Virtual photon contribution to frictional drag in double-layer devices

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Abstract

Frictional drag between coupled two-dimensional charge systems is commonly viewed as a second order effect arising either from screened Coulomb interaction, or phonon exchange. We point out that for single-photon exchange the first order contribution does not have to vanish even at $T = 0$, and evaluate this contribution for simple models.

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1. Introduction

The double layer configuration of the fermion gas, especially under the influence of a strong magnetic field, allows one to study many unusual transport phenomena. Pogrebinskii [1] and Price [2] advanced the idea that a current driven in an electron gas should manifest itself as a potential difference in a spatially separate system because of Coulomb scattering with a preferred direction of exchanged momentum. The effect, originally analyzed for bulk electrons, was quantitatively measured in double well heterostructures at low temperature [3], allowing for direct comparison with theory.

The measured quantity is transresistance, the ratio of potential difference due to dragged charges and driving current. It was the main subject of several theoretical works based on transport equations [4] or Kubo formula for conductivity [5,6]. Transresistance resulting from Coulomb drag depends on temperature as $T^2$ and decreases as $d^{-4}$ ($d$ is the distance between the wells). The theory was refined by including phonon exchange [7,8], which explained the observed low temperature deviations from the $T^2$ behaviour. A “current drag” effect was also

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proposed [9], and originates from the Van der Waals attraction between relative current flows. With strong magnetic field, the bilayer geometry allows the observation of new QHE phenomena, for a review see [10].

In the Kubo formalism transconductivity is given by the retarded correlator of currents in different layers. As a consequence of charge conservation, it is a second order quantity in the electrostatic and phonon interlayer interaction. In this Letter we discuss the effect of photon exchange, in the absence of external magnetic field, by considering the coupling of the photon field to currents, in the Coulomb gauge. Photon exchange is potentially important, given that photon drag arises as a first order effect. Although e.m. corrections are usually quite small, it is not obvious at the outset how they compare with second order effects.

The effective e.m. interaction was evaluated in RPA by Holstein et al. [11] with the significant feature of being unscreened at zero frequency. Reizer [12] investigated its influence on the Fermi surface, the low temperature specific heat, and homogeneous transport, and found small but interesting effects. Gauge-invariant response functions were studied by Kim et al. [13], and confirmed the Fermi liquid behaviour.

In this Letter the photon polarization is evaluated in RPA for the interlayer e.m. interaction, in the limit of thin layers. Single layer properties and disorder are accounted for in the diffusive regime.

We model the system as two inﬁnite parallel layers of electron gas, conﬁned in narrow potential wells centered in \( z = 0 \) and \( z = d \), with negligible tunnelling because of low temperature and sufﬁciently large layer-separation.

The Hamiltonian is

\[
H = H_1 + H_2 + H_{\text{ph}} + U_{\text{Cou}} + U_{\text{em}},
\]

where \( H_\ell \) is the kinetic energy of the electrons in layer \( \ell = 1, 2 \), \( H_{\text{ph}} \) is the energy of free e.m. ﬁeld, \( U_{\text{Cou}} \) is the Coulomb interaction. The e.m. minimal coupling with the vector potential in the Coulomb gauge is

\[
H_{\text{em}} = \frac{1}{c} \int d^3x \left( \mathbf{j}(\mathbf{x}) \cdot \mathbf{A}(\mathbf{x}) - \frac{e}{2mc^2} \int d^3x \rho(\mathbf{x}) A^2(\mathbf{x}) \right),
\]

where \( \rho \) is the electron charge density and \( \mathbf{j} \) is the paramagnetic part of the charge current

\[
\mathbf{j} = \frac{1}{m} \epsilon \mathbf{A}
\]

that enters in the equation for charge conservation.

To derive the Kubo formula for conductivity [8,15], one perturbs the Hamiltonian with a term \( \delta H = \frac{1}{c} \int d^3x \delta A^\text{ext} (\mathbf{x}, t) \cdot \mathbf{j}(\mathbf{x}) \), which couples the total current to a weak external electric ﬁeld. Linear response in \( \delta A^\text{ext} \) gives the conductivity tensor. For a time independent Hamiltonian (1) the response is a function of the time difference only, and its Fourier transform is

\[
\sigma_{ij}(x, x', \omega) = \frac{\tilde{h}}{\beta} \sum_n e^{-i\omega n} \langle \mathbf{T} \tau_j J_\mu(x, \tau) J_\nu(x', \tau') \rangle,
\]

where \( i, j \) are space directions, and \( \pi_{ij}^\text{Ret}(\omega) \) is the connected retarded current–current correlator. Evaluation of \( \pi^\text{Ret} \) is our main task.

It is convenient to use imaginary time \( (it = \tau) \) and consider \( \tau \)-ordered correlators

\[
\pi_{\mu\nu}^\text{C}(x, x', \tau - \tau') = \langle \mathbf{T} \tau_j J_\mu(x, \tau) J_\nu(x', \tau') \rangle
\]

using a four-dimensional notation \( J_\mu = (\epsilon \sigma \cdot \mathbf{j}) \), \( \mu = 0, 1, 2, 3 \) (since we use the Matsubara formalism, the metric is Euclidean). Brackets indicate equilibrium thermal-average. The Fourier transform of \( \pi_{\mu\nu}^\text{C} \) is a sum over discrete Matsubara frequencies

\[
\pi_{\mu\nu}^\text{C}(x, x', \omega_n) = \frac{1}{\beta} \sum_n e^{-i\omega_n \tau} \pi_{\mu\nu}^\text{C}(x, x', \omega_n).
\]

The correlator of interest \( \pi^\text{Ret}(\omega) \) can be written in terms of \( \pi^\text{C}(\omega_n) \) by analytic continuation.
A delicate issue is the presence of disorder. In the present Letter we assume that the impurities are uncorrelated in the two subsystems (for correlated impurities see, for example, [14]) and hence consider independently averaged correlation functions, which are translationally invariant in the plane \( r = (x, y) \). The static and homogeneous limit of the transconductivity is:

\[
\sigma_{ij}(z, z') = \lim_{\omega \to 0} \lim_{q \to 0} \frac{i}{\hbar \omega} \pi_{ij}^{\text{Ret}}(q, z, z', \omega),
\]

where \( z \) and \( z' \) belong to different layers, and \( q \) is FT of \( r \). Since there is no flow of charge in the \( z \)-direction, we require \( \sigma_{3j} = \sigma_{j3} = 0 \).

2. The polarization tensor

The thermal Green functions for the photon field are

\[
D_{ij}(x, \tau, x', \tau') = -\frac{1}{\hbar c} \langle T_{\tau} A_i(x, \tau) A_j(x', \tau') \rangle,
\]

where \( i, j = 1, 2, 3 \). The propagators for free photons and the bare Coulomb interaction are [15]

\[
D^{(0)}_{ij}(k, \omega_n) = -\left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \frac{4\pi c^2}{\omega_n^2 + c^2 k^2}, \quad D^{(0)}_{00}(k, \omega_n) = \frac{4\pi}{k^2},
\]

\[
D^{(0)}_{0i}(k, \omega_n) = D^{(0)}_{i0}(k, \omega_n) = 0.
\]

When the interaction with matter is included, the dressed photon propagator and the effective Coulomb interaction are components of a tensor \( D_{\mu\nu} \) which differs from the bare one by polarization insertions:

\[
D_{\mu\nu}(x, x', \omega_n) = D^{(0)}_{\mu\nu}(x, x', \omega_n) + D^{(0)}_{\mu\rho}(x, x_1, \omega_n) P_{\rho\sigma}(x_1, x_2, \omega_n) D^{(0)}_{\sigma\nu}(x_2, x', \omega_n).
\]

Summation and integration of repeated variables are understood hereafter. The polarization insertion and the density–density correlator are related via [15]:

\[
P_{00}(x, x', \omega_n) = \frac{1}{\hbar c^2} \pi_{00}^C(x, x', \omega_n).
\]

A perturbative analysis to all orders shows the further exact relations among the polarization insertion and the current–current correlator:

\[
P_{ij}(x, x', \omega_n) = -\frac{e}{mc^2} \rho(x) \delta_{ij} \delta_3(x - x') + \frac{1}{\hbar c^2} \pi_{ij}^C(x, x', \omega_n),
\]

\[
P_{0i} = \frac{1}{\hbar c^2} \pi_{0i}^C, \quad P_{i0} = \frac{1}{\hbar c^2} \pi_{i0}^C.
\]

Thus, the conductivity tensor is proportional to the retarded photon polarization:

\[
\sigma_{ij}(x, x', \omega) = i \frac{e^2}{\omega} P_{ij}^{\text{Ret}}(x, x', \omega).
\]

3. The Dyson equation

To evaluate approximately the polarization, we start by writing an exact Dyson equation in coordinate space. Due to the geometry of the problem, we find it convenient to rearrange the terms according to the two layer
configuration:

$$P_{\mu\nu}(x, x', \omega_n) = P_{\mu\nu}^*(x, x', \omega_n) + P_{\mu\nu}^*(x, x'', \omega_n)D_{\rho\sigma}^{(0)}(x'', x', \omega_n)P_{\sigma\nu}(x''', x', \omega_n),$$  \hspace{1cm} (16)

where $D_{\rho\sigma}^{(0)}$ is an interlayer bare propagator ($x''$ and $x'''$ in different layers) and $P^*$ is the irreducible polarization tensor, given as the sum of e.m. polarization insertions that cannot be disconnected by cutting a single interlayer photon or Coulomb line.

Next, we take the average over disorder, and assume the same Dyson equation for averaged correlators, and find

$$\bar{P}_{\mu\nu}(q, z, z', \omega_n) = \bar{P}_{\mu\nu}^*(q, z, z', \omega_n) + \bar{P}_{\mu\nu}^*(q, z, z'', \omega_n)D_{\rho\sigma}^{(0)}(q, z'' - z', \omega_n)\bar{P}_{\sigma\nu}(q, z''', z', \omega_n).$$  \hspace{1cm} (17)

Here,

$$D_{\mu\nu}^{(0)}(q, z - z', \omega_n) = \int_{-\infty}^{\infty} \frac{dk_3}{2\pi} D_{\mu\nu}^{(0)}(k, \omega_n)e^{ik_3(z-z')},$$  \hspace{1cm} (18)

With the definition of the auxiliary function

$$\bar{D}(q, z - z', \omega_n) = 2\pi e^{-\frac{1}{4}q|z-z'|/\sqrt{\omega_n^2 + q^2 c^2}}$$  \hspace{1cm} (19)

we find

$$D_{00}^{(0)} = 2\pi e^{-q|z-z'|} q, \quad D_{ab}^{(0)} = -\delta_{ab} \bar{D} - \frac{q a q b c^2}{\omega_n} (\bar{D} - D_{00}^{(0)}),$$

$$D_{33}^{(0)} = \frac{q^2 c^2}{\omega_n^2} (\bar{D} - D_{00}^{(0)}), \quad D_{0j}^{(0)} = 0,$$

$$D_{3a}^{(0)} = D_{a3}^{(0)} = 2\pi i \text{sign}(z - z') \frac{q a c^2}{\omega_n^2} (e^{-\frac{1}{4}q|z-z'|/\sqrt{\omega_n^2 + c^2 q^2}} - e^{-q|z-z'|}),$$

where the last equation is imposed by the gauge fixing condition $\partial_j A_j = 0$. To proceed further, we consider the limit of thin layers. We consider single particle states of the form $u_\ell(r, z) = \phi_\ell(z)\Psi_\ell(r)$, and in the thin layer limit $|\phi_\ell(z)|^2 \to \delta(z - z_\ell)$. Introducing the fermionic field operators $\psi_{\ell\sigma}(r)$ in each layer, we have

$$\rho(x) = \sum_\ell \rho_\ell(r) \delta(z - z_\ell), \quad \rho_\ell(r) = -e \sum_\sigma \psi_{\ell\sigma}^+(r) \psi_{\ell\sigma}(r),$$

$$J_\ell(x) = \sum_\ell J_{\ell\ell}(r) \delta(z - z_\ell), \quad J_{\ell\ell}(r) = j_{\ell\ell}(r) - e mc \rho_\ell(r) A_\ell(r),$$

$$j_{\ell\ell}(r) = \frac{i \hbar}{2m} \sum_\sigma \psi_{\ell\sigma}^+(r) \partial_\sigma \psi_{\ell\sigma}(r) - (\partial_\sigma \psi_{\ell\sigma}^+(r)) \psi_{\ell\sigma}(r).$$  \hspace{1cm} (20)

The index $\alpha$ will hereafter denote transverse space components $(x, y)$. The absence of tunnelling ensures the conservation of charge in each layer, which reads:

$$\frac{1}{i\hbar} [H, \rho_\ell(r)] = \text{div}_{xy} J_\ell(r).$$  \hspace{1cm} (21)

In the thin layer limit the Dyson equations (11) become algebraic:

$$P_{\mu\nu}(\ell\ell') - P_{\mu\nu}^*(\ell\ell') = P_{\mu\nu}^*(\ell_1\ell_2)D_{\rho\sigma}^{(0)}(\ell_1 \ell_2)P_{\sigma\nu}(\ell_2\ell').$$  \hspace{1cm} (22)
The variables \( \mathbf{q} \) and \( \omega_n \) are omitted for brevity. Recall that the Dyson equation was constructed with the requirement that \( D^{(0)} \) connects different layers, thus \( \ell_1 \neq \ell_2 \). Here we put

\[
D^{(0)}_{\mu\nu}(q, z\ell - z\ell', \omega_n) = P_{\mu\nu}(q, \omega_n, z\ell, z\ell').
\]

They are respectively the entries of two \( 4 \times 4 \) matrices \( D^{(0)}(\ell\ell') \) and \( P(\ell\ell') \). Therefore, the Dyson equations correspond to \( 4 \) matrix equations (\( \ell, \ell' = 1, 2 \)):

\[
P(\ell\ell') = P^*(\ell\ell') + P^*(\ell1)D^{(0)}(12)P(2\ell') + P^*(\ell2)D^{(0)}(21)P(1\ell').
\]

The structure of the polarization tensor is greatly constrained by symmetry and charge conservation. The latter implies the following exact relations:

\[
i\frac{\omega_n}{c} P_{0\nu}(\mathbf{q}, \omega_n, \ell, \ell') = q_a P_{a\nu}(\mathbf{q}, \omega_n, \ell, \ell'), \quad \nu = 0, 1, 2, 3.
\]

The same relation holds when the indices are exchanged. These relations correspond to the Ward identity relating the vertex functions for Coulomb and e.m. coupling to the electron field. In absence of external magnetic field, rotational symmetry requires the tensor structure

\[
P_{ab}(\mathbf{q}, \omega_n, \ell, \ell') = \delta_{ab} A(\mathbf{q}, \omega_n, \ell, \ell') + \frac{q_a q_b}{q^2} B(\mathbf{q}, \omega_n, \ell, \ell').
\]

From charge conservation we find:

\[
P_{0a} = P_{a0} = i \frac{\omega_n}{cq^2} P_{a0}, \quad A + B = -\frac{\omega_n^2}{c^2 q^2} P_{00}.
\]

4. The interlayer polarization in RPA

We solve Dyson’s equation in RPA and for identical layers. In this approximation \( P^* \) is independent of all interlayer interaction lines and therefore

\[
P^*(\ell\ell') = \delta_{\ell\ell} P^{(0)}_{\ell},
\]

where \( P^{(0)}_{\ell} \) is the exact polarization matrix of the layer \( \ell \) with its internal dynamics. The index \( \ell \) keeps track of the charge of the carriers. However, in the present approximation scheme (no charged impurities and no interlayer interaction) the charge appears always with even powers. Thus the index \( \ell \) can be neglected. The various components of \( P^{(0)}_{\ell} \) fulfill the relations of rotational symmetry and charge conservation, such as (index \( \ell \) is here suppressed)

\[
P_{ab}^{(0)} = A^{(0)} \delta_{ab} + B^{(0)} \frac{q_a q_b}{q^2}, \quad P_{0a}^{(0)} = P_{a0}^{(0)} = i q_a \frac{\omega_n}{q^2 c} P_{00}^{(0)},
\]

\[
A^{(0)} + B^{(0)} = \frac{q_a q_b}{q^2} P_{ab}^{(0)} = -\frac{\omega_n^2}{c^2 q^2} P_{00}^{(0)}.
\]

\( P_{ab}^{(0)} \) is directly linked to the conductivity tensor \( \sigma_{ab}^{(0)} \) of the isolated layer

\[
\sigma_{ab}^{(0)} = i\frac{e^2}{\omega} P_{ab}^{(0)}.
\]

The coupled Dyson equations (24) provide a matrix equation for the interlayer polarization:

\[
P(12) = P^{(1)}(12) + P^{(1)}(12)D^{(0)}(21)P(12),
\]
where $P^{(1)(12)} = P^{(0)}D^{(0)(12)}P^{(0)}$. By using charge conservation for the polarization we obtain after some algebra

$$P^{(1)(12)}_{ab} = -\tilde{D} \left[ A^{(0)2} \left( \delta_{ab} - \frac{q_0 q_b}{q^2} \right) + \frac{q_0 q_b}{q^2} \left( P^{(0)}_{00} \right)^2 \frac{\omega_n^2}{c^2 q^4} (\omega_n^2 + c^2 q^2) \right].$$

where $\tilde{D} = \tilde{D}(d)$ is the function in Eq. (19) with $|z - z'| = d$. From (32), taking the DC limits $q \to 0$ and than $\omega \to 0$ respectively we obtain transconductance to first order (Fig. 1) in interlayer interaction:

$$\sigma (12) = \frac{E^{(2)}}{J^{(1)}} = -\frac{\sigma (12)}{\sigma (11)\sigma (22) - \sigma (12)\sigma (21)}.$$

Under the condition $\sigma (12) \ll \sigma (0)$, we approximate the intralayer conductance $\sigma (\ell \ell)$ by the value $\sigma (0)$ of the isolated layer.

We then find $\rho (12) \approx \sigma (12)/\sigma (0)^2$ and obtain a universal result for transresistivity:

$$\rho (12) = \frac{2\pi}{c} = \alpha R_H,$$

where $R_H = \hbar/e^2$ is the Hall resistance and $\alpha$ is the fine structure constant corrected for the interlayer medium.

While $P^{(1)(12)}$ represents the inter-layer polarization with a single one-photon exchange, the solution of (31) is the polarization with an effective interlayer interaction, in RPA. $P^{(0)}$ corresponds to disorder averaged single layer polarization where the second layer is absent. If we consider also the second layer, then our approximation neglects those contributions to the disorder average that correlate various $P^{(0)}$ insertions. One finds:

$$P^{(12)00}(q, \omega_n) = 1 - \frac{\rho^{(0)2}_{00} \left( \frac{4\pi}{\epsilon q^2} \omega_n + c^2 q^2 \right) e^{-\frac{4\pi}{\epsilon q^2} \sqrt{\omega_n^2 + c^2 q^2}}}{1 - \rho^{(0)2}_{00} \left( \frac{4\pi}{\epsilon q^2} \omega_n + c^2 q^2 \right) e^{-\frac{4\pi}{\epsilon q^2} \sqrt{\omega_n^2 + c^2 q^2}}}.$$

Fig. 1. The first order contribution to the current–current correlator: coupling mediated by (a) screened Coulomb interaction, (b) four-dimensional electromagnetic propagator. $\rho$ is the electron density, $J$ the paramagnetic current density and $J = J - \frac{i\epsilon}{mc^2} \rho A$. The number inside the bubbles indicates the subsystem. The intralayer interaction is included within RPA in each bubble. (a) Vanishes in the DC limit and corresponds to the $\mu = \nu = 0$ component of (b). The space-like components of (b) are non-vanishing in this limit.
In the diffusive regime, we use

\[ P_{00}^{(0)}(q, \omega) = \sigma^{(0)} q^2 \frac{Dq^2 - i\omega}{Dq^2}. \]  

(37)

where \( D \) is the diffusion constant. In this approximation, the limits \( q \to 0, \omega \to 0 \) of transconductivity yield a small correction to (35) for transresistance:

\[ \rho(12) = \frac{2\pi}{c} \frac{1}{1 - \sigma^{(0)}2(2\pi/c)^2}. \]  

(38)

5. Discussion

In the present Letter we have considered the virtual photon contribution to the frictional drag in two parallel layers. In contrast to the Coulomb drag the contribution of the single virtual photon survives in the limit of \( q \to 0 \) and \( \omega \to 0 \). The RPA approximation allows to get the result in terms of the full single layer polarization even in presence of disorder. The result is valid also at finite temperature, which enters in the single layer conductivity. Eq. (35) gives the contribution of one single-photon-exchange and Eq. (38) yields the sum over all possible single-photon-exchanges. We emphasize that the results (35) and (38) are obtained within a simple model, and therefore the numerical value of the obtained transresistance should not be viewed quantitatively (in fact, (35) and (38) lead to a resistance which is larger than the measured values). Work is in progress to include finite-size corrections which may lead to significant numerical changes. Nevertheless, the main conclusions remain: (i) first order contribution is non-zero and (ii) it remains finite at \( T = 0 \).

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