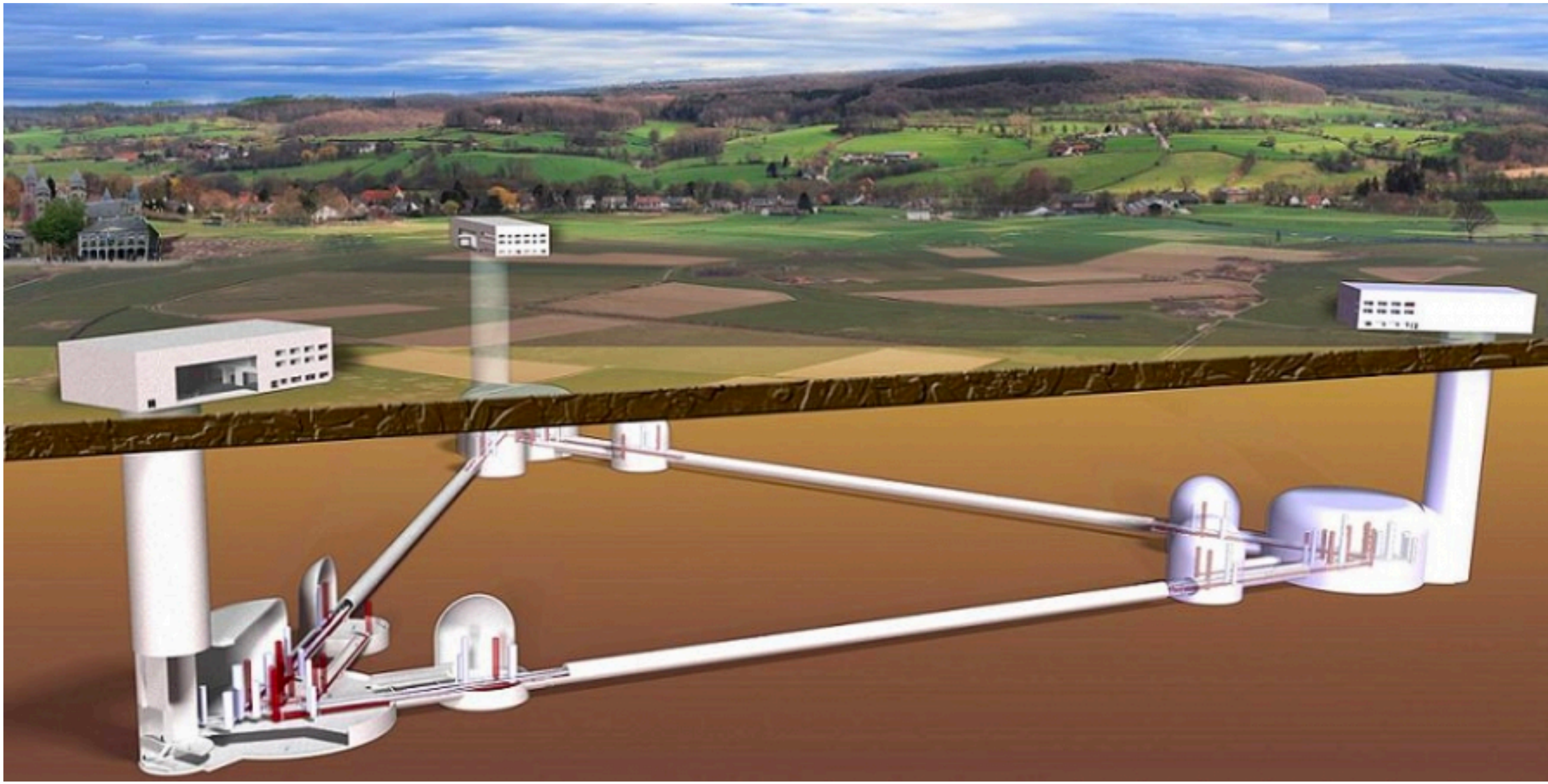
Detecting gravitational waves

Strong **Dutch** involvement in Gravitational Wave detection!



***Einstein Telescope**: a next-generation gravitational wave observatory in Limburg*

Special Relativity for elementary particles

Four-momenta

In Special Relativity **energy** and **linear momenta** are reference-frame dependent

Their description when changing reference frames is facilitated using **four-momenta**

$$p^\mu = \left(E/c, p_x, p_y, p_z\right) \qquad \mu = 0,1,2,3$$

The **norm** of a four-vector is the **same in all reference frames**

$$A^\mu = \left(A^0, A^1, A^2, A^3\right) \to |A| = \left((A^0)^2 - (A^1)^2 - (A^2)^2 - (A^3)^2\right)^{1/2}$$

Note the differences as compared to the **Euclidean form** (Pythagoras Theorem)

For a particle with **mass m** moving at a **speed v** (in 1D), then E and p_x given by

$$E = \left(1 - v^2/c^2\right)^{-1/2} mc^2$$

$$p_x = \left(1 - v^2/c^2\right)^{-1/2} mv$$

exercise

Four-momenta

For a particle with **mass m** moving at a **speed v** (in 1D)**,** then E and p_x given by

$$E = \left(1 - v^2/c^2\right)^{-1/2} mc^2$$
$$p_x = \left(1 - v^2/c^2\right)^{-1/2} mv$$

Compute the norm of its four-momentum. Why it makes sense that this norm is exactly the same in every reference frame?

exercise

Four-momenta

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$$E = \left(1 - v^2/c^2\right)^{-1/2} mc^2$$

$$p_x = \left(1 - v^2/c^2\right)^{-1/2} mv$$

Compute the norm of its four-momentum. Why it makes sense that this norm is exactly the same in every reference frame?

$$p^\mu = \left(E/c, p_x, 0, 0\right) = \left(1 - v^2/c^2\right)^{-1/2} (mc, mv, 0, 0)$$

$$|p^\mu| = \left(1 - v^2/c^2\right)^{-1/2} \left(m^2c^2 - m^2v^2\right)^{1/2} = mc$$

For a single particle, the **norm of its four-momentum is its mass**, which of course should be the same in all reference frame

Time dilation

In special relativity, **time intervals** depend on the observer's reference frame

Assume that in a given reference frame some event lasts for a **period Δt**, as measured by a **co-moving clock**

An observer moving with velocity **u** with respect to this reference frame will measure a different time interval **Δt'** for exactly the same event

$$\Delta t' = \gamma \Delta t = \left(1 - u^2/c^2\right)^{-1/2} \Delta t > \Delta t$$

hence this phenomenon is called **time dilation**

Time dilation also affects elementary particles: for example, a particle with a given **half-life in its rest-frame** will have a **different half-life** in the lab frame

$$\tau|_{\rm lab} = \left(1 - u^2/c^2\right)^{-1/2} \tau|_{\rm rest}$$

exercise

Time dilation

$$\tau|_{\text{lab}} = \left(1 - u^2/c^2\right)^{-1/2} \tau|_{\text{rest}}$$

Example: assume that you have produced a pion moving at ***v=0.9983c*** in the laboratory frame. Evaluate the average decay length of this pion in the lab.

$$\tau|_{\text{rest}} = 2.6 \times 10^{-8}\,\text{s}$$

exercise

Time dilation

$$\tau|_{\mathrm{lab}} = \left(1 - u^2/c^2\right)^{-1/2} \tau|_{\mathrm{rest}}$$

Example: assume that you have produced a pion moving at ***v=0.9983c*** in the laboratory frame. Evaluate the average decay length of this pion in the lab.

$$\tau|_{\mathrm{rest}} = 2.6 \times 10^{-8}\,\mathrm{s}$$

Its **average decay length** will be given by

$$\Delta x = v \times \tau|_{\mathrm{lab}} = v \times \gamma \times \tau|_{\mathrm{rest}} \simeq 70\,\mathrm{m}$$

Without accounting for time dilation, you would have found that the pion should decay on average after 7 m, so your **detector would be all wrong!**

Summary and next steps

- We have presented a general overview of the world of **elementary particles and interactions**

- The theory of elementary particles provides a mathematical language allowing the description of a wide variety of phenomena, from the **structure of nucleons** to **particle colliders** and **high-energy astrophysics**

- After the Christmas break, we will present this language and to explore systematically the **basic properties of elementary particles and interactions,** which determine the outcome of scattering processes

- Please review the basic concepts of **Special Relativity** and **Quantum Theory** (see appendices in lecture notes) since in the following we will use them extensively