Problem Set 2

MMathPhys: The Standard Model

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Quantum Chromodynamics in deep-inelastic scattering and in hadron-hadron collisions

Problems marked with * are more advanced.
To be completed and handed in by **Tuesday Week 6**.

1. **Splitting functions**

   (a) The plus prescription is defined by

   $$\int_0^1 dz f(z)[g(z)]_+ = \int_0^1 [f(z) - f(1)]g(z). \tag{1}$$

   Show that

   $$\left(\frac{1+z^2}{1-z}\right)_+ = \frac{\alpha}{(1-z)_+} + \beta\delta(1-z) + y(z), \tag{2}$$

   where $y(z)$ is an ordinary function, and α and β are rational numbers.

   (b) Consider the splitting functions

   $$P_{q\leftarrow q} = C_F\left(\frac{1+z^2}{1-z}\right)_+ \tag{3}$$

   $$P_{q\leftarrow g} = T_R(z^2 + (1-z)^2). \tag{4}$$

   The anomalous dimensions are given by the moments of the splitting functions,

   $$\gamma_{ij}(N, a_s) = \sum_{n=0}^{\infty} \gamma_{ij}^{(n)}(N)\left(\frac{a_s}{2\pi}\right)^{n+1}, \tag{5}$$

   $$\gamma_{ij}^{(0)}(N) = \int_0^1 dz\, z^{N-1} P_{ij}(z). \tag{6}$$

Show that

$$\gamma^{(0)}_{q\leftarrow q}(N) = C_F \left[-\frac{1}{2} + \frac{1}{N(N+1)} - 2\sum_{k=2}^{N} \frac{1}{k} \right] \tag{7}$$

$$\gamma^{(0)}_{g\leftarrow q}(N) = T_R \frac{2+N+N^2}{N(N+1)(N+2)}. \tag{8}$$

2. **Parton distribution functions and Higgs production in gluon-gluon fusion**

For this exercise, you will need to use the virtual machine that you have configured in the previous problem set. The first part of the exercise is meant to get familiar with parton distribution functions (PDFs) and their uncertainties. In the second part of the exercise you will study Higgs production in gluon fusion at LHC in order to quantify the impact of PDF uncertainties in precision measurements at hadron colliders.

(a) In the folder `problem_set_3` you will find the file `PDF.cc`. Have a look at the file, and understand some of the basic calls of `LHAPDF`. You will find a more detailed documentation at the website `http://lhapdf.hepforge.org`. Compute $g(x)$, $u(x)$, $d(x)$ as a function of x for $Q = 2, 10, 100, 1000$ GeV. With the help of `PDFplotter.py` plot $xg(x)$, $xu(x)$, $xd(x)$ as a function of x at different values of the energy.

(b) Now we will introduce PDF errors. Let O be a generic observable: in this part of the exercise it will be the PDF, but it could be something more generic, such as the total cross section as you will see later. We want to evaluate its central value O_0 and the associated PDF uncertainty, where the total uncertainty can be, in general, not symmetric. There are several collaborations which provides PDFs, and depending on the collaboration the uncertainties have to be calculated following different recipes. You will have to compute PDF uncertainties using the PDF sets `NNPDF30_nnlo_as_0118`, `MMHTnnlo68cl`, and `CT14nnlo`. Each of the PDF sets contains several members: NNPDF30 has 101 members; MMHT14 has xx members, and CT14 as xx members. In the Hessian approach, which is the one used by CT and MMHT collaborations, the central value O_0 is provided by `mem=0`, and the (asymmetric) PDF uncertainty is computed according to the formula

$$\sigma_+ = \sqrt{\sum_{i=1}^{N_{\rm mem}/2} \left(\max[O_{2i-1} - O_0, O_{2i} - O_0, 0]\right)^2}$$

$$\sigma_- = \sqrt{\sum_{i=1}^{N_{\rm mem}/2} \left(\max[O_0 - O_{2i-1}, O_0 - O_{2i}, 0]\right)^2} \tag{9}$$

where O_i is the value of the observable evaluated with the PDF set `mem=i`. In the NNPDF approach, the central value is obtained by computing the mean value

$$O_0 = \langle O \rangle_{\rm rep} = \frac{1}{N_{\rm rep}} \sum_{j=1}^{N_{\rm rep}} O_j \tag{10}$$

and the (symmetric) error is obtained by computing the standard deviation

$$\sigma = \left[\frac{1}{N_{\text{rep}} - 1} \sum_{j=1}^{N_{\text{rep}}} (O_j - O_0)^2 \right]^{1/2} . \quad (11)$$

Compute $g(x)$, $u(x)$, $d(x)$ as a function of x for the same values of point (a) including PDF uncertainties. Typing now in the terminal `./PDFplotter.py -b` plot $xg(x)$, $xu(x)$, $xd(x)$ showing also the PDF uncertainty bands. Comment your results.

(c) In the last part of the exercise you will compute the cross section for Higgs production in gluon fusion at NNLO with the program `ggHiggs`. In the folder `problem_set_3/Higgs` you will find the program `ggHiggs.cc`.

- You can obtain the predictions for Higgs production with E_{CM} = Energy [GeV] and a given PDF set by typing in the terminal

  ```
  ./ggHiggs -p PDF_SET_NAME -E Energy.
  ```

  Obtain the predictions for the following PDF sets: `NNPDF30_nnlo_as_0118`, `MMHTnnlo68cl`, `CT14nnlo` at 13 and 14 TeV.
- Now you will have to compute the PDF uncertainties following the recipe of point (b). Compute central value and uncertainties for `NNPDF30_nnlo_as_0118` at 13 and 14 TeV.
- Compute the cross section for Higgs boson production in gluon fusion as a function of E_{CM} between 7 and 100 GeV with different PDF sets, including the PDF error. Print your results in the files `gg_PDF_SET_NAME.dat`. Then type in the terminal

  ```
  ./plotggHiggs.py -p PDF_SET_NAME_1 PDF_SET_NAME_2 ...
  ```

  where you should replace `PDF_SET_NAME_i` with the PDF sets that you have used to compute the predictions. Using the option `-b` you will also plot the PDF uncertainties. You should obtain a file `gg_predictions.pdf`. Have a look at the figure produced. What is the behaviour of the cross section as a function of the energy? What is the behaviour of the PDF uncertainty? Are the predictions obtained with different PDF sets compatible? Can you relate what you have obtained to the behaviour of the gluon PDF that you have studied in point (a) and (b)?

3*. **Photoproduction of heavy quarks**

Consider the process of heavy quark pair photoproduction, $\gamma + p \to Q\bar{Q} + X$, for a heavy quark of mass M and electric charge Q. If M is large enough, any diagram contributing to this process must involve a large momentum transfer; thus a perturbative QCD analysis should apply. Work out the cross-section to the leading order in QCD. Choose the parton subprocess that gives the leading contribution to this reaction, and write the parton-model expression for the cross section. You will need to compute the relevant subprocesses cross section in order to do so, but you might find useful to take the results from the analogous QED calculations (e.g. Peskin-Schroeder Chapt. 5). Use the result to write an expression for the cross section for γ-proton scattering.