

# Towards NNPDF4.0: The Structure of the Proton to One-Percent Accuracy

XXVIII International Workshop on Deep-Inelastic Scattering and Related Subjects

Emanuele R. Nocera  
School of Physics and Astronomy, The University of Edinburgh  
on behalf of the NNPDF Collaboration

April 13, 2021



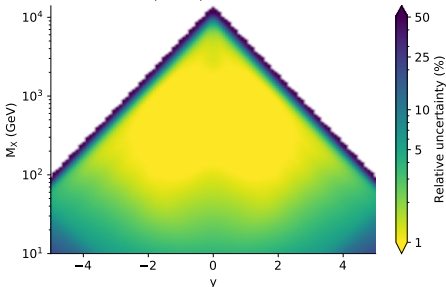
# From NNPDF3.1 to NNPDF4.0

$$\mathcal{L}_{ij}(M_X, y, \sqrt{s}) = \frac{1}{s} \sum_{i,j} f_i \left( \frac{M_X e^y}{\sqrt{s}}, M_X \right) f_j \left( \frac{M_X e^{-y}}{\sqrt{s}}, M_X \right)$$

SINGLET

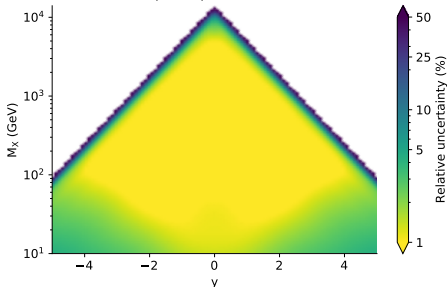
NNPDF3.1 (NNLO)

Relative uncertainty for qq-luminosity  
NNPDF3.1 (NNLO) -  $\sqrt{s} = 14000.0$  GeV



NNPDF4.0 (NNLO)

Relative uncertainty for qq-luminosity  
NNPDF4.0 (NNLO) -  $\sqrt{s} = 14000.0$  GeV



Steady progress towards 1% relative uncertainties on  $\mathcal{L}_{ij}$  on a broad kinematic range

How are we getting there?

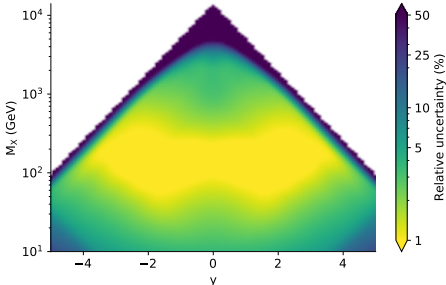
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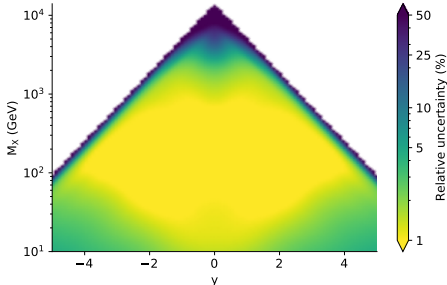
### NNPDF3.1 (NNLO)

Relative uncertainty for  $q\bar{q}$ -luminosity  
NNPDF3.1 (NNLO) -  $\sqrt{s} = 14000.0$  GeV



### NNPDF4.0 (NNLO)

Relative uncertainty for  $q\bar{q}$ -luminosity  
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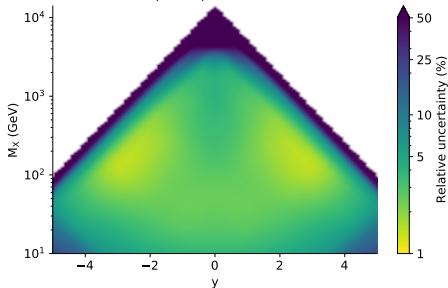
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## FLAVOURS

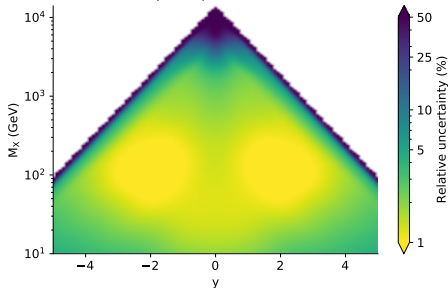
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Relative uncertainty for  $\bar{u}$ -luminosity  
NNPDF3.1 (NNLO) -  $\sqrt{s} = 14000.0$  GeV



### NNPDF4.0 (NNLO)

Relative uncertainty for  $\bar{u}$ -luminosity  
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Steady progress towards 1% relative uncertainties on  $\mathcal{L}_{ij}$  on a broad kinematic range

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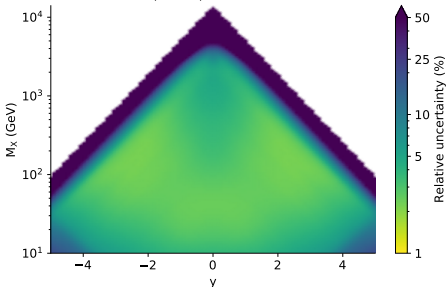
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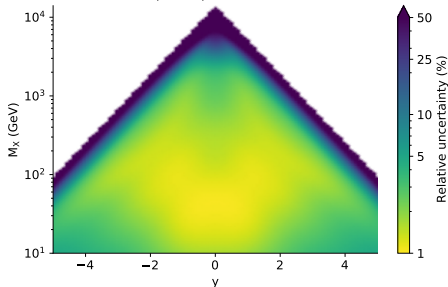
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Relative uncertainty for  $d\bar{u}$ -luminosity  
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Steady progress towards 1% relative uncertainties on  $\mathcal{L}_{ij}$  on a broad kinematic range

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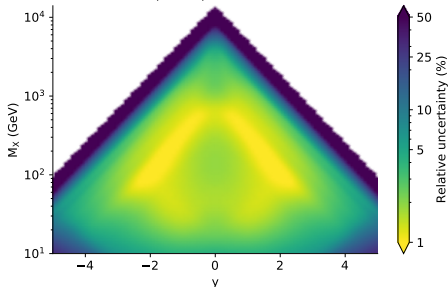
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## GLUON

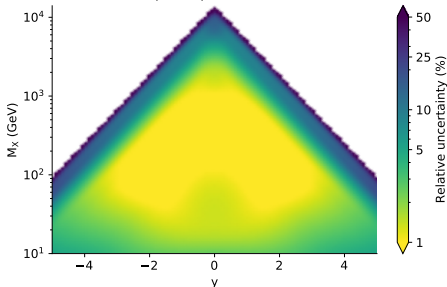
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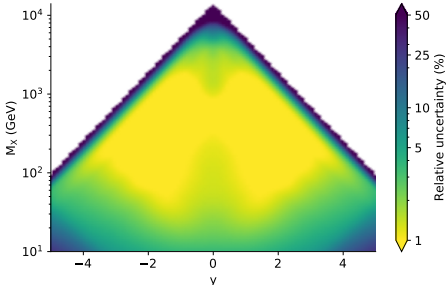
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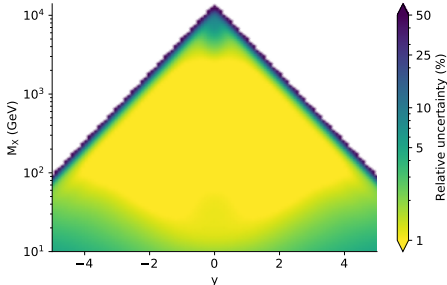
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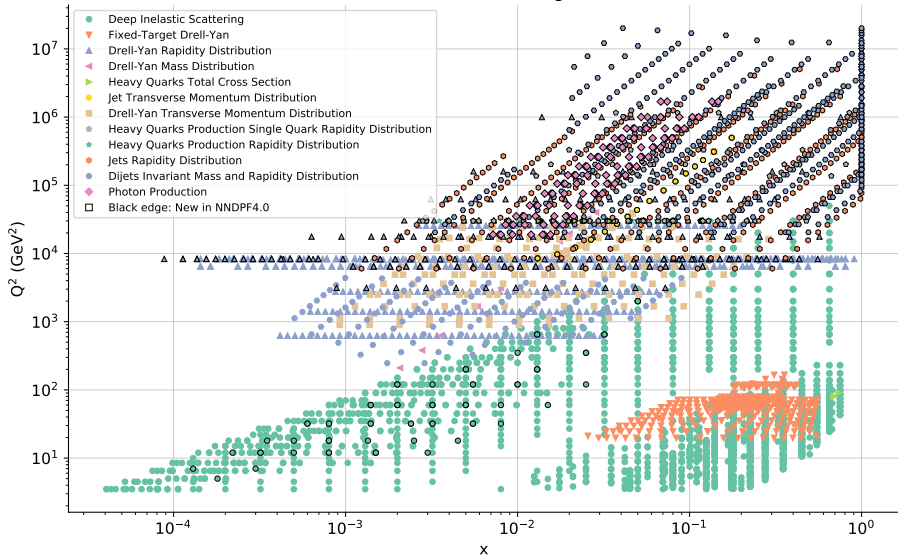


Steady progress towards 1% relative uncertainties on  $\mathcal{L}_{ij}$  on a broad kinematic range

How are we getting there?

# NNPDF4.0: data set extension

## Kinematic coverage



$\mathcal{O}(50)$  data sets investigated;  $\mathcal{O}(400)$  data points more in NNPDF4.0 than in NNPDF3.1



# NNPDF4.0: new data sets

| Process           | Experiment | Description   | Reference                                  |
|-------------------|------------|---|--|
| DIS               | HERA       | Combined reduced $c$ and $b$ cross sections                                 | [EPJ C78 (2018) 473]                       |
|                   | NOMAD*     | $\mathcal{R}_{\mu\mu}(E) = \sigma_{\mu\mu}(E)/\sigma_{CC}(E)$               | [NPB 876 (2013) 339]                       |
| DY                | ATLAS      | $W, Z$ central/forward rapidity distr., <b>7 TeV</b>                        | [EPJ C77 (2017) 367]                       |
|                   | ATLAS      | $\{m_{\ell\ell},  y_{\ell\ell} \}$ $Z$ high-mass distribution, <b>8 TeV</b> | [JHEP 08 (2016) 009]                       |
|                   | ATLAS      | $\{m_{\ell\ell},  y_{\ell\ell} \}$ $Z$ distribution, <b>8 TeV</b>           | [JHEP 12 (2017) 059]                       |
|                   | ATLAS      | $W$ rapidity distr., <b>8 TeV</b>   | [EPJ C79 (2019) 760]                       |
|                   | ATLAS      | $W$ and $Z$ total cross section, <b>13 TeV</b>                              | [PLB 759 (2016) 601]                       |
|                   | LHCb       | $y_Z$ distribution, $2e$ and $2\mu$ , <b>13 TeV</b>                         | [JHEP 09 (2016) 136]                       |
| $W+c$             | ATLAS†     | $ \eta^\ell $ distribution <b>7 TeV</b>                                     | [JHEP 05 (2014) 068]                       |
|                   | CMS†       | $ \eta^\mu $ distribution <b>13 TeV</b>                                     | [EPJ C79 (2019) 269]                       |
| single-jet        | ATLAS      | $\{p_T,  y \}$ distribution, <b>8 TeV</b>                                   | [JHEP 09 (2017) 020]                       |
| $t\bar{t}$        | CMS        | total inclusive cross section, <b>5 TeV</b>                                 | [JHEP 03 (2018) 115]                       |
|                   | CMS        | normalised $\{m_{t\bar{t}}, y_t\}$ distribution, <b>8 TeV</b>               | [EPJ C77 (2017) 459]                       |
|                   | CMS        | normalised $y_t$ distribution (dilepton), <b>13 TeV</b>                     | [JHEP 02 (2019) 149]                       |
|                   | CMS        | normalised $y_t$ distribution (lepton+jet), <b>13 TeV</b>                   | [PRD 97 (2018) 112003]                     |
| <b>single top</b> | ATLAS      | $R_t$ <b>7, 8, 13 TeV</b>   | [JHEP 04 (2017) 086]                       |
|                   | ATLAS      | normalised $y_t$ and $y_{\bar{t}}$ distributions, <b>7, 8 TeV</b>           | [PRD 90 (2014) 112006; EPJ C77 (2017) 531] |
|                   | CMS        | $t + \bar{t}$ cross section, <b>7 TeV</b>                                   | [JHEP 12 (2012) 035]                       |
|                   | CMS        | $R_t$ <b>8, 13 TeV</b>  | [JHEP 06 (2014) 090; PLB 772 (2017) 752]   |
| $W$ +jet          | ATLAS      | $p_T$ distribution, <b>8 TeV</b>  | [JHEP 05 (2018) 077]                       |
| isolated photon   | ATLAS      | $\{E_T^\gamma,  \eta^\gamma \}$ distribution, <b>13 TeV</b>                 | [PLB 770 (2017) 473]                       |
| di-jets           | ATLAS      | $\{m_{12}, y^*\}$ distribution <b>7 TeV</b>                                 | [JHEP 05 (2014) 059]                       |
|                   | CMS        | $\{m_{12},  y_{\max} \}$ distribution <b>7 TeV</b>                          | [PRD 87 (2013) 112002]                     |
|                   | CMS        | $\{p_{T,\text{avg}}, y_b, y^*\}$ distribution <b>8 TeV</b>                  | [EPJ C77 (2017) 746]                       |
| <b>DIS+jets</b>   | H1*        | Single- and di-jet differential distributions                               | [EPJ C75 (2015) 65; C77 (2017) 215]        |

\* Not in baseline fit; studied via reweighting

† Only NLO fit

Processes highlighted in **red** correspond to processes NOT in NNPDF3.1

# NNPDF4.0: theoretical and methodological features

- Refined theoretical framework [[EPJ C79 \(2019\) 282](#); [EPJ C81 \(2021\) 37](#); [EPJ C80 \(2020\) 1168](#)];
  - nuclear uncertainties for both deuteron and heavy nuclei included by default
  - NNLO charm-quark massive corrections implemented (a bug in the NLO corrected)
  - EW corrections not included to ensure consistency with data, but carefully checked
  - charm PDF parametrised on the same footing as other PDFs
- Improved implementation of PDF properties [[JHEP 11 \(2020\) 129](#)]
  - extended positivity constraints for light quark/antiquark and gluon PDFs
  - extended integrability constraints of non-singlet light quark PDF combinations
- New PDF parametrisation and optimisation [[EPJ C79 \(2019\) 676](#)]
  - single neural network to parametrise eight independent PDF combinations
  - check of the independence of the results from the chosen parametrisation basis
  - new optimisation strategy based on gradient descent rather than genetic algorithms
  - scan of the hyperparameter space to find the optimal minimisation settings
- Complete statistical validation of PDF uncertainties [[Acta Phys.Polon. B52 \(2021\) 243](#)]
  - (multi-)closure tests to validate PDF uncertainties in the data region
  - future tests to check the sensibleness of PDF uncertainties in extrapolation regions
- More efficient compression tool for PDF set delivery [[arXiv:2104.04535](#)]

[See also talks by R.L. Pearson later today and by C. Schwan and F. Heckhorn tomorrow.]

# NNPDF4.0: Fit quality – NNLO

| Data set                 | $N_{\text{dat}}$ | $\chi^2/N_{\text{dat}}$ |
|--------------------------|------------------|-------------------------|
| Fixed-target DIS         | 1881             | 1.10                    |
| HERA                     | 1208             | 1.21                    |
| $\sigma_c$               | 37               | 2.11                    |
| $\sigma_b$               | 26               | 1.48                    |
| Fixed-target Drell-Yan   | 189              | 1.00                    |
| CDF                      | 28               | 1.31                    |
| D0                       | 37               | 1.00                    |
| ATLAS                    | 621              | 1.18                    |
| Drell-Yan, 7, 8, 13 TeV  | 153              | 1.32                    |
| $W$ +jet, 8 TeV          | 32               | 1.15                    |
| single top, 7, 8, 13 TeV | 14               | 0.36                    |
| di-jets, 7 TeV           | 90               | 1.93                    |
| jets, 8 TeV              | 171              | 0.61                    |
| top pair, 7, 8, 13 TeV   | 16               | 2.30                    |
| $Zp_T$ , 8 TeV           | 92               | 0.86                    |
| direct photon, 13 TeV    | 53               | 0.72                    |
| CMS                      | 411              | 1.40                    |
| Drell-Yan, 7, 8 TeV      | 154              | 1.34                    |
| single top, 7, 8, 13 TeV | 3                | 0.43                    |
| di-jets, 7 TeV           | 54               | 1.67                    |
| di-jets, 8 TeV           | 122              | 1.50                    |
| top pair, 5, 7, 8 TeV    | 29               | 0.84                    |
| top pair, 13 TeV         | 21               | 0.67                    |
| $Zp_T$ , 8 TeV           | 28               | 1.42                    |
| LHCb                     | 116              | 1.53                    |
| Total                    | 4491             | 1.17                    |

Overall good description of the data sets

Two exceptions:

HERA  $\sigma_c$  and ATLAS top pair

Weighted fits analysis:

in case of HERA  $\sigma_c$ :

lack of small- $x$  resummation

in case of ATLAS top pair:

slight tension with (di-jet) data sets

poor fit if all distributions are included

normalised rapidity distributions retained

although their  $\chi^2/N_{\text{dat}}$  of order 3

CMS top pair data almost insensitive to all this

General remark:

as statistical uncertainties become smaller

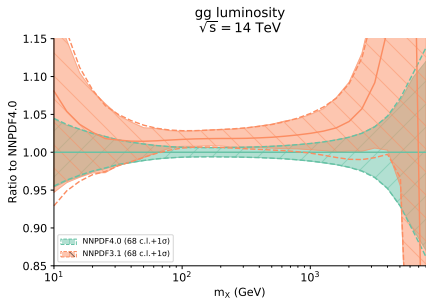
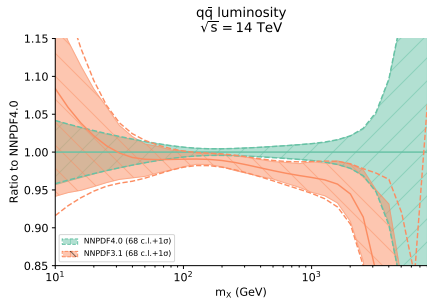
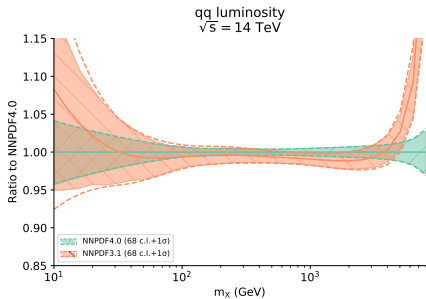
a good control of systematic uncertainties

and their correlations becomes fundamental

to interpret the sensibleness of the fit

All results in the sequel are obtained at NNLO

# From NNPDF3.1 to NNPDF4.0



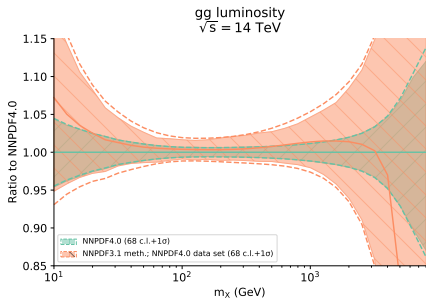
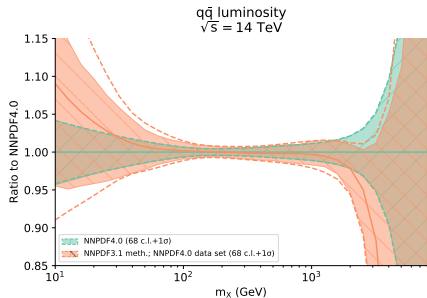
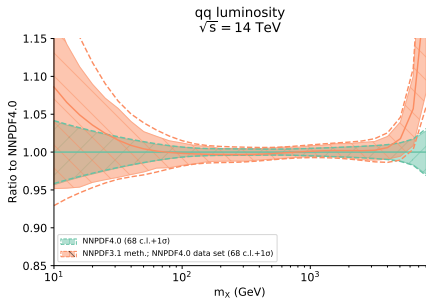
|                               | methodology |             |
|-------------------------------|-------------|-------------|
| data set ( $N_{\text{dat}}$ ) | NNPDF3.1    | NNPDF4.0    |
| NNPDF3.1 (4093)               | <b>1.19</b> | 1.12        |
| NNPDF4.0 (4491)               | 1.25        | <b>1.17</b> |

Consistency between PDF sets

NNPDF4.0 more precise  
(combination of data set and methodology)

NNPDF4.0 more accurate  
(superiority of the NNPDF4.0 methodology)

# From NNPDF3.1 to NNPDF4.0



|                               | methodology |             |
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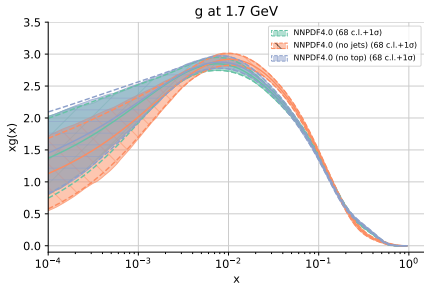
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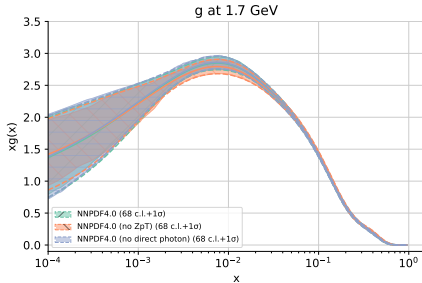
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# The gluon PDF: impact of data

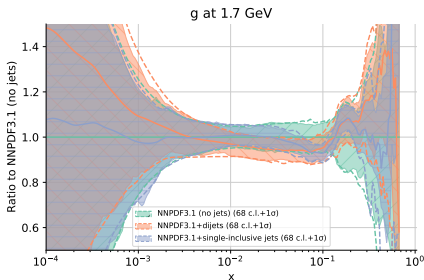
jet and  $t\bar{t}$  data



$Zp_T$  and direct photon data



di-jets vs single-inclusive jets



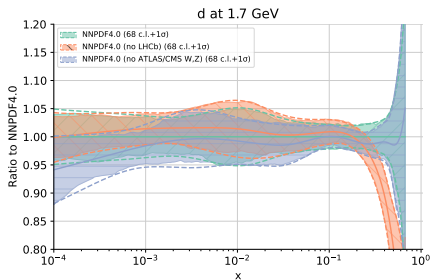
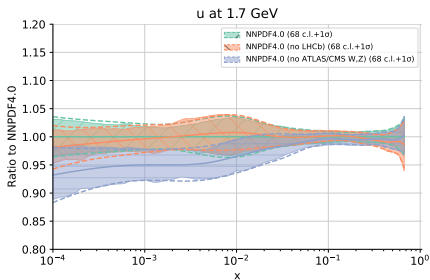
Hierarchical impact of different data sets  
di-jet measurements have the largest pull  
 $t\bar{t}$  and  $Zp_T$  measurements have a comparatively small pull, which is consistent with the global fit  
direct photon measurements almost immaterial

Inclusion of di-jet measurements is preferred over single-inclusive jet measurements given their greater theoretical accuracy and the avoidance of decorrelation models

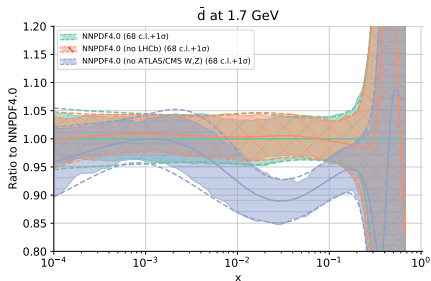
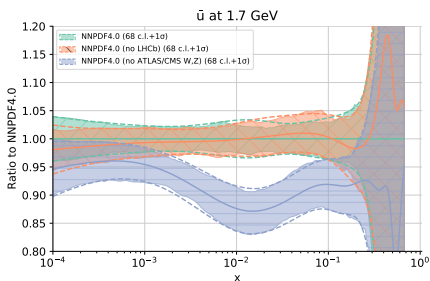
For details, see [EPJ C80 (2020) 8]

# Quark flavour decomposition: impact of data

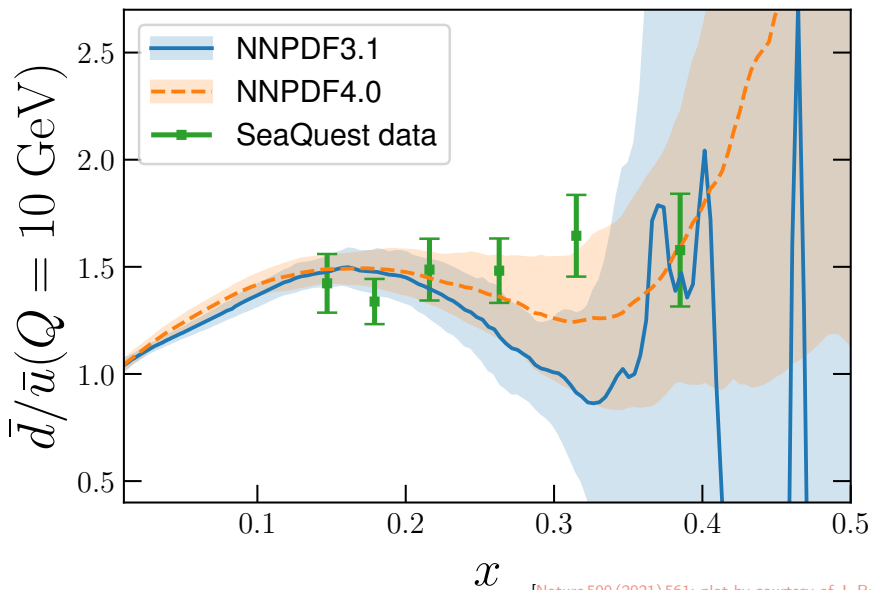
## Quarks



## Antiquarks



# Sea quark asymmetry: SeaQuest

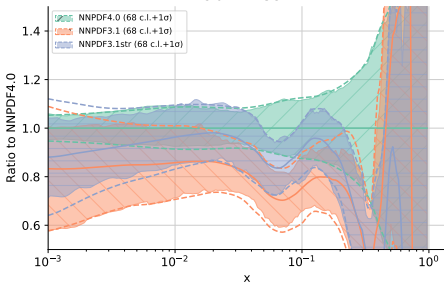


[Nature 590 (2021) 561; plot by courtesy of J. Rojo]

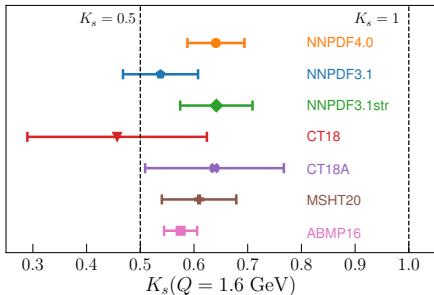
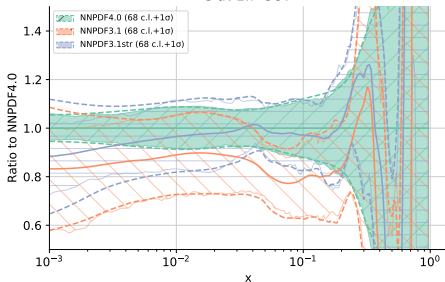


# The strange PDF: impact of data

$s$  at 1.7 GeV



$\bar{s}$  at 1.7 GeV



Enhanced  $s$  and  $\bar{s}$  PDFs w.r.t. NNPDF3.1  
effect of ATLAS  $W, Z$  and  $W$ +jet data

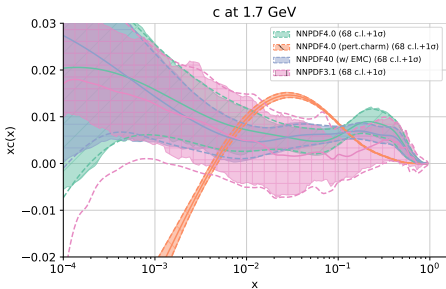
Good consistency with NNPDF3.1str  
no nuclear uncertainties in NNPDF3.1str  
no NOMAD data in NNPDF4.0

Good consistency of  $K_s$  across PDF sets

$$K_s(Q^2) = \frac{\int_0^1 dx [s(x, Q^2) + \bar{s}(x, Q^2)]}{\int_0^1 dx [\bar{u}(x, Q^2) + \bar{d}(x, Q^2)]}$$

See also [EPJ C80 (2020) 1168]

# Impact of theory: perturbative vs fitted charm

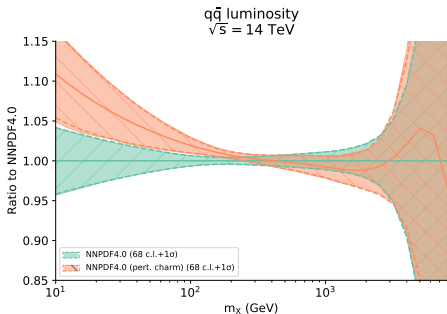
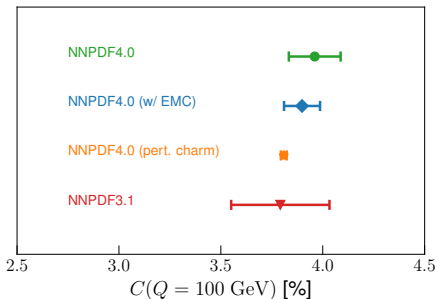


Striking evidence of intrinsic charm  
even w/o EMC  $F_2^c$  data

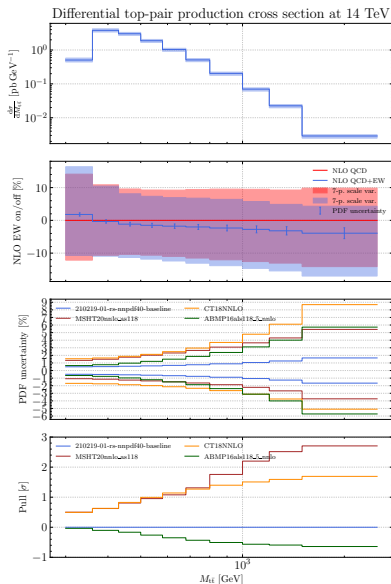
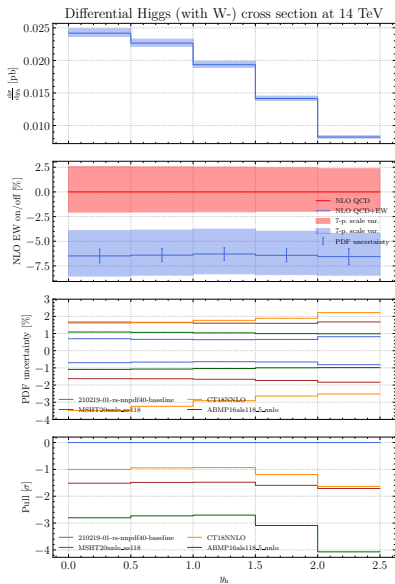
Perturbative charm alters the flavour  
decomposition and deteriorates the fit

$$\chi_{\text{fitted charm}}^2 = 1.17 \rightarrow \chi_{\text{pert. charm}}^2 = 1.19$$

mainly due to a worsening  
of the LHC  $W, Z$  and top pair data sets



# NNPDF4.0: implications for LHC phenomenology



[Plots by courtesy of C. Schwan]

# Conclusions

NNPDF4.0 is the next generation parton set of the NNPDF family.

It achieves 1% accuracy in an unprecedentedly broad kinematic range by consistently improving the previous NNPDF3.1 parton set.

This result builds upon an extensive LHC data set combined with deep-learning optimisation models.

Its faithfulness in representing PDF uncertainties is completely validated by closure tests.

1% PDF uncertainties challenge the accuracy of theoretical predictions and demand an increasing effort towards the systematic inclusion in the fit of theoretical uncertainties (nuclear, higher orders, physical parameters, ...) and higher-order QCD and EW corrections.

The **NNPDF code** used to produce the NNPDF4.0 parton set **will be made publicly available** with its documentation.

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## Thank you

# Individual data sets: $\chi^2$ breakdown

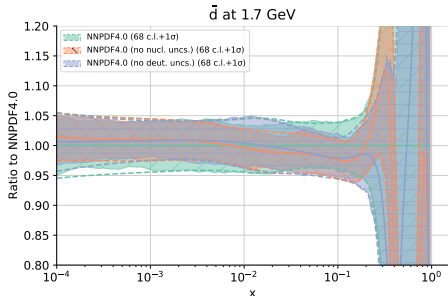
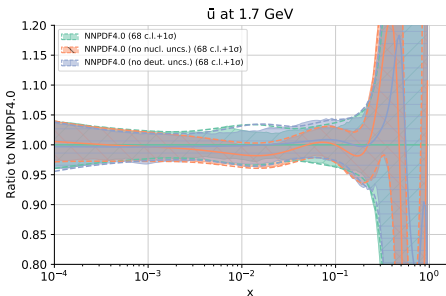
| data set<br>fit | ATLAS jets | CMS jets | ATLAS top | CMS top | ATLAS $Zp_T$ | CMS $Zp_T$ | ATLAS dir. phot. | total |
|-----------------|------------|----------|-----------|---------|--------------|------------|------------------|-------|
| NNPDF4.0        | 1.06       | 1.55     | 2.29      | 0.77    | 0.86         | 1.41       | 0.71             | 1.17  |
| (no jets)       | [1.71]     | [3.70]   | 1.54      | 1.00    | 0.86         | 1.35       | 0.72             | 1.14  |
| (no top)        | 1.08       | 1.57     | [3.51]    | [0.91]  | 0.86         | 1.43       | 0.74             | 1.18  |
| (no $Zp_T$ )    | 1.08       | 1.57     | 2.30      | 0.76    | [0.99]       | [1.41]     | 0.69             | 1.14  |
| (no dir. phot.) | 1.06       | 1.55     | 2.30      | 0.77    | 0.86         | 1.42       | [0.71]           | 1.18  |

| data set<br>fit                  | ATLAS 2j | CMS 2j | ATLAS 1j (7 TeV) | ATLAS 1j (8 TeV) | CMS 1j | $Zp_T$ | top  | total |
|----------------------------------|----------|--------|------------------|------------------|--------|--------|------|-------|
| NNPDF4.0                         | 1.93     | 1.56   | [1.28] [3.42]*   | 0.61 [2.82]*     | [1.31] | 0.99   | 1.17 | 1.17  |
| (single-jets instead of di-jets) | [2.41]   | [2.68] | 1.23 [3.36]*     | 0.85 [3.10]*     | 1.07   | 0.99   | 1.19 | 1.14  |

\* No decorrelation model

| data set<br>fit        | FT DIS | HERA | FT DY  | Tevatron | ATLAS $W, Z$ | CMS $W, Z$ | LHCb   | single top | total |
|------------------------|--------|------|--------|----------|--------------|------------|--------|------------|-------|
| NNPDF4.0               | 1.10   | 1.21 | 1.00   | 1.14     | 1.28         | 1.33       | 1.54   | 0.37       | 1.17  |
| (no LHCb)              | 1.08   | 1.21 | 0.97   | 1.27     | 1.34         | 1.35       | [2.60] | 0.34       | 1.16  |
| (no ATLAS/CMS $W, Z$ ) | 1.05   | 1.20 | 0.85   | 1.02     | [2.14]       | [1.36]     | 1.39   | 0.37       | 1.11  |
| DIS-only               | 1.03   | 1.21 | [1.40] | [1.22]   | [4.15]       | [3.83]     | [2.96] | [0.33]     | 1.10  |

# Quark flavour separation: nuclear uncertainties

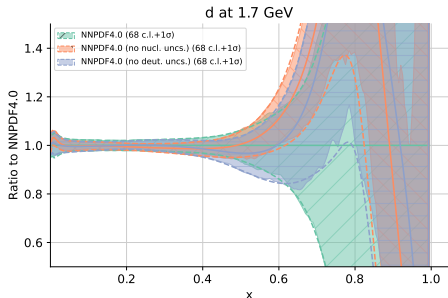


Effect of nuclear uncertainties relevant  
at large  $x$

to reconcile FT DIS with LHC DY data

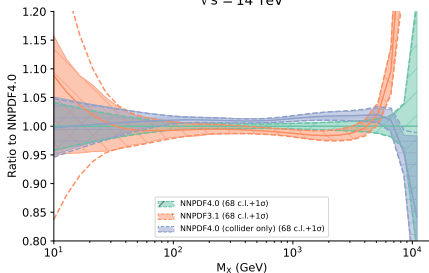
$$\chi_{\text{tot}}^2 = 1.17 \rightarrow \chi_{\text{tot}}^2 = 1.26 \text{ (no nucl. uncs.)}$$
$$\chi_{\text{LHCb}}^2 = 1.54 \rightarrow \chi_{\text{tot}}^2 = 1.76 \text{ (no nucl. uncs.)}$$

The bulk of the effect is due to nuclear  
uncertainties for heavy nuclei  
deuteron uncertainties have a comparatively  
smaller effect at intermediate values of  $x$

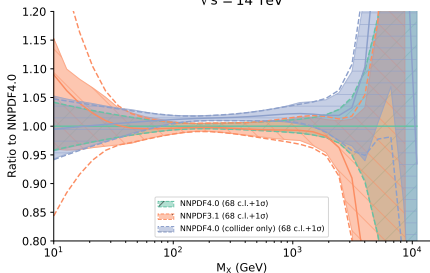


# NNPDF4.0: parton luminosities

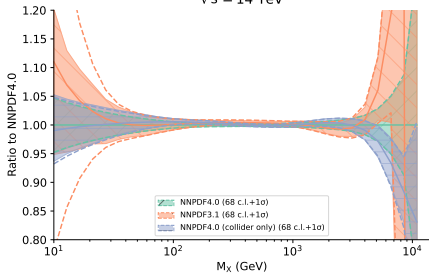
qq luminosity  
 $\sqrt{s} = 14$  TeV



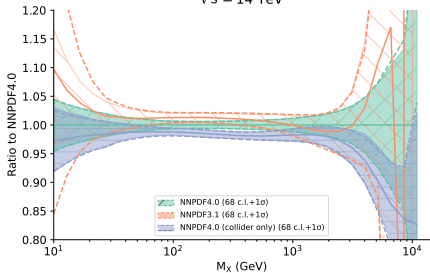
q $\bar{q}$  luminosity  
 $\sqrt{s} = 14$  TeV



gg luminosity  
 $\sqrt{s} = 14$  TeV



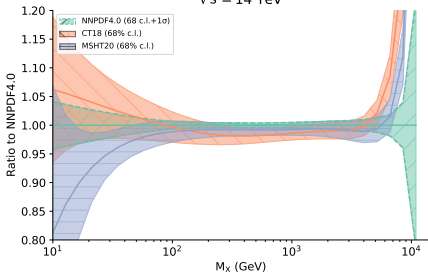
g $\bar{g}$  luminosity  
 $\sqrt{s} = 14$  TeV



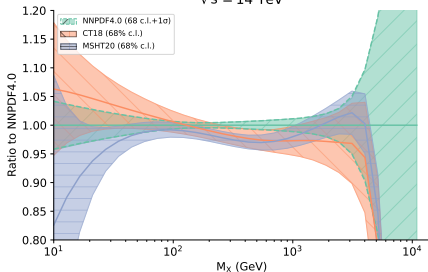


# NNPDF4.0: parton luminosities

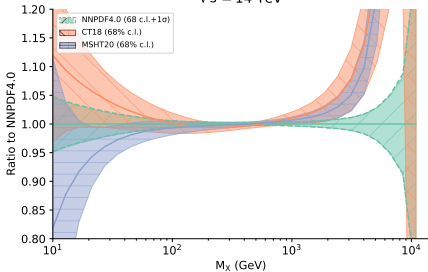
qq luminosity  
 $\sqrt{s} = 14$  TeV



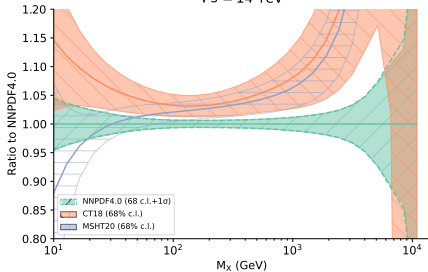
q $\bar{q}$  luminosity  
 $\sqrt{s} = 14$  TeV



gg luminosity  
 $\sqrt{s} = 14$  TeV



gg luminosity  
 $\sqrt{s} = 14$  TeV

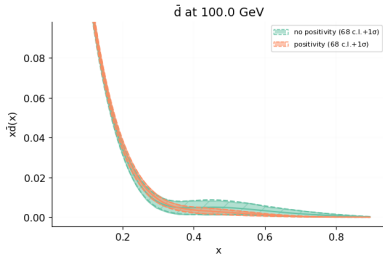
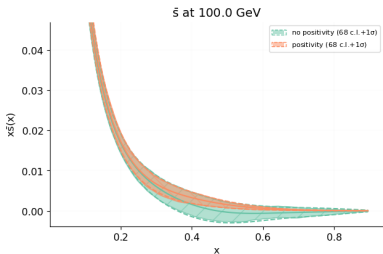


# Positivity - Implementation

Quarks, anti-quarks and gluon  $\overline{MS}$  PDFs  $q_k$  have to be positive: we add a term in the  $\chi^2$  penalizing negative distributions

$$\chi_{tot}^2 = \chi_{exp}^2 + \sum_k \chi_{k,pos}^2,$$

$$\chi_{k,pos}^2 = \Lambda_k \sum_i \Theta(-q_k(x_i, Q^2)), \quad \text{with} \quad \Theta(t) = \begin{cases} t & \text{if } t > 0 \\ 0 & \text{if } t < 0 \end{cases}.$$



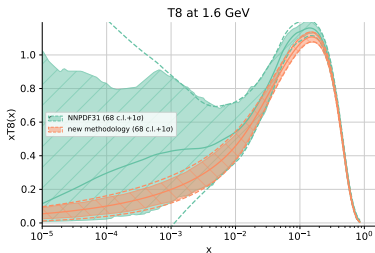
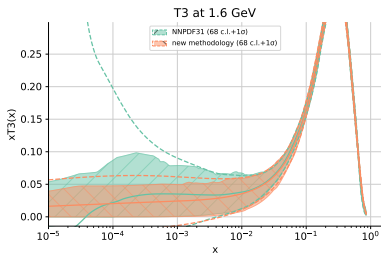
# Integrability

In order to satisfy valence and Gottfried sum rules the distributions  $q_k = V, V_3, V_8, T_3, T_8$  have to be integrable at small- $x$

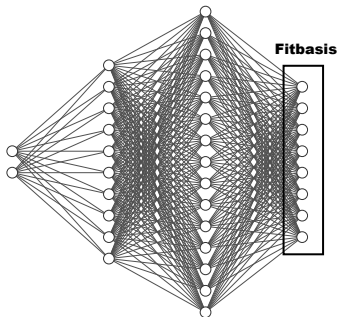
$$\lim_{x \rightarrow 0} x q_k(x, Q_0^2) = 0.$$

Similarly to what done for positivity, we add to the total  $\chi^2$  a penalty of the form

$$\chi_{k,integ}^2 = \Lambda_k \sum_i [x_i q_k(x_i, Q^2)]^2.$$



# Fitbasis



## Flavour basis:

$g, u, \bar{u}, d, \bar{d}, s, \bar{s}, c$

## Evolution basis:

$g, \Sigma, V, V_3, V_8, T_3, T_8, T_{15}$

- independently on the basis choice the same physical constraints have to be satisfied: positivity and integrability
- NNPDF4.0 will be hyper-optimized in the evolution basis
- the final results should not depend on the details of the methodology  
→ fitbasis independence studies

# Key differences with respect to the 3.1 methodology

## NNPDF 3.1 code

- **Genetic Algorithm optimizer**
- One network per flavour
- Physical constraints imposed independently of optimization
- Preprocessing fixed per each of the replicas
- C++ monolithic codebase
- In-house Machine Learning optimization framework
- Fitting times of up to various days



**Fit parameters manually chosen  
(manual optimization of  
hyperparameters)**

## NNPDF 4.0 code

- **Gradient Descent optimization**
- One network for all flavours
- Physical constraints integrated in the optimization
- Preprocessing can be fitted within replicas
- Python object oriented codebase
- Freedom to use external libraries (default: TensorFlow)
- Results available in less than an hour



**Fit parameters chosen automatically  
(hyperparameter scan)**

# Beyond the PDF fit: fitting the methodology

The main objective of NNPDF is to minimize choices that can bias the PDF:

- ✗ Functional form  $\rightarrow$  Neural Networks
- ✗ However: NN are defined by set of parameters!

Humans are good at recognising patterns but selecting the best set of parameters is a slow process and systematic success is not guaranteed

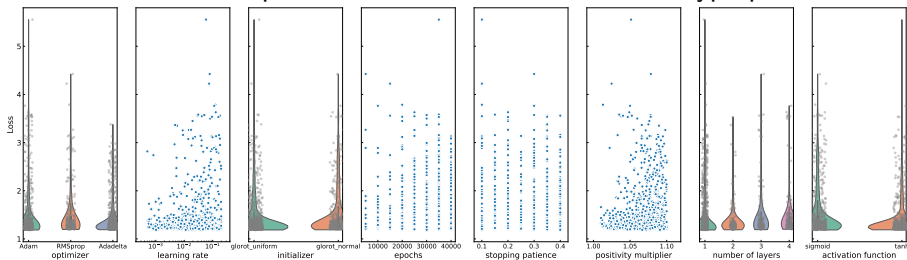


To overcome this selection problem we implement a **hyperparameter scan**: let the computer decide automatically

- ✓ Scan over thousands of hyperparameter combinations
- ✓ Define a reward function to grade the model
- ✓ Check the generalization power of the model

# Hyperparameter scan

Each blue dot corresponds to a fit of a different set of hyperparameters:



Thousands of fits for the hyperoptimization algorithm to choose:

- ✓ Optimizer
- ✓ Learning Rate
- ✓ Initializer
- ✓ Epochs
- ✓ Stopping Patience
- ✓ Positivity Multiplier
- ✓ Number of Layers
- ✓ Activation Function

# Hyperoptimization: reward and generalization

If we use as hyperoptimization target the  $\chi^2$  of the fitted data, we risk finding the hyperparameter set that better overfits.

We avoid this problem by adopting ***k*-folding**:

- Divide the data into  $k$  sets.
- Leave one set out and fit the  $k - 1$  sets left.
- Optimize the average  $\chi^2$  of the  $k$  non-fitted sets.



Example of function to hyperoptimize:

$$\text{Loss}(\text{optimizer\_name}, \text{depth\_of\_network}) = \frac{1}{k} \sum_k^i \frac{\chi_i^2}{N_i}$$

Where we are computing the  $\chi^2$  for the data that did not enter the fit. This ensures that the methodology can accommodate well even data that has never been seen by the fit.

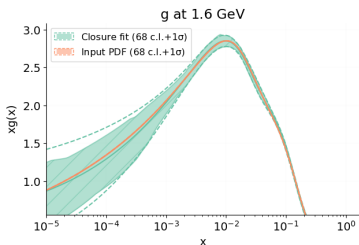


# Closure Tests

Fit replicas to pseudodata in usual way

$$(1) \quad \begin{aligned} \mathbf{y} &= \mathbf{f} + \boldsymbol{\eta} + \boldsymbol{\epsilon} \\ &= \mathbf{z} + \boldsymbol{\epsilon}, \end{aligned}$$

where  $\boldsymbol{\eta} \sim \mathcal{N}(0, C)$  and  $\boldsymbol{\epsilon} \sim \mathcal{N}(0, C)$  are sampled independently.  
Use predictions from an input PDF as proxy for  $\mathbf{f}$ .



Example closure fit and input PDF.

Allows testing of methodology, if the input assumptions hold.

For example:

**Bias:** difference between central prediction and true observable

**Variance:** uncertainty of replica predictions

Bias is a stochastic variable. If PDF uncertainty is faithful then

$$\mathbf{E}_{\boldsymbol{\eta}}[\text{bias}] = \text{variance} \quad (2)$$

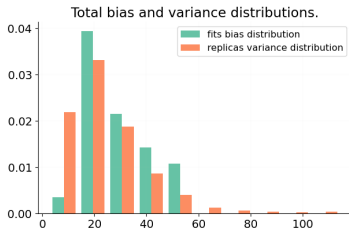
High demand on resources - made feasible with next generation fitting code.

# Preliminary results

Compare first moments:

|  |                |
|--|----------------|
| $\sqrt{\mathbf{E}_\eta[\text{bias}]/\mathbf{E}_\eta[\text{variance}]}$ |                |
| Total  | $1.11 \pm 0.5$ |

Alternatively look at the respective distributions

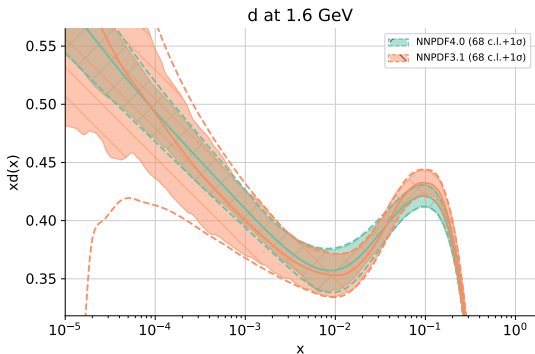


Bias distribution sampled with 25 fits, 40 replicas each.

# How can we future-proof the methodology?

Do we trust our errorbands?

The smaller error bands in the NNPDF4.0 fits are driven both by the increased amount of data and the improved methodology. But there are still kin. regions not covered by data!



Ideally: design an experiment for the regions not covered by fitted-data!

Problem: we want the results before 2050...



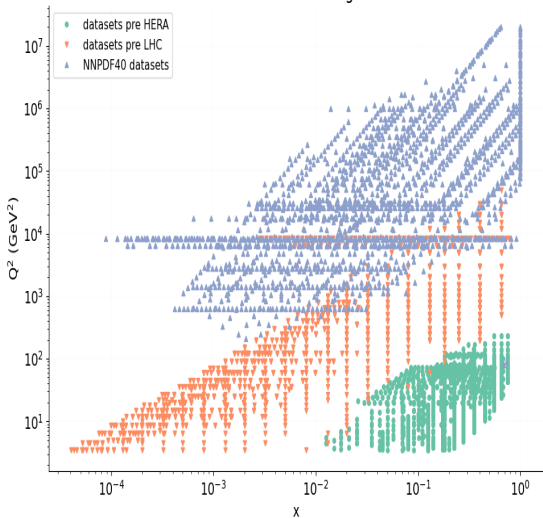
Fig: Other valid and certified future-testing methods

Solution: chronologically ordered subsets of data to test unseen regions, we named this “future tests”.

# Future tests

for more information see [arxiv:2103.08606](https://arxiv.org/abs/2103.08606)

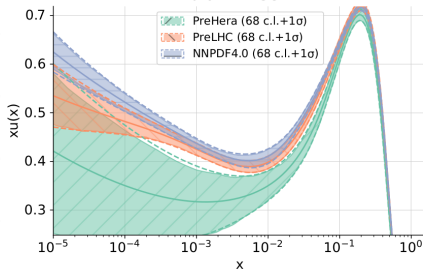
Kinematic coverage



$\chi^2/N$  (only exp. covmat)

| (dataset) | NNPDF4.0 | pre-LHC     | pre-Hera    |
|-----------|----------|-------------|-------------|
| pre-HERA  | 1.09     | 1.01        | 0.90        |
| pre-LHC   | 1.21     | 1.20        | <b>23.1</b> |
| NNPDF4.0  | 1.29     | <b>3.30</b> | <b>23.1</b> |

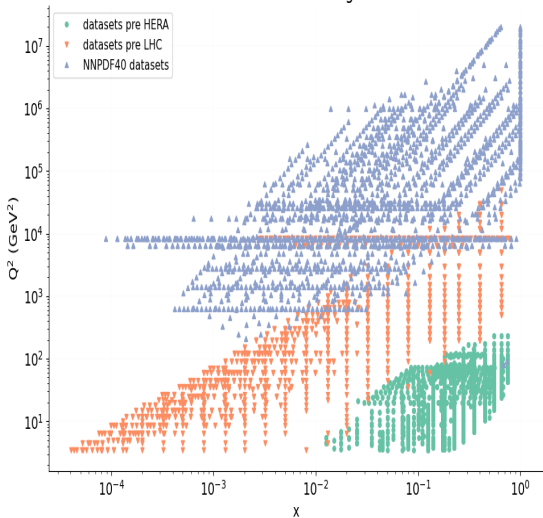
u at 1.7 GeV



# Future tests

for more information see [arxiv:2103.08606](https://arxiv.org/abs/2103.08606)

Kinematic coverage



$\chi^2/N$  (exp. and PDF covmat)

| (dataset) | NNPDF4.0 | pre-LHC     | pre-Hera    |
|-----------|----------|-------------|-------------|
| pre-HERA  |          |             | 0.86        |
| pre-LHC   |          | 1.17        | <b>1.22</b> |
| NNPDF4.0  | 1.12     | <b>1.30</b> | <b>1.38</b> |

u at 1.7 GeV

