Problem Set 1

MMathPhys: The Standard Model

Lecturer: Juan Rojo
Teaching Assistant: Luca Rottoli

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Quantum Chromodynamics in e^+e^- collisions

Problems marked with * are more advanced.
To be completed and handed in by **Tuesday Week 4**.

1. **Two-loop renormalization group relations**

   (a) In higher order of perturbation theory, the expression for the QCD β function is a series

   $$\beta(g) = -\frac{b_0}{(4\pi)^2}g^3 - \frac{b_1}{(4\pi)^4}g^5 - \frac{b_2}{(4\pi)^6}g^7 + \dots . \quad (1)$$

   Integrate the renormalization group equation and show that the running coupling constant is now given by

   $$\alpha_s(Q^2) = \frac{4\pi}{b_0}\left[\frac{1}{\ln(Q^2/\Lambda^2)} - \frac{b_1}{b_0^2}\frac{\ln\ln(Q^2/\Lambda^2)}{(\ln(Q^2/\Lambda^2))^2} + \dots\right], \quad (2)$$

   where the omitted terms decrease as $(\ln(Q^2/\Lambda^2))^{-2}$.

   (b) Combine this formula with the perturbation series for the e^+e^- annihilation cross section:

   $$\sigma(e^+e^- \to \text{hadrons}) = \sigma_0 \cdot \left(3\sum_f Q_f^2\right) \cdot \left[1 + \frac{\alpha_s}{2\pi} + a_2\left(\frac{\alpha_s}{2\pi}\right)^2 + O(\alpha_s^3)\right]. \quad (3)$$

   The coefficient a_2 depends on the details of the renormalization conditions defining α_s. Show that the leading two terms in the asymptotic behaviour of $\sigma(s)$ for large s depend only on b_0 and b_1 and are independent of a_2 and b_2. Thus the first two coefficients of the QCD β function are independent of the renormalization prescriptions.

2*. $\sigma(\mathrm{e^+e^-} \to \mathrm{q\bar{q}g})$

(a) Show that the phase space for the unpolarized decay into three massless objects can be written as:

$$d^5\phi_3 = \frac{1}{(2\pi)^5}\frac{s}{32}dx_1dx_2d\phi_{12}d\Omega_1 \tag{4}$$

where s is the c.m.s. energy and $x_i = 2E_i/\sqrt{s}$ are the fractional energies for the quark and anti-quark.

(b) Calculate the matrix element squared for $e^+e^- \to q\bar{q}g$. Use the fact that we are interested only in azimuthal averaged quantities and therefore we neglect angular correlations between the initial state plane and the final state one, so one can write

$$|M|^2 = \frac{1}{s^2}L^{\mu\nu}H_{\mu\nu} \to \frac{1}{s^2}(L^{\mu\nu}g_{\mu\nu})(H^{\rho\sigma}g_{\rho\sigma}) \tag{5}$$

where $L^{\mu\nu}$ and $H^{\mu\nu}$ are the leptonic and hadronic tensors that come from the squaring of the the corresponding currents. The result to be found is:

$$\sigma^{q\bar{q}g} = \sigma^{LO}C_F\frac{\alpha_s}{2\pi}\int dx_1dx_2\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}. \tag{6}$$

3. **Jet Algorithms**

The last exercise is meant to get acquainted with the event generator `PYTHIA` and to try different jet algorithms. You will use two different routines, FastJet and SlowJet. Their results should be equal, but their performances are very different.

(a) In the folder `problem_set_1` you will find the file `example_jets.cc`. Run the program a first time. It will produce four plots in the folder `problem_set_1/plots`. They will contain the distributions of the number of jets N_{jets} and of the transverse momentum $p_{T,\text{jet}}$ for SlowJet and for FastJet. Compare the plots.

(b) Using the existing histograms as a template, create and plot an additional histogram, $N_{\text{jets}}^{\text{Fast}} - N_{\text{jets}}^{\text{Slow}}$. Are the two routines really identical?

(c) Plot the histograms `tSlow` and `tFast`, which are already filled in the program. Is FastJet considerably faster than SlowJet?

(d) Using the FastJet routine, compare the distributions of $p_{T,\text{jet}}$ obtained using the anti-k_T, Cambridge-Aachen and k_T jet algorithms.

(e) Using the FastJet routine, plot N_{jets} and $p_{T,\text{jet}}$ for different values of R: $0.4,\ 0.6,\ 0.8$.